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The Bell Journal of Economics, Volume 8, Issue 2 (Autumn, 1977), 394-418.

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# The rat race and internal labor markets

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The labor market is viewed as a market for labor contracts. A firm is identified as having an internal labor market if the efficient mode of production requires that it employ heterogeneous worker types by offering a wage structure as a set of subsidizing contracts. If the firm is free to offer its choice of wage structures, and if the Wilson notion of equilibrium is considered, then certain irreducible combinations of wage-job contracts will obtain. Depending upon the distribution of worker types, wages in these equilibrium wage structures may not correspond to the marginal productivities of individual workers, but the firm breaks even because the wages of high productivity types subsidize low productivity types within the firm. The theoretical framework of our model is based on recent contributions to the theory of self-selection screening.

#### 1. Introduction

■ Recently there has been considerable growth in research addressing problems pertaining to the internal organization of firms. As Spence (1975) indicates, one important approach is to examine the economic conditions under which an internal organization may emerge as an efficient mode of production and organization. This line of inquiry suggests that firms tend to make use of internal resource allocation mechanisms in situations where the market performs poorly. Undoubtedly, replacement of the market by such alternative allocative mechanisms is an appropriate economic paradigm for study of the existence of internal organizations. Care must be taken however, for this paradigm may be a somewhat two-edged sword. In particular, the question arises as to where the process of market replacement will end.

It may be desirable in such analyses, on both theoretical and empirical grounds, to examine the extent to which internal organization displaces the market process. In other words, theory should address itself simultaneously to problems of the mode of economic activity within firms and to the interrelationships between the market and internal organizations (cf. Williamson, 1975, and Stiglitz, 1975). A theory of internal organization cannot be considered fully complete unless it enables us to delimit the boundary of the firm within the

I would like to acknowledge the helpful comments of D. Cass, G. Duguay, S. Goldman, J. Harris and R. Winter. I would like especially to thank G. Akerlof and D. McFadden to whom I owe intellectual debts. I would also like to thank H. Neary, the Editorial Board, and an anonymous referee of this Journal for their many invaluable comments. The responsibility for any shortcomings in this paper remains entirely mine. Financial help extended by the National Science Foundation (NSF-SOC 75-18919, NSF-SOC 75-23076) is gratefully acknowledged.

market. The present paper is an attempt to model an internal organization in the framework of this extended paradigm. In particular, the question we are concerned with may be phrased as follows: Are there simple conditions under which an internal labor market may *emerge* and persist as an efficient mode of organizing production in a *competitive* market environment?

There are three important reasons why the study of labor allocation is relevant and interesting. First, the recent advances in "new institutional economics' indicate that to understand the workings of an internal organization, the domain of inquiry may have to be broadened from output markets alone to include the transactional modes of input, and especially labor, markets (Williamson, 1975). Secondly, organizational theorists emphasize the interaction of "variable" human factors with environmental factors in an attempt to grapple with problems of employment relations. Thirdly, industrial relations economists have long studied "internal labor markets" and have explained the rules and hierarchies that regulate the job ladder largely in terms of legal and sociological arguments; in addition, the new labor economists have reinterpreted internal labor markets in terms of firm-specific human capital and the costs of training and turnover in the skilled labor force (Becker, 1962). We can thus complement and draw lessons from the internal labor market literature in interpreting an internal labor market as an operative and possibly efficient mode of labor contracts.

It should be emphasized that the present paper is designed to be complementary to the existing literature. In particular, the term "internal labor market" usually connotes a quasi-judicial nexus of relations and an institutional richness in modes of industrial-labor management, little of which our model is designed to capture. Nevertheless, the term internal labor market is used to designate our notion of an internal labor allocation mechanism, because this paper can be viewed as an effort to give analytic structure to the elusive statement that there is an economic rationale for the existence of internal labor markets. Our study has been motivated by the distinct world view expressed by Williamson, Wachter, and Harris in their recent article (1975) in which they apply an "organizational failures framework" to the analysis of internal labor markets. In the concluding section, we shall indicate how our model can be seen as complementary to the deeper set of issues discussed by them.

