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The Economic Journal, Vol. 102, No. 413. (Jul., 1992), pp. 896-905.

Stable URL:
http://links.jstor.org/sici?sici=0013-0133%28199207%29102%3A413%3C896%3ATDCFGG%3E2.0.CO%3B2-F

The Economic Journal is currently published by Royal Economic Society.

THE DEMAND CURVES FOR GIFFEN GOODS ARE DOWNWARD SLOPING*

Yoram Barzel and Wing Suen

The Giffen paradox is one of the most prominent curiosities in economics. Few price theory textbooks fail to mention the possibility that a utility-maximising consumer may consume more of a good at a higher price. Fewer textbooks, however, show awareness as to why the precise nature of the price increase is a crucial element in the Giffen curiosity. This paper will demonstrate that the Giffen curiosity is an artifact of simplification. It only arises when people do not exploit available options and do not plan their consumption in anticipation of (certain or probabilistic) price changes. In a fully specified model, a maximising individual will never buy more of a good when its price increases.

Imagine, for example, that a consumer's demand for fresh vegetables displays the textbook Giffen property. We will show that this same person's demand is downward sloping once the analysis properly accounts for the optimal response to the seasonal price pattern of vegetables. In order to maximise utility, the marginal utility of income between the seasons has to be equalised. This paper demonstrates that the price of a Giffen good is positively related to the marginal utility of income. The consumer will therefore structure his expenditures such that he will spend more in the higher price period and less in the lower price period. Giffen goods being inferior goods, the consumer will consume less vegetables when the price is high and more when the price is low.

In this paper we first demonstrate that where the Giffen good would be present, individuals can gain by buying the appropriate price insurance which will eliminate the Giffen outcome. Even if such insurance markets are not available, individuals can operate on their asset holdings or on their consumption allocation over time. Each alone will also eliminate the Giffen paradox. Thus, if our theory is right, there is a strong presumption that people will find ways to avoid the Giffen good traps.

I. OPTIMAL INSURANCE AGAINST PRICE RISKS

The received mathematical analysis of the Giffen paradox can be stated as follows. If the optimal demand for a good at price p^0 is x^0 and the optimal demand for the good at price $p' > p^0$ is x', and if the price of other goods and money income are the same in the two situations, then the postulate of utility maximisation alone is not sufficient to rule out the possibility that $x' > x^0$. Thus stated, the Giffen paradox is a comparative statics relationship; it shows how the quantity demanded would vary at hypothetically different prices. In the

analysis the consumer is assumed to respond passively to variations in price in the sense that x^0 is chosen without regard to p', and x' is chosen without regard to p^0. Money income is also assumed to be exogenous. Such an analysis would be appropriate if the price change were completely unanticipated. If consumers can at least assign subjective probabilities to possible price changes, they will make consumption plans that take into account the price risk. In that case optimal consumption at p^0 and at p' cannot be determined independently of one another. Moveover, money income in different states is not longer exogenous to the consumer given the possibility of insurance. An explicit analysis using a framework of expected utility maximisation is necessary.

The theory of consumer behaviour under uncertainty suggests that, under certain conditions (see for example Luce and Raiffa, 1957), consumer preferences can be represented by the utility function:

$$U = \sum_{j=1}^{J} \pi_j u(x_{1j}, \ldots, x_{nj}), \tag{1}$$

where π_j is the probability of state j and x_{ij} is the consumption of good i in state j. For a given money income in state j, denoted y_j, the consumer will choose $x_{1j}, \ldots, x_{nj}$ to maximise the von Neumann-Morgenstern utility function u subject to the budget constraint $\Sigma_i p_{ij} x_{ij} = y_j$. The solution to this problem yields the indirect utility function $v(p_{1j}, \ldots, p_{nj}, y_j)$, and quantity demanded in each state can be obtained by using Roy's identity, $x_{ij} = -v_i(j)/v_y(j)$.

