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AN ECONOMIC THEORY OF CENTRAL PLACES*

B. Curtis Eaton and Richard G. Lipsey

Since the seminal work of Walter Christaller (1966) and August Lösch (1954), central place theory has become an important, perhaps the most important, theoretical tool of economic geography. No attempt seems to have been made, however, to deduce its propositions from a rigorously stated set of assumptions concerning the behaviour of buyers and sellers. Thus, existing central place theory is not really a theory of spatial economic behaviour. Instead, it is a series of brilliant conjectures about the locational configurations that will result from such behaviour. The brilliance of these conjectures is illustrated by the successful applications of the theory to such diverse phenomena as the locational patterns of retailing activity in cities and of service centres in rural areas, the size distribution of cities and the diffusion of information and diseases.¹

In this paper, we outline an economic model of central places.2 Our theorising has two distinct purposes. First, we wish to begin the development of a theory of central places that is based on maximising behaviour of economic agents. Any theory of economic behaviour from which central places can be derived is necessarily both difficult and cumbersome. Non-convexities in the activities of buying and selling drive the model, and thus non-differentiabilities and discontinuities abound. As one seeks more generality in such circumstances, difficulties multiply rapidly. In order to begin the job, we have used assumptions that are specific and sometimes even crude. The second purpose of our theorising is to illustrate the limitations of current central place theory by providing counter examples to some propositions commonly accepted to follow from it. For this purpose we need not worry about lack of generality in any of our counter examples. Of course we are not solely concerned to refute accepted generalisations. By doing so, we hope to make alternative possibilities apparent. What we hope to attain, therefore, is a theoretical development inspired by the extremely fruitful conjectures of existing central place theory.

* This paper is a revised version of Eaton and Lipsey (1979a). Preliminary work on it was done while the authors held visiting appointments at the University of Colorado at Boulder in 1974-5. We are grateful to the Killam Foundation for support, and to Gernot Kofler, David McGechie, Douglas West and Myrna Wooders for comments and suggestions.

¹ The most common, and perhaps the most successful, applications have seen studies of spatial patterns of retailing. The classic studies are those by Berry and his colleagues at the University of Chicago (see, e.g. Berry, 1958 and 1963 and Simmons, 1964 and 1966), and those by economic geographers at the University of Iowa (see, e.g. Golledge, Rushton and Clark, 1966 and Rushton, 1971). For models concerned specifically with the size distribution of cities, see Beckmann (1958) and Berry (1961). The basic paper on diffusion processes in central place systems is Hudson (1969).

² This paper is the third in a series dealing with the clustering of firms. In our first paper (Eaton and Lipsey, 1975) we showed that Hotelling's explanation of clustering is applicable only to duopolies. In the second paper (Eaton and Lipsey, 1979 b) we showed that clustering of firms throughout the market can result from comparison shopping among firms selling similar goods. In the present paper we show that clustering can also result from scale economies for purchases of dissimilar goods. In Hotelling's model clustering is universally wasteful. In the models of comparison shopping and central places much of the clustering of firms is cost reducing and hence socially beneficial even though optimal configurations do not always result.

I. A REVIEW OF EXISTING CENTRAL PLACE THEORY

Central place theory begins with an analysis of the geographic network of trade or market areas for a single good, X_i . Consumers purchase X_i from the firm that offers the lowest delivered price. The theory asserts that in market equilibrium, producers of X_i will be located on a regular lattice of points, servicing identical hexagonal market areas and charging a common price. (Although the details of the single-industry case are still controversial, the major theoretical problems that concern us here arise only in the multi-industry case.) For each good X_i , let R_i denote the size of the regular hexagonal market area that is required for a firm selling all of the X_i demanded within that area to be able to cover its costs. In the jargon of the theory, R_i is the 'range' of X_i . Index the n goods so that $R_1 < R_2 \ldots < R_{n-1} < R_n$.

In Christaller's analysis, the interrelationships among the locations of sellers of different goods are derived in the following manner. Let producers of X_n be located in a network with hexagonal market areas of size R_n . Since R_n exceeds R_i , $i \neq n$, Christaller argued that all n goods will be offered in these centres, central places of 'order n'. Now consider a set of central places located at the centroid of each of the equilateral triangles defined by the central places of order n. These locations, along with the original central places of order n, define a new network of hexagonal market areas of size $R_n/3$. All goods with $R_i \leq R_n/3$ will be offered in these new central places of order n-1, while all goods with $R_i > R_n/3$ will be offered only in central places of order n. Replications of this geometric argument gives rise to a system of central places which exhibits the hierarchial principle: any goods supplied in a central place of order i is also supplied in all central places of order j > i.

There is no formal analysis of any economic force that causes firms in different industries to agglomerate in this fashion, nor could there be in a model that first determines the locational pattern of firms within each industry and only then considers the interrelationships among industries that are implied by each industry's locational pattern. The pattern of central places and the hierarchical principle are simply products of Christaller's geometric argument.

Analysis of the economic incentives that cause agglomeration is also absent from modern statements of the theory. For example, Dacey et al. (1974) give a treatment of central place theory in a one-dimensional market which has virtually no reference to purposive economic behaviour. In the statement of the theory by Alao et al. (1977, p. 150), the Christallerian structure is obtained by invoking 'a weak agglomeration axiom'. This axiom assumes the basic result under study rather than deducing it from a behaviourally motivated analysis of the interaction of economic agents.

¹ Papers dealing with the one industry locational problem in the Löschian landscape include Mills and Lav (1964), Hartwick (1973), and Eaton and Lipsey (1976).