The organization of the paper is as follows. Section 2 gives an overview of the rat race model and the results obtained. Precise specification of the model and a taxonomy of equilibria are presented in Section 3. The characterization of efficient wage structures is spelled out in Section 4. Section 5 interprets the rat race model as a maximization program, by means of which the existence and the uniqueness of equilibrium are proved. Section 6 contains a numerical example of the model which, it is hoped, will elucidate the taxonomy and characterization of equilibria presented in Sections 3 and 4. Finally, Section 7 summarizes and discusses the results obtained and indicates some qualifications to which the model is subject. The proofs of the various propositions and lemmata are available from the author on request.

<sup>&</sup>lt;sup>1</sup> The phrase "variable human factors" was coined by Thompson (1967) to describe the managerial-human environment of an organization.

# 2. The rat race and internal labor markets

■ Before proceeding, we shall sketch our view of an internal labor market. We regard the labor market as a market for labor contracts; a labor contract specifies the wage rate to be paid on a given job. We shall identify a firm as having an *internal labor market* if the efficient and profitable operation of the firm requires that it employ heterogeneous worker types, each to be assigned to a different job. In other words, an internal labor market firm finds it necessary to provide a *set* of distinct wage-job contracts, on which it can at least break even. Our characterization falls short of the usual view of an internal labor market, which deals primarily with the hierarchical relations that regulate a job ladder. This reflects our main interest, which is to investigate requisite market conditions for the emergence of such a hierarchical structure.

The theoretical framework of our model has borrowed heavily from recent contributions to the theory of screening. In particular, we closely follow Akerlof's seminal model (1973, 1976) of a *rat race* on assembly lines, and exploit its *self-selection* properties within the framework of Rothschild and Stiglitz' (1975, 1976) analysis of insurance markets.<sup>2</sup> Hereafter, we shall use the phrase "rat race" to describe the self-selection screening process which occurs in the labor market. The demonstration and characterization of equilibrium will be carried out by means of a graphical exposition similar to that employed by Rothschild and Stiglitz.

We shall assume that the economy consists of a large number of firms and workers. The firm tries to maximize profits by designing efficient labor contracts; the worker searches for the firm which offers a utility maximizing labor contract. We also assume that the labor force is heterogeneous in worker types, but that all firms are technologically identical. It is further assumed that the technology exhibits constant returns to scale, with additive separability in inputs of the different worker types. Under this assumption the role of technology on the mode of organization becomes less than deterministic. In particular, it enables us to rule out the case of purely technological nonseparability, upon which Alchian and Demsetz (1972) rely to explain the rationale of internal labor markets.<sup>3</sup> Instead, an informational constraint on market performance enters our model in the form of uncertainty about given individual workers.

Let us start unfolding our story of the rat race. Employers may well possess a sufficient statistic on the population distribution of all existing worker types. Yet the individual worker's traits and ability are not directly observable. Employers are most likely to be ignorant about the potential productivity of workers who have just entered the labor force. Workers, knowing themselves, or at least knowing more about themselves than does the employer, have an incentive to pretend to be better than they in fact are. Because of this asymmetry of information, employers seek to devise mechanisms to screen the "good" workers from the "bad."

The assumption of asymmetric information seems appropriate in

<sup>&</sup>lt;sup>2</sup> The logical equivalence of Akerlof's rat race and the Rothschild and Stiglitz self-selection model of insurance markets is recognized in Rothschild and Stiglitz (1975–1976). Williamson (1975) notes that the insurance problem is an interesting paradigm for studying the employment relation.

<sup>&</sup>lt;sup>3</sup> Alchian and Demsetz (1972) present a thesis that nonseparability in the production function can account for the existence of an internal allocation mechanism.

an industrial economy where workers earn wages by working for someone else. Furthermore, it is often impractical, perhaps due to the high costs involved, to monitor each individual's performance; instead, supervision and control are generally based on the average productivity of the worker group assigned to a given job. Thus, the idea is that wage bargaining is done with respect to jobs, not individuals, and that there is a collective consumption dimension to most jobs. The rat race model is especially useful to our purpose, since it provides a plausible way of connecting the informational asymmetry faced by the firm to the presence of wage-job competition in the labor market.