Let state j correspond to the state in which the price of good 1 is equal to p_{1j} (and the prices of other goods are the same across states). Then the utility function (1) can also be written as

$$U = \sum_{j=1}^{J} \pi_j v(p_{1j}, p_2 \ldots, p_n, y_j), \tag{2}$$

If the consumer has money income M and if he can purchase actuarially fair income insurance (i.e. state-contingent claims to financial resources), he will choose $y_1, \ldots, y_J$ to maximise (2) subject to the constraint:

$$\sum_{j=1}^{J} \pi_j y_j = M. \tag{3}$$

The left side of equation (3) is the cost of an actuarially fair insurance policy that pays an amount y_j in state j. Necessary conditions for the solution to this optimal insurance problem include:

$$\left. \begin{array}{l} v_y(j) = \mu, \quad \text{for all } j; \\ \pi_j v_{yy}(j) + \pi_k v_{yy}(k) \leqslant 0, \quad \text{for all } j \neq k. \end{array} \right\} \tag{4}$$

The first order conditions imply that the marginal utility of income (v_y) is the same across all states. The second order conditions require that the utility function U be quasi-concave in $y_1, \ldots, y_J$ at the optimum. They imply that the marginal utility of income is increasing $(v_{yy} > 0)$ in at most one state. However, if $p_{1j} = p_{1k}$, then $y_j = y_k$ and v_{yy} must be negative. By continuity v_{yy} is negative

if the difference between p_{1j} and p_{1k} is small. Moreover, if the consumer can take fair gambles, he will gamble out of the convex region of the von Neumann-Morgenstern utility function so that the marginal utility of income is always decreasing (Friedman and Savage, 1948).

With optimal insurance against price risks, total spending in the two states must be such that the marginal utilities of income in these two states are equal, i.e.,

$$v_y(p_{1j}, \ldots, y_j) = v_y(p_{1k}, \ldots, y_k). \tag{5}$$

Taking a first order Taylor expansion on (5), and letting $p_{1k} - p_{1j} = dp_1$, $y_k - y_j = dy$, we get

$$dy = \left(\frac{-v_{1y}}{v_{yy}} \right) dp_1. \tag{6}$$

Let $dx_1 = x_{1k} - x_{1j}$ be the difference in the quantity of good 1 purchased in the two states. Using Roy's identity and equations (4) and (6), we have

$$dx_1 = \left[\frac{-v_1(k)}{v_y(k)} \right] - \left[\frac{-v_1(j)}{v_y(j)} \right]$$

$$= \frac{[v_1(j) - v_1(k)]}{\mu}$$

$$= \frac{(-v_{11} dp_1 - v_{1y} dy)}{\mu}$$

$$= \left[\frac{(v_{1y}^2 - v_{11} v_{yy})}{v v_{yy}} \right] dp_1. \tag{7}$$

Thus if x_1 is a Giffen good (by this we mean that the demand function for x_1 resulting from the maximisation of the von Neumann-Morgenstern utility function u subject to the constraint $\Sigma_i p_{ij} x_{ij} = y$, has a positive own price derivative), it will have a downward sloping demand curve under our specification if and only if

$$v_{1y}^2 - v_{11} v_{yy} > 0. \tag{8}$$

To prove that condition (8) is true, we need the following lemma.

LEMMA. If good 1 is a Giffen good, then

(a) $-v_y v_{11} + v_1 v_{1y} > 0;$

(b) $-v_y v_{1y} + v_1 v_{yy} < 0;$

(c) $-v_y^2 v_{11} - v_1^2 v_{yy} + 2 v_1 v_y v_{1y} < 0.$

Proof. Roy's identity gives $x_1 = -v_1/v_y$. Since x_1 is a Giffen good, $\partial x_1 / \partial p_1 > 0$, which implies part (a) of the lemma. Since a Giffen good is also an inferior good, $\partial x_1 / \partial y < 0$, which implies part (b). Since the substitution effect is negative, $\partial x_1 / \partial p_1 + x_1 (\partial x_1 / \partial y) < 0$, which implies (c).