² In contrast to Christaller, Lösch begins his analysis with the lowest order central place and works up. We do not review Lösch's reasoning since our paper is concerned with the retail sector of the economy and is really in the Christaller-Berry tradition.

Christaller's and Lösch's treatments contain much fruitful intuition about the economic processes that might give rise to central places. Their formal analysis, however, is based on mechanistic, geometric arguments. Modern treatments have refined the mechanistic arguments, stripping away all of the discussion of behaviour that might produce agglomeration. It seems not unfair to say, therefore, that existing formal central place theory is a theory of the location and the agglomeration of firms in which no firm ever chooses its location and in which there are no economic forces that create agglomeration.

We seek to root our model in economic behaviour, and its behavioural engine arises from our answer to the basic question of agglomeration: Why do central business districts, or shopping centres, or suburban shopping districts exist? Why, in other words, do firms retailing different goods tend to cluster together? The explanation that leaps to mind is that, because the clustering of heterogeneous firms facilitates multipurpose shopping, it allows consumers to economise on shopping costs,

Direct observation reveals that the activity of 'shopping' - finding goods, purchasing, and transporting them - is constrained by some important indivisibilities that imply decreasing average total costs over some range of activity. First, there is the indivisibility of the shopper. Shoppers who combine, say, a trip to the butcher with a trip to the baker economise on the time-costs of shopping. A second indivisibility lies with the automobile. It is not to times as costly to transport 10 bags of groceries as it is to transport one bag; indeed, it is more accurate to regard the total costs in this example as constant. A third indivisibility relates to the goods themselves: consumer goods are usually available only in discrete units.

Abstracting from these indivisibilities greatly simplifies the analysis of shopping behaviour. If shopping were characterised by constant returns to scale, and if goods were infinitely divisible, consumers would buy and transport goods at a rate equal to the desired rate of consumption. To do otherwise would effect no

¹ Lösch (1954, p. 76) appears to cite multipurpose and comparison shopping as the 'first advantage of association':

^{&#}x27;First, under any given market situation: The preference of consumers for combining small purchases or comparing various qualities of differentiated products is hardly less important for the formation of towns than for the existence of special business districts within a town and of department stores in these districts. The mere fact of their proximity not only lowers the cost of production, especially general costs, but at the same time increases the share of the demand. Christaller (1966, page 50) appears to have had similar insights:

^{&#}x27;The fact that a central place is larger or smaller has an immediate influence on the range of a

central good, because more types of central goods are offered at a central place of a higher order than at a central place of lower order. This means that, on the basis of a single trip (round-trip costs), one may simultaneously obtain several types of central goods. This has an effect similar to that of a general price decline of the central goods offered in the larger towns. It will be shown in the following discussion of prices that the range of a good is greater when it is offered in a smaller central place.

But these insights are not an integral part of Christaller's or Lösch's analyses. In modern formal treatment of Dacey et al. (1974) and Alao et al. (1977), the topic of agglomeration economies receives virtually no attention. Many other modern writers have conjectures that multi-purpose shopping would provide the behavioural underpinning of a theory of central places, but none have succeeded in demonstrating this result. Bollabas and Stern (1972) give a rigorous demonstration that a hexagonal configuration of homogenous firms would be the planners solution to the locational problem in two-dimensional space. This important result has, however, no bearing on our problem: what is the market's solution to the locational problem where atomistic firms selling different kinds of products make individual locational decisions?

savings in the costs of shopping and would entail unnecessary costs of holding inventories. Furthermore, shoppers would have no incentive to engage in multipurpose shopping since this would not reduce shopping costs. Abstracting from the indivisibilities of shopping would, however, rob us of the ability to understand patterns of location. These indivisibilities imply that consumers who wish to minimise shopping costs will engage in multipurpose shopping.

Firms will then find that they can increase their profits by offering purchasers the chance to indulge in multipurpose shopping. Indeed, it is the interaction of multipurpose shopping and firms' profit-maximising behaviour that provides the core of our theory of central places.¹

II. THE MODEL

We develop our basic theory as well as the counter examples that we require by concentrating on the two-commodity case. The first step is to set out the assumptions of our behavioural model.

(a) Households

- (H-1) Households consume goods A and B at constant rates per unit of time. Units of A and B are such that their rates of consumption are unity.
- (H-2) A and B are marketed in indivisible bundles of size t/α and t/β units respectively.
- (H-3) At regular intervals of time, each household surveys its current inventory of goods, and we choose our unit of time to be equal to this time interval. If the household's current inventory of either good is insufficient for its consumption over the next time period it makes a shopping trip.
- (H-4) Households never buy more than one bundle of any good on any shopping trip. This requires the restrictions $1/\alpha$, $1/\beta \ge 1$ to allow households to satisfy their consumption demands.²
- (H-5) The time and money costs of shopping are an increasing function of distance travelled and of the number of stops the shopper must make, but they are independent of the number of commodities purchased. The cost of each stop is ϵ , which is positive but arbitrarily small.
 - (H-6) Shoppers minimise transport costs on each shopping trip.
- (H-7) Shoppers choose randomly among alternative shopping trips that offer them equal costs.

The household behaviour required by these assumptions is as follows. At the beginning of each time interval, shoppers survey their inventories of goods. If the stock of at least one good is insufficient for consumption needs over the

- ¹ In their analyses, both Christaller and Lösch employ the assumption that transport costs are constant per unit of distance per unit of product. Although Alao et al. (1977, p. 94) do not assume linear transport costs, they do assume that transport costs can be independently defined as a function of distance for each good. These assumptions with respect to transport costs assume away the indivisibilities that drive our model, and they obviously rule out any reason for multipurpose shopping. To the extent, therefore, that multipurpose shopping plays a role in central place theory, it is used as a 'deus ex machina', the mere mention of which justifies the formation of central places.
- ² If costs of holding inventories are sufficiently large, the household would never want to buy more than one bundle of A or B on any trip. Assumption (H-4) can then be thought of as the assumption that inventory holding costs are large.

period, a trip is made to purchase one bundle of each such good. The shopper chooses the retail shops to visit so as to minimise transport costs.