If firms are competitive both in the labor market and in the output market, then, in the long run at least, the average wage rate each firm pays must be equal to the average marginal productivity of its entire work force. If workers cannot be separated according to their ability and attitudes towards work, then each worker receives a wage rate exactly equal to the average marginal productivity of the entire work force. High productivity workers lose and low productivity workers gain. Thus, there is an obvious incentive for the higher productivity workers to have themselves identified. This is how the self-selection mechanism is put into motion.

It is assumed that workers' productivity differences can be distinguished on the basis of their differential preferences for varying speeds on the assembly line (i. e., for varying jobs); high productivity workers are more willing to accept a faster work speed than lower productivity workers. Employers exploit this self-selection incentive and sort out worker types by offering them different wage-job contracts. However, unless the firm's contracts are efficiently designed, mismatches of jobs and workers will result. Such mismatches may cause production inefficiency, which could be remedied by superior design of wage-job combinations. Worse, the firm may not break even if it loses good workers to rival firms that are competitively offering more attractive labor contracts. Thus, the firm will be eager to offer contracts which are, on the one hand, efficient in their operation as screening devices to exploit the workers' self-selection incentives, and which are, on the other hand, attractive enough to ensure the employer a viable supply of labor. Competition among firms will ensure elimination of the less efficient wage-job contracts. This completes a competitive loop which links the competitive (external) labor market and the internal "race" of self-selection.

The main results of the Akerlof-Rothschild-Stiglitz [hereafter A-R-S] rat race analysis are simply stated. First, a rat race equilibrium may not exist in the model. However, and secondly, if an equilibrium does exist, then workers will be paid their marginal products, but not in the Pareto-optimal wage-job configurations which would prevail under full information. We now discuss these results in the light of two key assumptions which underlie the A-R-S model, and with which we take issue; these assumptions concern, on the one hand the Nash notion of equilibrium, and on the other, the single-contract restriction, under which each firm is permitted to offer only one wage-job contract.

The nonexistence of a rat race equilibrium arises mainly because of the Nash equilibrium concept. A set of wage-job contracts is called a Nash equilibrium set if each firm at least breaks even on that set and if there exists no other contract which, if offered, would make a nonnegative profit. A Nash firm is therefore myopic in the sense that it assumes that the set of contracts offered by other firms is independent of its own actions. However, even under the single-contract restriction, it has been demonstrated that the model always has an equilibrium if an alternative concept of equilibrium is applied.<sup>4</sup>

The alternative notion, which we adopt below, is that of a Wilson equilibrium. This notion implies a greater degree of nonmyopic rationality on the part of the firm than is involved in the Nash equilibrium concept in the following sense. When a new contract is introduced, it may make some of the existing contracts no longer profitable. The firm offering the new contract anticipates that these unprofitable contracts will be simply withdrawn or eliminated from the market. A Wilson firm is just far-sighted enough to recalculate correctly the effect which such elimination has, in turn, on the profitability of the new contract. The Wilson equilibrium then requires that there be no new contract which would remain profitable after the elimination of all contracts rendered unprofitable. Thus, unlike the Nash notion, the Wilson notion assumes that an individual firm, even in a competitive environment, takes into account certain effects of its own action upon the actions of the other firms.<sup>5</sup>

The nonoptimality of the equilibrium wage-job configurations is generally interpreted as the result of a negative informational externality in which the high productivity workers lose by the presence of lower productivity workers. Despite this, the Nash equilibrium, if it exists, will entail a wage rate exactly equal to the average of the marginal productivity of a worker group assigned to a given job. Hence, at least in theory, each wage-job contract, breaking even on its own, can be offered by different firms; a firm can exist simply by creating one job position and paying its marginal productivity wage. Now it is clear that the single-contract restriction and the marginal productivity wages are mutually enforcing. The fact is that, within the context of the rat race, marginal productivity wages make the notion of internal labor markets quite superfluous.