With the above lemma it is not difficult to prove the main proposition of this paper.

PROPOSITION. For any Giffen good x_1 and any two states j and k, holding money income M and the price of other goods $p_2, \ldots, p_n$ constant, $p_{1k} > p_{1j}$ implies $x_{1k} < x_{1j}$.

Proof. Part (c) of the lemma can be written as

$$v_y(-v_y v_{11} + v_1 v_{1y}) < v_1(-v_y v_{1y} + v_1 v_{yy}), \tag{9}$$

and part (b) can be written as

$$v_1 v_{yy} < v_y v_{1y}. \tag{10}$$

Since the four terms in (9) and (10) are all positive, we can multiply the two inequalities to get

$$v_{yy}(-v_y v_{11} + v_1 v_{1y}) > v_{1y}(-v_y v_{1y} + v_1 v_{yy}), \tag{11}$$

which simplifies to condition (8).

Our proposition demonstrates that, when consumers can insure against price risks, they will purchase a relatively low quantity of a Giffen good when its price is relatively high. *A fortiori*, if Giffen goods have downward sloping demand curves, other goods also have the same property. For completeness, we prove this assertion in an Appendix (available upon request from the authors). Note that our proposition also implies factor supply curves are always upward sloping (see Rosen, 1985).

Let us consider why Giffen goods have downward sloping demand curves in our formulation. What is crucial is that marginal utility of income is high when the price of a Giffen good (or, more generally, the price of an inferior good) is high (see also Marshall, 1895). From part (b) of our lemma, we have

$$v_{1y} > \frac{v_1 v_{yy}}{v_y} > 0. \tag{12}$$

Silberberg and Walker (1984) demonstrate that the compensated change in the marginal utility of income in response to a change in price is always of opposite sign to the income effect. The result in (12) is consistent with this. In Fig. 1 we
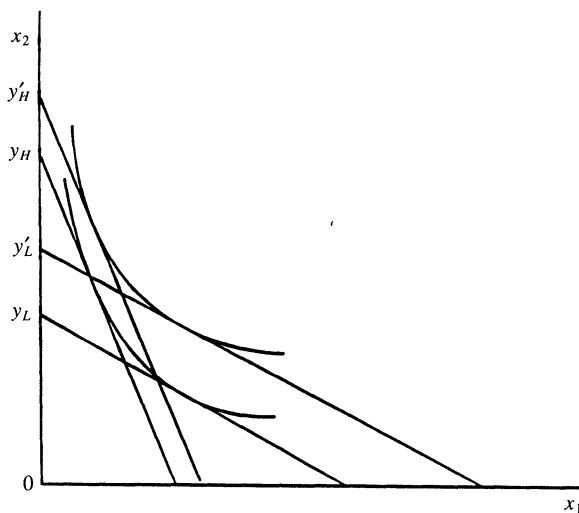

Fig. 1. The marginal cost of utility is low when the price of x_1 is high.

have drawn the indifference map for two goods, x_1 and x_2, where x_1 is a Giffen good. The indifference curves are more tightly packed together in the northwest than in the southeast part of the quadrant. When the price of x_1 is high, the budget line needs to shift only slightly to achieve a higher utility level (from y_H to y'_H). When the price of x_1 is low, the budget line needs to shift a great deal to achieve the same increase in utility (from y_L to y'_L). This means that the marginal cost of utility is low when the price of x_1 is high—in other words, the marginal utility of income is high when the price of the Giffen good is high. Optimal insurance then implies that the consumer will reallocate total spending from situations when p_1 is low to situations when p_1 is high. Formally,

$$\frac{dy}{dp_1} = \frac{-v_{1y}}{v_{yy}} > 0. \tag{13}$$

When p_1 is high, the consumer receives a high income from his insurance policy. Since x_1 is inferior, the high income will cause the consumer to reduce the purchases of x_1. The income effect *reinforces* the substitution effect to produce a downward sloping demand curve.

