Notes on CV, EV and Consumer Surplus

ECON 402

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Kevin Wainwright[1]
Simon Fraser University
Burnaby, BC

1 Utility Maximization

Consider a consumer with the utility function $U = xy$, who faces a budget constraint of $B = P_x x P_y y$, where all variables are defined as before.

The choice problem is

Maximize

$$U = xy \tag{1}$$

Subject to

$$B = P_x x + P_y y \tag{2}$$

The Lagrangian for this problem is

$$Z = xy + \lambda(B - P_x x - P_y y) \tag{3}$$

The first order conditions are

$$\begin{aligned} Z_x &= y - \lambda P_x = 0 \\ Z_y &= x - \lambda P_y = 0 \\ Z_\lambda &= B - P_x x - P_y y = 0 \end{aligned} \tag{4}$$

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Solving the first order conditions yield the following solutions

$$x^M = \tfrac{B}{2P_x} \quad y^M = \tfrac{B}{2P_y} \quad \lambda^M = \tfrac{B}{2P_xP_y} \tag{5}$$

where x^M and y^M are the consumer's Marshallian demand functions. Substituting x^M and y^M into the utility function yields the indirect utility function

$$V(P_x, P_y, B) = \left(\frac{B}{2P_x}\right)\left(\frac{B}{2P_y}\right) = \frac{B^2}{4P_xP_y} = U_0 \tag{6}$$

If we denote the maximum utility by U_0 and re-arrange the indirect utility function to isolate B

$$\frac{B^2}{4P_xP_y} = U_0$$

$$B = (4P_xP_yU_0)^{\frac{1}{2}} = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U_0^{\frac{1}{2}} = E(P_x, P_y, U_0) \tag{7}$$

We have the expenditure function

Let $P_x^o = 2$, $P_y = 1$ and $B = 40$. From (6) we find that $U_0 = 200$.

2 CV and EV for a Price Fall

Suppose the price of x were to fall to $P_x^N = 1$.

The consumer's utility becomes

$$\frac{B^2}{4P_xP_y} = \frac{(40)^2}{4(1)(1)} = 400 = U_N$$

The budget necessary to achieve the original utility ($U_0 = 200$) is found from the the expenditure function (7)

$$(4P_x^NP_yU_0)^{\frac{1}{2}} = (4 \cdot 1 \cdot 1 \cdot 200)^{1/2} = 28.28$$

CV is found by

$$CV = B - E(P_x^N, P_y, U_0) = 40 - 28.28 = 11.72$$

Alternatively, EV can be found by using the expenditure function with the original prices but the new utility

$$EV = E(P_x^o, P_y, U_N) - B = (4 \cdot 2 \cdot 1 \cdot 400)^{1/2} - 40 = 56.57 - 40 = 16.56$$

In this case EV is bigger than CV; x is a normal good.

3 CV and EV for a Price Rise

Now, instead, suppose the price of x were to rise to $P_x^N = 3$.

The consumer's utility becomes

$$\frac{B^2}{4P_xP_y} = \frac{(40)^2}{4(3)(1)} = 133.3 = U_N$$

The budget necessary to achieve the original utility ($U = 200$) is found from the the expenditure function (7)

$$(4P_x^N P_y U_0)^{\frac{1}{2}} = (4 \cdot 3 \cdot 1 \cdot 200)^{1/2} = 48.99$$

CV is found by

$$CV = B - E(P_x^N, P_y, U_0) = 40 - 48.99 = -8.99$$

Alternatively, EV can be found by using the expenditure function with the original prices but the new utility

$$EV = E(P_x^o, P_y, U_N) - B = (4 \cdot 2 \cdot 1 \cdot 133.3)^{1/2} - 40 = 32.66 - 40 = -7.34$$

Note that, again, EV is greater than CV.