The existing A-R-S rat race model falls short of capturing our notion of an internal labor market, and it fails to identify fully certain inefficiencies of the neoclassical market which might be attenuated by an internal mode of allocation. We remedy this situation by explicitly allowing the firm to offer a set of multiple wage-job contracts, i. e., a wage structure. Competition in terms of wage structures captures the essence of the internal labor market literature wherein wages are attached to jobs and not to individuals. Under this condition, it will be shown that an internal labor market can prevail, paying nonmarginal productivity wages. The equilibrium will always be guaranteed to exist under the Wilson equilibrium concept, even when the firms offer more than one wage-job contract.

We thus modify the original A-R-S rat race model by combining two notions: 6 the Wilson concept of equilibrium and the idea that a

<sup>&</sup>lt;sup>4</sup> See, for instance, Riley's (1976) "reactive" equilibrium.

<sup>&</sup>lt;sup>5</sup> See Rothschild and Stiglitz (1975, 1976) and Wilson (1973, 1976) for further discussion on the notion of Wilson equilibrium.

<sup>&</sup>lt;sup>6</sup> Rothschild and Stiglitz (1975) discuss these two notions. It is our innovation to combine them. Wilson (1976) also mentions the possibility of such a combination in the analysis of insurance markets.

firm offers a wage structure rather than a single contract. Despite these modifications, the model retains the essential features of a competitive labor market. Firms are a little more far-sighted than under the Nash assumption, but there is no explicit collusive behavior among them, and free entry and exit are permitted. The point is that for the purpose of analyzing labor markets, the rat race framework permits competition with respect to wages as well as competition among jobs, and this is one interpretation of what Rothschild and Stiglitz calls "price-quantity" competition. This aspect of the labor market is further clarified by the notion of a wage structure, which allows of the interpretation that firms may partially internalize the market inefficiency caused by the informational asymmetry.

Our main result is the following. If the firm is free to offer its choice of wage structure, and if the Wilson notion of equilibrium is considered, then certain irreducible combinations of wage-job contracts will obtain. Depending upon the distribution of worker types. wages in these equilibrium wage structures may not correspond to the marginal productivities of individual workers, but the firms break even because some workers' wages cross subsidize others. In fact the high productivity types subsidize the low productivity types within the firm. The firm is stuck offering all of the wage-job contracts as its internal wage structure, because if it withdrew the losing contracts, then the released low productivity workers would just come back and take over the jobs designed for high productivity types. The idea of Wilson equilibrium is that the firm takes this into account and keeps the low productivity losers on the payroll. This is our notion of an internal labor market equilibrium. In this equilibrium the informational externality is still present, but its effect is partially attenuated by the internal wage structure. The modified rat race model captures the efficiency of an internal allocation that could not be replaced by neoclassical mechanisms, and demonstrates the existence of nonmarginal productivity wages in internal labor markets.

■ We make an extreme abstraction by assuming that all firms are identical, producing a homogeneous output, and that human capital is the only capital necessary for production. In this way we incorporate the new labor theorists' emphasis on heterogeneous labor and human capital aspects. For simplicity there are only two types of workers  $L_i$ , i = 1, 2. These two types of workers differ in their ability, measured in terms of productivity, and their attitude towards work. In addition, we shall consider a steady-state flow of the labor force in which workers are entering and retiring from the labor force at a constant rate, thereby keeping a stationary population distribution of worker types. Type 2 workers are more productive than type 1 workers. Let  $\lambda$  be the proportion of type 1 workers and 1 -  $\lambda$  be that of type 2 workers. Throughout the analysis  $\lambda$  will be taken as a fixed exogenous datum. The population distribution of the worker types is known to all, and every worker knows to which type he belongs. However, there is no *direct* way for potential employers to verify the type of any given individual, and thus the market contains an informational asymmetry.