II. GIFFEN GOODS AS ASSETS

Our result in the previous section depends on the argument that consumers can acquire an optimal portfolio in anticipation of possible price changes. In this section we will show that holding commodity bundles as assets will serve as an imperfect substitute to a perfect insurance market. When a person's portfolio consists of commodities instead of money income, a rise in the price of a good does not necessarily entail a fall in real income. If a person holds an amount of a Giffen good greater than what he consumes, his real income will actually rise when the price of the Giffen good rises. Since the marginal utility of income is positively correlated with the price of Giffen goods, the person's optimal policy is to hold an amount of a Giffen good greater than the amount he consumes. The income effect will then always reinforce the substitution effect to produce a downward sloping demand curve. There are many ways to include a Giffen good in one's assets besides physically holding the commodity. If bread is a Giffen good, for example, the consumer can buy wheat futures or invest in the stock of bakeries. Alternatively, the consumer can arrange to have his wage indexed to the price of bread. In fact, when the prices of consumption goods are variable, consumption decisions and, portfolio decisions are not separable. Individuals tend to prefer assets whose values are positively correlated with the price of inferior or income-inelastic goods (see also Breeden (1979) and Besley (1989)). The analysis that follows shows that optimal portfolio decisions will avert the Giffen good trap.

Suppose the price of good 1 in state j is equal to p_{1j}. We assume that the price of good 1 before the state of nature is revealed is equal to the expected price:

$$p_1 = \sum_{j=1}^{J} \pi_j p_{1j}. \tag{14}$$

The consumer first purchases a_i units of good i at price p_i and holds them as assets. After the state of nature is revealed, his final income at state j will be equal to

$$y_j = p_{1j} a_1 + \sum_{i=2}^{n} p_i a_i, \tag{15}$$

and he can then spend y_j to achieve indirect utility $v(p_{1j}, p_2, \ldots, p_n, y_i)$. At the first stage the consumer will choose an asset holding $a_1, \ldots, a_n$ to maximise expected utility subject to a budget constraint:

$$\text{maximise} \quad \sum_{j=1}^{J} \pi_j v(p_{1j}, p_2, \ldots, p_n, y_j)$$

$$\text{subject to} \quad p_1 a_1 + \sum_{i=2}^{n} p_i a_i = M, \tag{16}$$

where p_1 is given by equation (14) and y_j is given by (15).

The first order conditions for the solution to problem (16) can be combined to get

$$\sum_{j=1}^{J} \pi_j v_y(j) (p_{1j} - p_1) = 0. \tag{17}$$

Equation (17) says that, at the optimum, the covariance between the marginal utility of income and the price of good 1 is zero. Thus, on average, marginal utility of income is equalised across high-price and low-price states. In other words, holding assets or buying futures in the Giffen good serves as a method of partial insurance against price risks. When the price of a Giffen good is high, other things equal, marginal utility of income is also high. Under that situation it is especially advantageous to have a high money income. This is achieved by a long position in the Giffen good because money income will rise automatically when the Giffen good appreciates in value. When a positive amount of the Giffen good is held as asset, the Slutsky equation is written as:

$$\frac{\partial x_1}{\partial p_1} = s_{11} + (a_1 - x_1) \frac{\partial x_1}{\partial y}. \tag{18}$$

Here s_{11} is the pure substitution effect, and it is negative in sign. In the Appendix available from the authors, we show that the optimal asset holding implies $a_1 - x_1 > 0$. Since $\partial x_1 / \partial y$ is negative when x_1 is a Giffen good, the expression in (18) is always negative. When the price of a Giffen good rises, people will consume less of it.

