## ECON 460 Suggested Answers for Questions 7, 8, 10 and 11

Suppose the government wishes to regulate mercury emissions of factories in a specific industry by either setting an emissions standard or imposing an emissions fee (per ton of mercury). The government is uncertain as to the marginal abatement costs, which may be high $\left(M C_{1}\right)$ or low $\left(M C_{2}\right)$.

$$
\begin{aligned}
& M C_{1}=15 M+500 \\
& M C_{2}=15 M-500
\end{aligned}
$$

where $M$ is the units of mercury abated. The government believes there is a $50 \%$ chance of each of the marginal abatement costs. The marginal benefit of abatement is known to be:

$$
M B=1500-10 M
$$

a. What is the optimal level of emissions for each of the cost curves above?
b. What is the expected marginal abatement cost (equation)?
c. What is the optimal emissions standard according to the expected abatement costs?
d. What is the optimal abatement fee according to the expected abatement costs?
e. Which regulation will result in a lower DWL in the presence of the uncertainty? Explicitly compute the expected DWL arising from each proposal.

## Answer:

a. For each marginal cost curve, determine the level of $M$ where $M B=M C$. If the costs are high:
$15 M+500=1500-10 M$
Solving yields $M=40$. If the costs are low, $M=80$.
b. The expected abatement cost is:

$$
\mathrm{E}(\mathrm{MC})=.5(15 M+500)+.5(15 M-500)=15 M
$$

c. The optimal level of abatement (standard) is where $E(M C)=M B$, or $M=60$.
d. The optimal fee is found by substituting $M=60$ into the $M B$ or $E(M C)$

$$
\mathrm{fee}=15(60)=\$ 900
$$

e. We need to estimate the DWL arising from each regulation.

When a standard is imposed of $M=60$ :
If costs are high, the standard is too high and the DWL is 5000
If costs are low, the standard is too low and the DWL is 5000
So the expected DWL from the standard is 5000 .
When the fee is imposed of $\$ 900$ :
If costs are high, the DWL is 2214.45
If costs are low, the DWL is 2214.45
So the expected DWL is 2214.45 .
The DWL from a fee in this case is lower

Consider the following pollution offset problem:
There are two firms, each with the same total savings functions:

$$
T S_{1}=200 e_{1}-\left(e_{1}\right)^{2} \text { and } T S_{2}=200 e_{2}-\left(e_{2}\right)^{2}
$$

The marginal savings functions, respectively, are

$$
M S_{1}=200-2 e_{1} \text { and } M S_{2}=200-2 e_{2}
$$

At receptor Station One, the diffusion coefficients are $\mathrm{a}_{11}=0.5$ and $\mathrm{a}_{12}=1$. The target ambient level is 100. Therefore, at Station One, the emission constraint is:

$$
0.5 e_{1}+e_{2}=100
$$

a) Determine the cost minimizing levels of emissions at Station One (hint: use the Lagrange method).

$$
\begin{aligned}
& \mathrm{L}=200 \mathrm{e}_{1}-\left(\mathrm{e}_{1}\right)^{2}+200 \mathrm{e}_{2}-\left(\mathrm{e}_{2}\right)^{2}+\lambda_{1}\left(100-0.5 \mathrm{e}_{1}-\mathrm{e}_{2}\right) \\
& 200-2 \mathrm{e}_{1}=0.5 \lambda_{1} \quad 200-2 \mathrm{e}_{2}=\lambda_{1} \\
& \mathrm{e}_{1}=80 \mathrm{e}_{2}=60
\end{aligned}
$$

Now suppose that there is a second receptor (Station Two) that has the following emissions constraint:

$$
2 \mathrm{e}_{1}+\mathrm{e}_{2}=120
$$

Where the diffusion coefficients are $\mathrm{a}_{21}=2$ and $\mathrm{a}_{22}=1$; with a target ambience of 120 .
b) Use the same approach as in part (a) to determine the cost minimizing level of emissions that satisfy Station Two.

$$
\begin{aligned}
& L=200 e_{1}-\left(e_{1}\right)^{2}+200 e_{2}-\left(e_{2}\right)^{2}+\lambda_{2}\left(120-2 e_{1}-e_{2}\right) \\
& 200-2 e_{1}=2 \lambda_{2} 200-2 e_{2}=\lambda_{2} \\
& e_{1}=28 e_{2}=65
\end{aligned}
$$

c) Carefully graph both constraints on a graph with e1 on the horizontal axis and e2 on the vertical axis. Indicate the region that satisfies both constraints.

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d) Label the solutions to (a) and (b) on your graph. Can you determine if each violates the other constraint? Is there a solution where both constraints are satisfied? If so, which receptor has the binding constraint? Receptor 2 is the binding constraint
e) Suppose the government had initially granted emission permits to each firm: firm one was given 13.33 permits and firm two was given 93.33 permits. (These are the initial allocations that satisfy both constraints). Given the information in your answers for (a) to (d), determine who will sell permits and who will buy permits. What will be the price ratio of the permits that are traded?

Firm 2 will sell permits to firm 1 at a trading ratio of 2:1

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## Question 8

a) Social Optimum is $\mathrm{MD}=\mathrm{MS}$ or $\mathrm{E}^{*}=120$
b) Uniform standard: $\mathrm{E}_{1}=\mathrm{E}_{2}=\mathrm{E}_{3}=40$

$$
\mathrm{TAC}_{1}=3200 \quad \mathrm{TAC}_{2}=2560 \quad \mathrm{TAC}_{3}=4272
$$

c) Tax is 80

$$
E_{1}=40, E_{2}=20, E_{3}=60
$$

TAC for each is:
TAC 1 \$3,200
TAC 2 \$4,000
TAC 3: \$2,400
d) For Marketable Permits the Equilibrium price will be $\$ 80$ (Same as the tax). The emissions and total abatement costs will be the same as the tax case. However, their NET total cost will differ, depending if they are net buyers or sellers:

## Question 10

a) $\mathrm{E}=20$
b) $\mathrm{Tax}=100$

Calculate change in costs for the firm from adopting the new technology when:
c) The government uses an emissions standard equal to your answer in (a) above If Standard set at $\mathrm{E}=\mathbf{2 0}$, Savings from switching is $\boldsymbol{\$ 2 0 0}$
d) The government uses an emissions tax equal to your answer in (b)

|  | Tech 1 (old) | Tech 2 (new) |
| :--- | :--- | :--- |
| TAC | 1000 | 1250 |
| TAX Bill | 2000 | 1500 |
|  | $\$ 3000$ | $\$ 2750$ |

Savings from switching is $\mathbf{\$ 2 5 0}$
Now suppose the government adjusts the standard and/or the tax such that MD = New MS. Calculate the change in total costs for the firm from adopting the new technology when:
e) The government adjusts the standard, and
f) The government adjusts the tax rate

## Under NEW technology:

With a standard equal to 17.8
TAC $=986.8$
Savings = 13.2

Under Tax rate of $\mathrm{t}=88.9$,
TAC + Taxbill $=2569.2$
Savings $=430.8$