# 3. A model of the internal labor market

 $\square$  **Production function.** We assume that there is a numerical index s of jobs, where s measures the work-effort intensity of a given job. The larger is the value of s, the more demanding is the task. The firm's production function f exhibits a constant returns to scale technology with linear additivity in  $L_1$  and  $L_2$  for a given  $(s_1, s_2)$ :

$$f(L_1, L_2; s_1, s_2) = f_1(L_1; s_1) + f_2(L_2; s_2),$$

where  $f_i(L_i; s_i)$  is the functional form of a linear activity in  $L_i$ , given  $s_i$ . Then, we can rewrite  $f_i(L_i; s_i)$ , with some notational abuse, as

$$L_i f_i(1; s_i) \equiv L_i f_i(s_i),$$

where  $f_i$  is assumed to be strictly concave and increasing in  $s_i$ . Note that  $f_i(s)$  is  $MPL_i(s)$ , the marginal productivity of the *i*th type on a given job s.

Since  $f'_i(s)$  is the output contributed by an incremental increase in the effort of the *i*th type, we stipulate in accordance with the type 1-type 2 distinction that

$$f'_{2}(s) > f'_{1}(s)$$

for all  $s \ge 0$  with the boundary condition that

$$f_2(0) \ge f_1(0)$$
.

These two conditions imply

$$MPL_2(s) = f_2(s) > f_1(s) = MPL_1(s)$$

for all s > 0. These MPL functions reflect the idea that (almost) everyone can learn how to do a given job, but that some people will be good at it while others are not so good.

In what follows, the only average productivity relevant to our analysis is the average productivity when both types take the same job s in the ratio  $\lambda/1 - \lambda$ . We shall denote this quantity by APL(s). Because of linear additivity of the production function,

$$APL(s) = \lambda MPL_1(s) + (1 - \lambda)MPL_2(s).$$

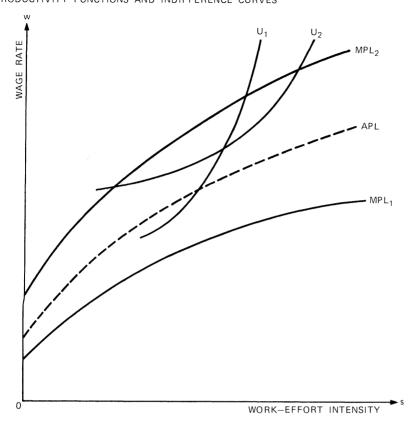
See Figure 1 for possible shapes and positions of the MPL and APL curves.

 $\Box$  Utility function. The worker's attitude toward jobs is summarized by a strictly concave utility function  $U_i$  whose arguments are  $(w_i, s_i)$ .  $w_i$  is the real wage paid to the *i*th type, and  $s_i$  is our measure of the effort required to execute the job assigned to the *i*th type. In accordance with the standard theory of utility, we specify that

$$\frac{\partial U_i}{\partial w_i} > 0$$
 and  $\frac{\partial U_i}{\partial s_i} < 0$ .

Following the existing literature on self-selection and screening, we assume that there is a correlation between ability and attitude. In our model it is assumed that the high productivity workers are more willing to trade wages w for more demanding jobs s.<sup>7</sup> This assumption can be expressed in terms of MRS, the marginal rate of substitution between w and s. That is,

<sup>&</sup>lt;sup>7</sup> In addition to Akerlof (1973, 1976), see Spence (1973) for a model of screening/signalling in education.



$$MRS_1(w, s) > MRS_2(w, s)$$

for all 
$$(w, s)$$
 where  $MRS_i(w, s) = \frac{\partial w_i}{\partial s_i} |_{dU_i = 0}$ .

See Figure 1 for the relative position of two indifference curves which show this *MRS* condition. An intended interpretation is that willingness to meet the challenge of an intensive assignment is an important trait, and it is this willingness that makes a good worker. This correlation between the shapes of utility functions and productivity curves is what pushes the self-selection mechanism in the desired direction. The firm can now offer an increasing sequence of wage-job combinations and screen out the less productive workers.