III. GIFFEN GOODS IN AN INTERTEMPORAL CONTEXT

When the price of a good is uncertain, consumers will take actions to mitigate the price risks. We have shown that buying insurance or holding Giffen goods as assets will eliminate the possibility of upward sloping demand curves. When the price of a good changes over time and such price variations are anticipated,

consumers can also respond to the change in relative prices across time through intertemporal substitution. Assuming lifetime utility is additively separable, we will show in this section that the time series observations of price-quantity pairs will produce downward-sloping demand curves.

We assume preferences are additively separable in time such that the utility function can be written as:

$$U = \sum_{t=1}^{T} u(x_{1t}, \dots, x_{nt}),\tag{19}$$

where x_{it} is the consumption of good i at time t. This is not the most general specification of preferences because it precludes habit formation and life-cycle variations in tastes. We have also ignored discounting for simplicity, but the analysis is unchanged as long as the subjective rate of discount is equal to the rate of interest.

The consumer maximises lifetime utility (19) subject to the lifetime budget constraint:

$$\sum_{t=1}^{T} \sum_{i=1}^{n} p_{it} x_{it} = W,\tag{20}$$

where p_{it} is the price of good i at time t and W is money wealth, and where we again ignore discounting. Suppose x_1 is a Giffen good. (That is, the demand function for x_1 resulting from the maximisation of the per-period utility function subject to a per-period budget constraint has a positive own price derivative.) If the price of x_1 jumps unexpectedly and permanently to a higher level, $p_{11}, \dots, p_{1T}$ will be proportionally higher than they were before. Using Hicks's composite commodity theorem, the demand for x_1 will jump permanently to a higher level since x_1 is a Giffen good. The situation, however, is markedly different when we consider the demand for x_1 if the consumer anticipates the price variations over time. Since the price of x_1 in one period relative to another period changes, and since prices at other periods also affect consumption at the current period, an explicit dynamic analysis is required.

The intertemporal maximisation problem can be solved in two stages because utility is additively separable in time. For a given total expenditure at time t, denoted y_t, the consumer will choose $x_{1t}, \dots, x_{nt}$ to maximise per-period utility u subject to the per-period budget constraint $\Sigma_i p_{it} x_{it} = y_t$. The solution to this problem will yield per-period indirect utility functions $v(p_{1t}, \dots, p_{nt}, y_t)$. Having solved this second stage problem, the consumer will choose $y_1, \dots, y_T$ in order to

$$\left.\begin{array}{l} \text{maximise} \quad \sum_{t=1}^{T} v(p_{1t}, \dots, p_{nt}, y_t), \\[2mm] \text{subject to} \quad \sum_{t=1}^{T} y_t = W. \end{array}\right\}\tag{21}$$

The conditions for the solution to this problem include:

$$\left.\begin{array}{l} v_y(t) = \mu, \quad \text{for all } t; \\[2mm] v_{yy}(t) + v_{yy}(s) \leqslant 0, \quad \text{for all } t \neq s. \end{array}\right\}\tag{22}$$

Conditions (22) are the same as those in the insurance model. Optimal intertemporal allocation of resources requires that the marginal utility of income be equal across all time periods. Since marginal utility of income is high when the price of a Giffen good is high, the consumer will allocate greater total spending to time periods when the Giffen good is relatively expensive. The higher total spending in turn depresses the consumption of the Giffen good since it is inferior. The formal analysis of the situation is identical to that in Section I. Thus time-series observations of price-consumption pairs will always yield a downward sloping demand curve even for a Giffen good. As in Hall (1978), our proposition describes the consumption pattern that maximising individuals must satisfy; it does not specify a structural demand equation. This specification of the demand curve is closest in spirit to the 'marginal-utility-of-income-constant' demand function of Frisch (1959) and MaCurdy (1981).