This representation of shopping behaviour catches much of the essence of multipurpose shopping while keeping the model analytically tractable. A more general treatment would formulate the household's problem as the minimisation of the sum of transport costs and costs of holding inventories of goods. Each household would then minimise the sum of transport and inventory costs, subject to the constraint that its consumption requirements be met at each point in time by choosing (1) the timing of shopping trips to purchase A only, and the quantity of A to purchase on such trips, (2) the timing of multipurpose shopping trips and quantities of A and B to purchase on such trips. The solution would be dependent on the locations of retailers of A and B and would be different for each household. This is a mixed integer/real programming problem that is extremely difficult to solve.

Assumptions (H-1) to (H-4) have a convenient implication. Consider a household's purchases of good A over a long period of time T. With a rate of consumption equal to unity, the household must purchase $T/(1/\alpha) = \alpha T$ bundles of A to meet its consumption needs, and this will require αT shopping trips since, by assumption, the household will purchase only one bundle of A on any one trip. Hence, in any one period, the probability that good A will be on the shopping list of a randomly chosen household is α . Similarly, the probability that B will be on the list is β .

(b) Firms

Since we are interested in the consequences of decisions on location taken by individual firms in two different industries, we assume that A and B are always sold by different firms. In order to concentrate on the locational aspects of our problem, we abstract from price competition. In addition, we capture the scale effects that are necessary for the very existence of firms in any spatial model by the assumption of an indivisibility in capital.

(F-1) Any firm retails A or B, but not both, and faces the following average total cost function: $ATC_I = K_I/Q_I + c_I, \quad I = A,B.$

 K_I is fixed costs associated with an indivisible unit of capital, Q_I is quantity retailed, and c_I is a constant marginal cost. For convenience, and without loss of generality, we assume throughout that c_I is zero.

- (F-2) Goods are sold at parametric prices, P_A and P_B .
- (F-3) Each firm chooses its location so as to maximise profits. Assumptions (F-1) and (F-2) imply that profit maximisation is equivalent to sales maximisation. Thus, firms choose their locations so as to maximise sales at the parametric price.

¹ In a path-breaking study of some agglomeration forces Bacon (1971) set up a consumer problem similar to the one just outlined. To solve the consumer's problem given the locations of firms he was, however, forced to rely on numerical simulation techniques. If Bacon's problem defied general analytical solution, the firm-location problem does so doubly since to choose its optimal location every firm must solve each customer's problem for each possible firm location and then aggregate these solutions to determine its demand as a function of its location.

(F-4) In choosing its location, each firm assumes that all other firms will maintain their current locations.

(F-5) For convenience, we assume that firms occupy no space and hence more than one firm can be located at the same point in space.

Assumption (F-2) is the one way in which our model is behaviourally more limited than traditional central place theory. The introduction of price formation would significantly complicate our analysis without, we believe, generating significant further insights into the phenomenon of agglomeration. If in building a theory of location and agglomeration we must choose between a behavioural theory of price and arbitrary assumptions about location on the one hand, and arbitrary assumptions on price and a behavioural theory of location on the other hand, we have no hesitation in choosing the latter combination.

Given our concerns in this paper, assumption (F-4) is convenient and, we believe, appropriate. When, however, a firm can foresee the locational reactions of other firms to its own locational choice, location becomes a strategic variable and assumption (F-4) is inappropriate. The strategic choice of location in a model of central places is an interesting theoretical problem but is beyond the scope of this paper.

(c) Completion of the Model

There is a one-dimensional market of unit length with a uniform density of households, D. Firms are numbered in ascending order from left to right along the market. The ith A firm is denoted by A_i and its location by a_i , and the jth B firm is denoted by B_j and its location by b_j . A bar over a location indicates that the location is fixed for the exercise in question, while the absence of a bar indicates that it is variable. When a shopper makes a shopping trip to buy only one good, we refer to the trip as an A-only or a B-only trip; and when a shopper makes a trip to buy A and B, we refer to the trip as an A-with-B trip. Sales made to shoppers on A-only trips) are referred to as A-only sales (B-only sales), while sales made to shoppers on A-with-B trips are referred to as A-with-B sales. A point in the market with at least one A and one B firm is called a CP2 (for 'central place of order two'). A point with only one type of firm is called a CP1 (for 'central place of order one'), and either a CP1A or CP1B when we wish to specify the type of firm.

III. EQUILIBRIUM CONFIGURATIONS

It is by now well known that equilibrium does not always exist in location models where the space is bounded. To keep the current paper manageable we focus only on equilibrium configurations.¹ There are three conditions that are necessary and, taken together, sufficient for an equilibrium of locations in this model.

¹ Non-existence is a problem in many models of location in bounded space. See Eaton and Lipsey (1975) for some examples where equilibrium does not exist, and Prescott and Visscher (1977) for a constructive response to non-existence that requires the strategic, forsightful behaviour that is ruled out of our model. In Eaton and Lipsey (1979a), we delineate the conditions in which equilibrium does and does not exist in the present model.

Equilibrium Condition (i): No existing firm can increase its sales by changing its location.

Equilibrium Condition (ii): All existing firms of type I must have revenues greater than or equal to K_I , I = A, B.