□ Wage structures. We denote a labor contract C by a vector (w, s) which specifies a payment w if the worker takes the job s.  $C_i = (w_i, s_i)$  refers to a contract taken by the *i*th worker type. We shall frequently use the term wage structure to denote a set of labor contracts offered by a single firm; a firm is then identified by its wage structure. Since there are only two types of workers, the firm offers at most a pair of contracts, a doubleton wage structure, at a given time. Because of the constant returns to scale technology, a measure of profitability is the profit rate defined as the average profit per employee. The profit rate

of a firm when it employs  $L_1$  under  $C_1$  and  $L_2$  under  $C_2$  is given by

$$\frac{L_1}{L_1 + L_2} \left[ MPL_1(s_1) - w_1 \right] + \frac{L_2}{L_1 + L_2} \left[ MPL_2(s_2) - w_2 \right].$$

We say that a wage structure is *feasible* if it can earn a nonnegative profit rate.8

We use the bracket  $[\ ]$  to denote a firm's wage structure. Thus,  $[C_1, C_2]$  means that a pair of contracts was offered by a single firm. Parentheses (), on the other hand, will denote a set of wage structures available in the economy. Thus, for example,  $([C_1], [C_2])$  means that the economy consists of some firms employing only the type 1 workers under  $[C_1]$  and others specializing in type 2 workers under  $[C_2]$ . Similarly,  $([C_1, C_2])$  means that all firms in the economy are adopting the wage structure  $[C_1, C_2]$ . A central point of our analysis is to investigate whether or not the economy  $([C_1, C_2])$  can be considered equivalent to  $([C_1], [C_2])$  under the Wilson notion of competition. Finally, we shall sometimes use  $(C_1, C_2)$  just to denote a set of contracts available in the economy without specifying firms' wage structures.

Following Rothschild and Stiglitz, we distinguish between separating and pooling sets of contracts. A pair of contracts  $(C_1, C_2)$  is said to be *separating* if it is *possible* that the type 1 workers can be induced to choose only  $C_1$  while the type 2 workers take  $C_2$ . In terms of vector notation,  $(C_1, C_2)$  is separating if

$$\begin{pmatrix} U_1(C_1) \\ U_2(C_2) \end{pmatrix} > \begin{pmatrix} U_1(C_2) \\ U_2(C_1) \end{pmatrix}.$$

We shall call a contract, *C*, *pooling* if it attracts both types of workers. Wage structures are defined as separating or pooling in analogous fashion.

We define a minimal wage structure as a feasible wage structure that contains the fewest number of contracts. If a minimal wage structure is a doubleton set  $[C_1, C_2]$ , then we identify the firm as having an internal labor market. The firm is called a neoclassical firm if its minimal wage structure is a singleton set.<sup>10</sup>

This set of definitions is motivated by the notion that an internal labor market should connote a hierarchical structure with at least a distinct two-step job ladder. Williamson, Wachter, and Harris (1975) contend that observed job ladders often have efficiency enhancing properties as compared with the unassisted (what I shall refer to as neoclassical) market process. We may say that the neoclassical concept of markets is closely related to a hypothetical economy in which each commodity (i. e., each labor contract) can be marketed independently and separately by different profit-maximizing firms. This particular aspect of the neoclassical economy is expressed by those feasible modes of production which entail ( $[C_1], [C_2]$ ). However, if the

<sup>&</sup>lt;sup>8</sup> Obviously, feasibility depends on the set of all contracts offered by other firms, and on the assumed foresight of the firm. The notion of Wilson-feasibility will be discussed shortly.

<sup>&</sup>lt;sup>9</sup> Vector inequalities are defined as follows: For any two vectors of the same dimension, (i) x > y, if  $x_i \ge y_i$  for all i, but  $x_i > y_i$  for some i, (ii)  $x \ge y$ , if  $x_i > y_i$  for all i, and (iii) x = y, if  $x_i = y_i$  for all i.

<sup>&</sup>lt;sup>10</sup> Those who are troubled with this language may refer to these firms as doubleton and singleton firms, respectively.

firm must maintain  $[C_1, C_2]$  to at least break even, and if it cannot be induced to operate with either only  $[C_1]$  or  $[C_2]$ , then such a firm must employ both worker types and internally allocate them over two different jobs according to the minimal doubleton wage structure  $[C_1, C_2]$ . The above definition is designed to express the notion of an internal wage structure, where each of the component contracts cannot be offered separately and profitably by different firms in the labor market. A firm with an internal wage structure is an internal labor market firm.