    In reality, price changes are not perfectly foreseen. Were there an unexpected increase in the price of a Giffen good in the last period, for example, the consumer would not be able to reallocate expenditures already spent towards the last period so that the income effect might give rise to an increase in quantity demanded. However a slight extension of the model along the lines of Section I would cover the case of uncertainty. Suppose consumers are able to purchase actuarially fair insurance. Then maximisation of expected utility implies that the marginal utility of income will be the same across all states of nature. If there is some chance that the price of the Giffen good is high in the last period, consumers will buy insurance to cover this possibility because the marginal utility of income will also be high. When the price increase actually occurs, consumers will receive income from their insurance policies. This additional income from insurance policies will wipe out the effect of the reduction in real income from the price increase. Combining lifetime utility maximisation with expected utility maximisation increases the domain of our proposition.

IV. CONCLUSION

The standard theory of utility maximisation is not strong enough to rule out the logical possibility of an upward sloping demand curve. Yet most economists do not take the alleged exception to the law of demand seriously. Economists' skepticism towards the Giffen paradox can be attributed to several reasons. (1) When the share of expenditure on a' good in total spending is small, the magnitude of the income effect is likely to be small (e.g. Hicks, 1946). (2) At the aggregate level, the income effect of a price change is approximately zero (e.g. Friedman, 1949; Heiner, 1974). (3) With an upward sloping demand curve, a decrease in supply would result in a lower price, which is inconsistent with reasonable market dynamics and is counter to the facts (e.g. Dougan, 1982; Dwyer and Lindsay, 1984). (4) Empirical research has not produced a convincing case of an upward sloping demand curve (e.g. Stigler, 1947). We believe the skepticism towards the Giffen good paradox is well-founded. Consuming more of a good when its price is high seems to be a pathological

response. In this paper we demonstrate that the Giffen paradox will not arise when consumers plan their consumption in anticipation of price changes. Using a framework of expected utility maximisation or of lifetime utility maximisation, we show that maximising behaviour is inconsistent with upward sloping demand curves. Our paper thus provides a theoretical justification for the empirical regularity known as the law of demand.

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Date of receipt of final receipt of final typescript: December 1991

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APPENDIX

(A) We want to show that

$$\frac{dx_{1t}}{dp_{1t}} = \frac{(v_{1y}^2 - v_{11} v_{yy})}{v_{yy}} < 0,$$

i.e., that condition (8) in the text is true for all goods. The condition is satisfied if $v_{11} \geqslant 0$, so we need only to consider the case when $v_{11} < 0$. Part (c) of our lemma in the text is true for all goods because it is derived from the negativity of the substitution effect. Rearranging part (c), we get

$$v_{1y} > \frac{(v_y^2 v_{11} + v_1^2 v_{yy})}{2 v_1 v_y} > 0,$$

for $v_{11} < 0$.

In the case of an inferior (but not Giffen) good, $\partial x_1/\partial p_1 < 0$ and $\partial x_1/\partial y < 0$. Thus our lemma will be modified to

(a') $$-v_1 v_{1y} > -v_y v_{11};$$

(b) $$v_y v_{1y} > v_1 v_{yy}.$$

These four terms are all positive and we can multiply the inequalities to obtain the desired result.

If x_1 is a normal good, the compensated and uncompensated price derivatives are both negative. Our lemma will be written as

(a') $$-v_1 v_{1y} > -v_y v_{11};$$

(c) $$-v_y(-v_y v_{11} + v_1 v_{1y}) > -v_1(-v_y v_{1y} + v_1 v_{yy}).$$

Again the four terms are positive and we can multiple the inequalities to get the desired result.

(B) to show that $a_1 - x_1 > 0$, we take a Taylor approximation around p_1 to equation (17) in the text. Let the variance of p_1 be σ^2, we get

$$(v_{1y} + a_1 v_{yy}) \sigma^2 = 0.$$

The expansion involves no covariance terms since we assume the prices of other goods are fixed. Using the above equation and Roy's identity,

$$a_1 - x_1 = \left(\frac{-v_{1y}}{v_{yy}}\right) - \left(\frac{-v_1}{v_y}\right)$$

$$= \frac{v_1 v_{yy} - v_y v_{1y}}{v_y v_{yy}},$$

which is positive by part (b) of our lemma.