Equilibrium Condition (iii): At all locations, anticipated revenues for a new entrant of type I must be less than K_I , I = A,B. Condition (i) implies that no existing firm wants to change its location; condition (ii) implies no existing firm wants to exit; condition (iii) implies that no new firm wants to enter.

(a) The Necessity of Agglomeration

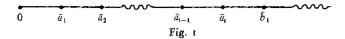
We begin by demonstrating that agglomeration (the existence of higher order central places) is necessarily a property of equilibrium in our model. Formally we show that Equilibrium Condition (i) implies

PROPOSITION 1. In market equilibrium, (1-a) there must exist at least one CP2, and (1-b) it is impossible for a CP1A and a CP1B to be neighbours.

To prove Proposition 1 we first show that Equilibrium Condition (i) implies.

PROPOSITION 2. Scanning the market from left to right, (2-a) there must be a first CP2, and (2-b) between the left-hand market boundary and the first CP2 there can be at most one type of CP1.

In order to avoid a tedious taxonomy, we assume that the market is large enough to support several firms of each type. Then when we scan the market from left to right, we will observe a first central place. If the first central place is a CP2, this does not contradict Proposition 2. If the first central place is not a CP2, then it must be a CP1 and we can assume without loss of generality that it is a CP1A. As we continue to scan from left to right, we will eventually encounter the first B firm, B_1 . If B_1 is in a CP2, this does not contradict Proposition 2. Accordingly, we assume that B_1 is not in a CP2 and prove by contradiction that the market cannot be in equilibrium.

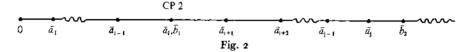


Formally, assume (I) that all firms in the market satisfy equilibrium condition (i) and (II) that, scanning the market from left to right, we observe i CP1As and then a CP1B. This locational configuration is illustrated in Fig. 1.

In the Appendix we show a contradiction between I and II, thus proving Proposition 2. Here we present an intuitive account of the argument. The location of A_i , a_i , is confined to the half-closed interval $[\bar{a}_{i-1}, \bar{b}_1)$ by assumption II. Observe next that a_i in the open interval $(\bar{a}_{i-1}, \bar{b}_1)$ does not satisfy assumption I. When $\bar{a}_{i-1} < a_i < \bar{b}_1$, the A-only sales of A_i are independent of a_i since the size of the market segment from which A_i attracts A-only shoppers is $(\bar{a}_{i+1} - \bar{a}_{i-1})/2$. However the A-with-B sales of A_i monotonically increase with a_i : for multipurpose shoppers to the right of a_i , the option of travelling to a_i and \bar{b}_1 (or some other B firm) becomes less expensive as a_i increases and accordingly the A-with-B sales of A_i from this region increase; for multipurpose shoppers located to the

left of a_i , the cost of travelling to a_i and $\bar{b_1}$ is independent of a_i . Thus, for a_i in the open interval $(\bar{a}_{i-1}, \bar{b_1})$, A_i 's sales are a monotonically increasing function of a_i , and assumption I is not satisfied for a_i in this open interval.

The only possible location for A_i which could satisfy assumptions I and II is thus \bar{a}_{i-1} . If assumption I is to be satisfied it is, of course, necessary that A_i prefer location \bar{a}_{i-1} to \bar{b}_1 . We will argue that the conditions which ensure that A_i prefers \bar{a}_{i-1} to \bar{b}_1 imply that A_{i-1} does not satisfy equilibrium condition (i). To do this we first ask what conditions would cause A_i to prefer \bar{a}_{i-1} to \bar{b}_1 . When $a_i = \bar{b}_1$, A_i captures all of the A-with-B business from the left of \bar{b}_1 since shopping at A_i and B_1 then involves only one stop, and it captures the A-only sales from a market segment equal to $(\bar{a}_{i+1} - \bar{a}_{i-1})/2$. When $a_i = \bar{a}_{i-1}$, A_i 's A-with-B sales are clearly smaller than they are when $a_i = \bar{b}_1$, but its A-only sales may be



larger. When $a_i = \bar{a}_{i-1}$, A_i and A_{i-1} equally share A-only sales from a market segment equal to $(\bar{a}_{i+1} - \bar{a}_{i-2})/2 = (\bar{a}_{i+1} - \bar{a}_{i-1})/2 + (\bar{a}_{i-1} - \bar{a}_{i-2})/2$. It is then clear that A_i will prefer to locate at \bar{a}_{i-1} if and only if $(\bar{a}_{i-1} - \bar{a}_{i-2})/2$ is 'sufficiently large'. That is, it is the prospect of sharing A-only sales from a 'large' market segment $(\bar{a}_{i-1} - \bar{a}_{i-2})/2$ which can lead A_i to prefer \bar{a}_{i-1} . But in these circumstances A_{i-1} does not satisfy assumption I since it would prefer to monopolise A-only sales from this 'large' segment by moving just to the left of A_i . Thus the circumstances in which A_i would be in equilibrium at \bar{a}_{i-1} (rather than \bar{b}_1) require that A_{i-1} not be in equilibrium. It follows that equilibrium condition (i) requires that A_i and B_1 form a CP2.

We can replicate the argument outlined above to prove Proposition 1. Scan the market from left to right beyond the first CP2. If the first central place we observe is a CP2, this is obviously consistent with Proposition 1. If we observe a string of CP1s of the same type (CP1As or CP1Bs) followed by a CP2, this is also consistent with Proposition 1. We must, however, demonstrate that in market equilibrium, we cannot observe a string of CPIs of one type followed by a CPIof the other type. A proof by contradiction will establish this result. Assume (I) that all firms satisfy equilibrium condition (i) and assume, without loss of generality, (II) that we observe a string of CP1As followed by a CP1B (the configuration illustrated in Fig. 2). The structure of the proof follows. The location of A_i , by assumption (II), is confined to $[\bar{a}_{i-1}, \bar{b}_2)$, and the only location for A_i in this interval that satisfies assumption (I) is $a_i = \bar{a}_{i-1}$. A necessary condition for this location to satisfy assumption (I) is that $\bar{a}_{j-1} - \bar{a}_{j-2}$ be 'sufficiently' large. But when it is 'sufficiently' large, A_{i-1} would prefer to locate just to the left of A_{i} , and A_{i-1} does not satisfy assumption I. Hence, there can be at most one type of CP1 between the first and second CP2s. Repetition of this argument throughout the length of market establishes Proposition 1.

We have now shown that in market equilibrium, CP1As and CP1Bs cannot be adjacent to each other. In addition we have shown that in market

equilibrium CP2s must exist. An immediate consequence of Propositions 1 and 2 is

PROPOSITION 3. In market equilibrium all of the A-with-B business will be transacted in CP2s, and the firms in CP1s will serve only single-purpose shoppers.

(b) A Taxonomy

Proposition 1 is consistent with a market served by (a) CP2s and CP1As, (b) CP2s and CP1Bs, (c) CP2s only, and (d) CP1As and CP1Bs in different intervals between C2Ps. In this section we use equilibrium conditions (ii) and (iii) to eliminate case (d), thus establishing the hierarchial principle of our model. In addition we derive necessary and sufficient conditions for cases (a), (b) and (c).

Define Y, Z and λ as follows:

$$Y = K_A/(\alpha P_A D), \tag{1}$$

$$Z = K_B/(\beta P_B D), \tag{2}$$

$$\lambda = Z/Y. \tag{3}$$

If an A firm could capture all of the A business from a market segment of length Y, expected revenues in any period would be αP_A DY since good A is on the shopping list of any shopper in any period with probability α . From (1), it is then clear that we can interpret Y as the length of a market an A firm must have in order to cover costs if it captures all of the A business from this segment. Z can be similarly interpreted.

We wish to prove

PROPOSITION 4. (4-a) a necessary condition for the existence of CP1As in market equilibrium is that $\lambda > 1/(1-\beta)$; (4-b) a necessary condition for the existence of CP1Bs in market equilibrium is that $\lambda < 1-\alpha$; (4-c) a necessary condition for the market to be served only by CP2s is that $(1-\alpha)/2 < \lambda < 2/(1-\beta)$.

In proving (4-a) we consider the existence of a CP1A in a market segment of length L bounded by CP2s. (The reader can verify that the necessary condition also applies to the existence of a CP1A in a peripheral market segment.) We begin by using equilibrium conditions (ii) and (iii) to establish bounds on L. L must be large enough so that equilibrium condition (ii) is satisfied for at least one CP1A. Proposition 3 dictates that the CP1A would attract only A-only shoppers. In any period the probability that any shopper will make an A-only trip is $\alpha(1-\beta)$. One CP1A in the segment of length L would attract the A-only shoppers from a market segment equal to L/2. The CP1A's revenues would then be $DP_A\alpha(1-\beta)L/2$. Condition (ii) then implies that

$$L \geqslant \frac{2K_A}{\bar{D}P_A\alpha(1-\beta)} = \frac{2Y}{1-\beta},\tag{4}$$

is necessary if CP1As are to exist in market equilibrium (in the segment of length L).

Suppose we have at least one CP_1A in the segment of length L. If a B firm were to enter this segment it would locate at a CP_1A , forming a new CP_2 , and it would

capture all of the B business from a market segment equal to L/2. Its revenues would be $DP_B \beta L/2$. It is obviously necessary for the existence of CP_IAs in market equilibrium (in the segment of length L) that a B firm not find the option of entering this interval attractive. That is, equilibrium condition (iii) dictates that

 $L < \frac{2K_B}{\beta DP_B} = 2Z,\tag{5}$

is necessary for the existence of CP1As.

Inequalities (4) and (5) are necessary for the existence of CP_1As in market equilibrium, and if they are both to be satisfied, we require that $2Z > 2Y/(1-\beta)$, or that

 $\lambda > \frac{1}{1 - \beta}.\tag{6}$

This establishes (4-a). An exactly analogous argument establishes (4-b).

To establish (4-c) we need to find conditions which ensure that at least one firm of each type can cover costs in a CP2 and which ensure that neither an A nor a B firm will find it profitable to establish a CP1. It is convenient to proceed in stages. First assume that $\lambda > 1$, then from (4-b) we are assured that a B firm will not establish a CP1. Reversing inequality (4) ensures that an A firm will not establish a CP1:

 $L < 2Y/(1-\beta). \tag{7}$

The firms in CP2s will attract all of the business from a market segment equal to L. Viability of at least one B firm in a CP2 requires that $\beta P_BDL \geqslant K_B$ or that

$$L \geqslant Z$$
. (8)

Since $\lambda > 1$, (8) also ensures the viability of at least one A firm in a CP2. For (7) and (8) to hold simultaneously requires that $\lambda < 2/(1-\beta)$, the second inequality in (4-c). An analogous argument, with $\lambda < 1$, establishes the first.

The conditions in (4-a) and (4-b) cannot simultaneously hold. Thus we have the hierarchial principle of our model:

PROPOSITION 5. If central places of orders one and two exist in market equilibrium, all central places of order one offer the same good.

It is clear from Proposition 4 that equilibrium is not unique in our model; that is, the necessary conditions in Proposition 4 are not also sufficient. The following sufficient conditions are immediately implied by Proposition 4.

PROPOSITION 6. (6-a) a sufficient condition for the existence of CP1As in market equilibrium is that $\lambda \ge 2/(1-\beta)$; (6-b) a sufficient condition for the existence of CP1Bs in market equilibrium is that $\lambda \le (1-\alpha)/2$; (6-c) a sufficient condition for the market to be served only by CP2s is that $(1-\alpha) \le \lambda \le 1/(1-\beta)$.

One further observation on the equilibrium of our model seems worthwhile. Suppose for purposes of illustration that $t < \lambda < (t - \beta)$ so that the market is served only by CP2s. Our assumption that prices are parametric implies that the number of firms of each type in each CP2 will be

$$\eta^A = INT[L/Y], \tag{9}$$

$$\eta^B = INT[L/Z], \tag{10}$$

3

where INT is the largest integer function. Using (7) and (8) we can establish bounds on η^A and η^B

$$\lambda \leqslant \eta^{\mathcal{A}} < 2/(1-\beta), \tag{11}$$

$$1 \leq \eta^B < 2/\lambda(1-\beta). \tag{12}$$

It is then clear that in this case there is no upper bound on the number of firms of each type that can exist in any CP_2 .

Indeed, as we show in Eaton and Lipsey (1979a), for any value of λ it is possible to construct equilibria in which either η^A or η^B is arbitrarily large. Given our assumptions, more than one firm of either type in a CP2 represents pure excess capacity – it increases the costs of retailing and does nothing to reduce the costs born by shoppers. We thus have

Proposition 7. Pure excess capacity is possible in market equilibrium. We comment on the possible significance of this result below.

III. SIGNIFICANCE

(a) Contrasts With Traditional Central Place Theory

In order to emphasise the contrasts between traditional central place theory and our versions of the theory we briefly compare the two in the context of an example.

Let Z = 2Y, or $\lambda = 2$. Our interpretation of Z and Y in section II-b means that, using the jargon of central place theory, Y is the 'range' of good A and Z is the 'range' of good B. According to traditional central place theory, we should expect a unique configuration of firms in these circumstances. There should be CP2s composed of one A and one B firm spaced Z units apart; there should also be a CP1A at the midpoint of the interval between each pair of CP2s.

In these circumstances, however, many equilibrium configurations are possible in our model. First, let $\beta > \frac{1}{2}$. Proposition (6-c) implies that there can be no CP1s, and the market will be served only by CP2s. The difference between the two theories arises because traditional central place theory is based solely on costs while our theory is driven by the interaction of costs and demand externalities between goods. In our model, CP2s impose a negative demand externality on any CP_{IS} by leaving merely the A-only business to the CP_{IA} s (Proposition 3). Thus, a CP1A requires a market segment larger than Y to survive. How much larger depends upon the magnitude of β , which can be thought of as an index of the negative demand externality between CP2s and CP1As. When $\beta > \frac{1}{2}$, a market large enough to support a CP1A is large enough to invite entry of a B firm, converting the CP1A into a CP21 Thus, if we set up the market in the sequence of alternating CP2s and CP1As suggested by central place theory, but with an interval between adjacent CP2s sufficient to support a CP1A, the isolated A firm will immediately be joined by a new B firm, and we will be left with only CP2s.

Secondly, let $\beta < \frac{1}{2}$ so that the condition 4-a is satisfied. Consider setting the firms down in the exact pattern suggested by central place theory $-CP_{2}$ s at intervals of Z with $CP_{1}A$ s at the midpoints between adjacent CP_{2} s. The $CP_{1}A$ s will obtain the A-only business over the market segment of Z/2, which by the

assumptions of our illustrative example is equal to Y. The revenue of each CP1A will thus be

$$DP_{A}\alpha(1-\beta)Y = DP_{A}\alpha(1-\beta)K_{A}/\alpha P_{A}D = (1-\beta)K_{A}.$$

It is clear that revenues will cover costs either if $\beta = 0$, which is uninteresting since there is no second good, or if by assumption there is no multipurpose shopping. Thus, while traditional central place theory sometimes invokes multipurpose shopping as a justification for overlaying the locational configuration for different types of outlets developed in isolation, it is clear that the precise locational pattern that results when the range of one good is twice the range of the other is inconsistent with the existence of any multipurpose shopping. Given multipurpose shopping, we can, in this example, have CP2s separated by some given distance and a CP1A at the mid-point between each pair of CP2s, but only if the CP2s are separated by a distance larger than Z = 2Y and if $\beta < \frac{1}{2}$.

The coexistence of multipurpose and single-purpose shopping follows from a rational model of household shopping behaviour. This means, however, that adjacent central places of higher order take some of the potential sales from central places of lower order. For this reason, even in something so simple as the two-good case, various patterns are possible and they depend both on costs and on the relative volumes of multipurpose and single-purpose shopping. Whatever the details of a particular case, the equilibrium configuration can never be established simply by finding the patterns that would exist (1) if there were only A firms and (2) if there were only B firms, and then overlaying these two patterns. This statement is true in any theory of central places where agglomeration is economically motivated. It is not dependent on the restrictive assumptions of our model.¹

(b) Capacity Relations

Above we illustrated the possibility of substantial pure excess capacity in market equilibrium. Given our price and cost assumptions it is a pure social waste to have more than one firm of each type in any GP2. Of course, if the existence of more than one firm gives rise to price competition or if we assume U-shaped cost curves, the existence of multi outlets for the same good in a CP2 would not necessarily be a waste. Nevertheless, we believe our model generates some insight into the phenomenon of excess capacity in retailing. The demand externalities that arise from multipurpose shopping serve to create something analogous to a spaceless market in the CP2. If a CP2 containing one firm of each type yields pure profits other profit-seeking firms may enter the CP2 rather than locating outside it. This is because the market for both goods from households on multipurpose shopping trips exists only in CP2s. As long as new firms expect profits, they will enter the CP2, and the final equilibrium will be akin to Chamberlin's - although as Kaldor (1935) long ago pointed out, the discreteness of entry due to capital indivisibilities means that existing firms may be making substantial profits while a new entrant expects losses.

¹ Some critics of traditional central place theory have obviously been aware of the problems outlined above (see Rushton (1971), Parr (1973), Clark and Rushton (1970)). The quotation from Christaller cited in Note 1 of page 58 above indicates that he was aware of these problems.

The existence of excess capacity gives rise to a second interesting possibility. Let there be a CP2 with multiple outlets for either or both types of firms. Now let the density of customers grow. Profits of existing firms will grow, and entry of further A and/or B firms into existing CP2s will occur. Eventually, customer density will grow enough so that a new CP2 will be formed between adjacent CP2s. The market of each existing CP2 now falls discretely, since a new entrant between adjacent CP2s will halve the range over which existing CP2s draw A-with-B business. In this case, there is a fall in the total revenues earned by all the firms in each of the original CP2s. If more than one firm is selling one kind of good, all may make losses. If there are N firms selling this good, then up to N/2 of them may be redundant after the new CP2s are formed. Exit will occur until the firms remaining can at least cover costs.

Thus in a market where demand is growing steadily, growth of central places may be oscillatory. First, more A and/or B firms are added to existing CP_{2S} , creating Chamberlinian excess capacity. This is an equilibrium phenomenon in the sense that if demand were held constant, the excess capacity would persist indefinitely. If demand goes on expanding, however, new CP_{2S} will be formed, rendering some firms in old CP_{2S} redundant. Exit will occur until the firms remaining in the old CP_{2S} can cover costs. Further growth of demand will then lead all CP_{2S} to grow once more until a further round of entry of new CP_{2S} causes the old ones to shrink in size once more.

(c) Shopping Centres as Central Places

Consider any of the CP2s in our model that contain more than one firm of either or both types. Now assume the central place is demolished and the developer of a shopping centre is allowed to exploit the opportunity thereby created. If we ignore the possibility of entry by independent retailers, it is obvious that, given the cost specification in our model, the developer would simply establish one A firm and one B firm in his shopping centre. Consumers would be no worse off, and the developer would quite obviously enjoy pure profits greater than those collectively earned by the independent retailers who formerly comprised the CP2. The shopping centre would not dissipate rents through pure excess capacity in the manner which independent retailers may do in our model. This result is similar to one that comes from models of a common property resource. When there exists no mechanism to limit entry into the CP2, independent retailers dissipate rents in much the same fashion as independent fisherman dissipate the rents available in a fishery.

¹ The new CP2 can arise in either of two ways. First, if there is no CP1A between existing CP2s, then eventually it will seem profitable for a CP1A to be formed; the A firm will immediately be joined by a B firm, thus creating a CP2s. Second, if the parameters are such that there is one or more CP1As between existing CP2s, then sooner or later, as demand rises, it will pay a B firm to join one of the A firms, converting an established CP1A into a CP2.

² Of course, in a model (such as Eaton and Lipsey (1979b)) that allows comparison shopping, the developer might choose to have more than one outlet for the same commodity. But he never permits the dissipation of rents by having more than the joint-profit-maximising number.

³ The classic references are Gordon (1954) and Scott (1955). Neher (1978) has recently argued that the 'common property problem' arises in many situations. One way of interpreting our arguments with respect to pure excess capacity is that the exploitation of a demand for a good in a central place is also

We have, of course, simply assumed away the potential for entry of independent retailers once the shopping centre is established. The developer can, however, effectively forestall entry by purchasing all the land within some radius of the location of the shopping centre. In this way, he can ensure that he will get all of the revenues enjoyed by the original *CP2*. If he does not buy up all such land, entry will occur in the same way as it occurs in the model of independent retailers, and the rents of location will be dissipated.

V. CONCLUSIONS

The existing theory of central places is simultaneously a theory of the location and agglomeration of economic activity in which there is no force creating agglomeration, in which agglomeration serves no purpose, and in which no firm ever chooses a location. Yet this 'theory' has proved useful in interpreting the economic landscape.

To develop an economic theory of central places, we have focused on the demand externalities created by multipurpose shopping. We demonstrate that these demand externalities must give rise to higher order central places, and that equilibrium satisfies a hierarchial principle. The model differs in important respects from the traditional model, and it yields insights into the phenomenon of excess capacity in retailing, into the dynamic process of expansion of the retail sector in a growing market, and into the role played by, and the motivation behind, shopping centres.

Our model is primitive in many respects, and it can be regarded as merely a beginning. Primitive as it is, however, it demonstrates the importance of providing a behavioural economic theory of central places and it illustrates the potential pay off to such a theory in terms of understanding real world phenomena.¹

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plagued with common property problems. The retailers can be thought of as 'fishing' for customers from a common pool - those who travel to the central place for their purchases. In the absence of a shopping centre, firms already serving the market would seem to have a very limited capacity to deterentry of competing firms.

¹ Referees and editors have asked us to investigate whether our theory has more empirical content than traditional central place theory and to show how comparative tests can be made between the two theories. In the present paper our concern is to develop a new micro behavioural underpinning for central place theory. We illustrate the importance of this by showing that our micro underpinnings produce some predictions that agree with, and others that conflict with, those of traditional central place theory. The next two urgent tasks are first to discover all of the interesting testable statements where our theory diverges from central place theory and, second, to make comparative tests of these divergent predictions. Each of these tasks is a major research project. We are pleased, therefore, that Professor D. West, who has already done empirical work on several spatial theories including some of our own (West, 1981 e) is directing his attentions to these tasks.

APPENDIX

Proof of Proposition 2

Assume (I) that all firms in the market satisfy equilibrium condition (i), and

(II) that, scanning the market from left to right, we observe i CP1As and then a CP1B. This locational configuration is shown in Fig. 1.

We now show a contradiction.

If i = 1, the contradiction is immediate since A_1 could obviously increase its sales by moving to the right (and A_1 thus does not satisfy equilibrium condition (i)).

Consider the case in which $i \ge 2$ and consider the location of firm A_i in $[\bar{a}_{i-1}, \bar{b}_1]$. Location anywhere in the open interval $(\bar{a}_{i-1}, \bar{b}_1)$ will generate an identical amount of A-only sales, given by $\frac{1}{2}D\alpha(1-\beta)(\bar{a}_{i+1}-\bar{a}_{i-1})$, and an identical amount of A-with-B trade from consumers located in the interval $[0, \bar{a}_{i-1}]$, given by $D\alpha\beta\Sigma_1$ where

$$\Sigma_1 = \frac{\bar{a}_1}{i} + \frac{\bar{a}_2 - \bar{a}_1}{i - 1} + \dots + \frac{\bar{a}_{i-1} - \bar{a}_{i-2}}{2}. \tag{1}$$

But A_i will obtain a greater share of A-with-B sales, from consumers to the right of \bar{a}_{i-1} the nearer is A_i located to B_1 . Thus for a_i in $(\bar{a}_{i-1}, \bar{b}_1)$, A_i 's sales increase with a_i , and since the interval is open A_i cannot satisfy equilibrium condition (i) in it. This leaves \bar{a}_{i-1} and \bar{b}_1 as the only locations which may be consistent with the equilibrium conditions.

(a) Location at b_1 . Total A-with-B sales are

$$D\alpha\beta\bar{a}_{i-1} + D\alpha\beta(\phi - \bar{a}_{i-1}), \tag{2}$$

i.e. all customers in $[0, \bar{a}_{i-1}]$ plus customers in (\bar{a}_{i-1}, ϕ) , where ϕ is the point at which a customer would be indifferent between visiting the CP2 at \bar{b}_1 and some other combination of A and B firms. Clearly $\phi > \bar{b}_1$.

(b) Location at \bar{a}_{i-1} . Total A-with-B sales are

$$D\alpha\beta\Sigma_1 + D\alpha\beta(\theta - \bar{a}_{i-1})/2 \tag{3}$$

i.e. a share of customers to the left of \bar{a}_{i-1} plus half the customers to the right of \bar{a}_{i-1} who would prefer the combination (B_1, A_i) (or (B_1, A_{i-1})) to some other combination of A and B firms. Clearly, $\theta < \phi$.

The loss of A-with-B sales involved in choosing location \bar{a}_{i-1} in preference to b_1 is:

$$L_1 = D\alpha\beta[(\bar{a}_{i-1} - \Sigma_1) + (\phi - \bar{a}_{i-1}) - \frac{1}{2}(\theta - \bar{a}_{i-1})]$$
 (4)

which is positive since $\bar{a}_{i-1} > \Sigma_1$, $\phi > \theta$. The change in A-only sales is:

$$C = D\alpha(\mathbf{1} - \beta) \, \frac{1}{4} (\bar{a}_{i+1} - \bar{a}_{i-2}) - D\alpha(\mathbf{1} - \beta) \, \frac{1}{2} (\bar{a}_{i+1} - \bar{a}_{i-1})$$

$$= D\alpha(\mathbf{1} - \beta) \, (\frac{1}{2} \bar{a}_{i-1} - \frac{1}{4} \bar{a}_{i+1} - \frac{1}{4} \bar{a}_{i-2}). \tag{5}$$

If assumption (II) is to be satisfied $C > L_1$ and A_i locates at \bar{a}_{i-1} . Now consider A_{i-1} . If A_{i-1} locates an arbitrarily small distance to the left of A_i , its A-with-B

sales are given by $D\alpha\beta\Sigma_1$, i.e., it loses A-with-B sales in $[\theta - \tilde{a}_{i-1}]$. The loss of A-with-B sales is then

 $L_2 = \frac{1}{2} D \alpha \beta (\theta - \hat{a}_{i-1}).$ (6)

The change in A-only sales is

$$D\alpha(\mathbf{1} - \beta) \, \frac{1}{2} (\bar{a}_{i-1} - \bar{a}_{i-2}) \, - \, D\alpha(\mathbf{1} - \beta) \, \frac{1}{4} (\bar{a}_{i+1} - \bar{a}_{i-2}) \\ = \, D\alpha(\mathbf{1} - \beta) (\frac{1}{2} \bar{a}_{i-1} - \frac{1}{4} \bar{a}_{i+1} - \frac{1}{4} \bar{a}_{i-2}) \, = C, \quad (7)$$

which is identical to that obtained by A_i in locating at \bar{a}_{i-1} rather than \bar{b}_i . But note that

$$L_1 - L_2 = D\alpha \beta [(\phi - \theta) + (\tilde{a}_{i-1} - \Sigma_1)] > 0$$
 (8)

hence
$$C > L_1 > L_2$$
. (9)

Thus if A_i chooses location at \bar{a}_{i-1} , A_{i-1} would wish to change location and so cannot satisfy the equilibrium condition (i). Hence Assumption (II) leads to a contradiction.

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