□ Equilibrium defined. The following story is suggestive of the nature of an equilibrium notion relevant to our model. Since we have been identifying the firm by its wage structure, we can view a withdrawal of the wage structure as the firm's exit from the market, and similarly, the introduction of a new wage structure as entry by a new firm. Thus, the situation under consideration is that of free entry and exit. Firms will continue to enter and exit until all the existing wage structures earn zero profits and there exists no new wage structure that would make nonnegative profits.

Imagine that a firm enters the labor market with a wage structure which bids away workers from other firms. A loss of workers, especially a reduction in the proportion of high productivity workers within the firm, will lead to a decline in profit rate. Outcompeted by the new firm, some firms will exit the market, dropping their old wage structures in the process. Under the assumptions of our model, 11 this exit of firms can result in adverse repercussions for the profitability of the new firm. First, this firm may absorb the low productivity workers discharged by the losing firms. Second, it may also catch low productivity workers who have just entered the labor force and who would otherwise have joined the exiting firms. Soon, then, such a new firm may obtain a composition of worker types that is quite different from that for which its wage structure was originally designed. Consequently, this maverick firm may not in fact break even.

The foregoing story can be paraphrased under the Wilson notion of equilibrium for the ex ante market in wage structures. Before a firm actually offers a new wage structure, it takes pause to contemplate the effect this will have on the existing set of wage structures. Specifically, the firm foresees which subsets of the existing wage structures will become unprofitable as the result of its new introduction. The firm thus takes into account the fact that it may have to absorb the low productivity workers being discharged by the losing wage structures. The firm will calculate whether such absorption will render its own proposed wage structure unprofitable. In this fashion the firm recalculates what its profitability will be after all the unprofitable wage structures have been withdrawn by other firms. The new wage structure will be offered if and only if it still earns nonnegative profit after all unprofitable wage structures are eliminated from the market. The set of existing wage structures is called a Wilson equilibrium set if each firm earns zero profits and if there exists no new wage structure which would make nonnegative profits after the elimination of all the existing wage structures thereby rendered unprofitable.

<sup>&</sup>lt;sup>11</sup> Recall our assumption that there is a steady flow of workers joining and leaving the labor force and that the firm's production function is constant returns to scale.

It will be shown later that an equilibrium results in a situation wherein all firms adopt the same wage structures. Thus, we can outline salient characteristics of three distinct types of equilibrium: neoclassical separating, internal labor market, and pooling. If an equilibrium entails wage structures that are neoclassical and separating, then it will be called a neoclassical separating equilibrium. Other equilibria are similarly defined.

Suppose that the firm's equilibrium wage structure  $[C_1, C_2]$  is separating in such a way that it breaks even on each worker type, i. e.,  $w_i = MPL_i(s_i)$ , i = 1, 2. Then, at least in theory, such a firm is decomposable into two independent firms; one firm would specialize in employing only type 1 workers with  $[C_1]$ , and the other would exclusively employ type 2 workers with  $[C_2]$ . Each new firm still earns zero profit and no workers are worse off. In other words, there is no particular advantage in establishing a wage structure with two distinct wage-job contracts. Since the minimal wage structure in this case is a singleton  $[C_i]$ , the firm is neoclassical. A necessary and sufficient condition for a neoclassical separating equilibrium is that each type of worker receive a differential wage equal to his own marginal productivity.

Opposed to the neoclassical separating case is the situation wherein the firm employs both worker types under a separating wage structure  $[C_1, C_2]$ , but gains on one contract and loses on the other. Now it may be that if the firm withdraws the losing contract, then the remaining profitable contract would become unprofitable as the firm is deluged by the discharged workers. If such is a consequence perceived by the Wilson firm, then there will be an equilibrium such that two subsidizing contracts constitute an irreducible wage structure on which a firm at least breaks even. The cross subsidization among contracts means that some workers receive wages above their own marginal productivity, while others receive wages below it. Such will be the case for an internal labor market equilibrium. A brief argument in the next section will demonstrate that the higher productivity worker must subsidize the lower productivity worker within the firm's internal labor market.

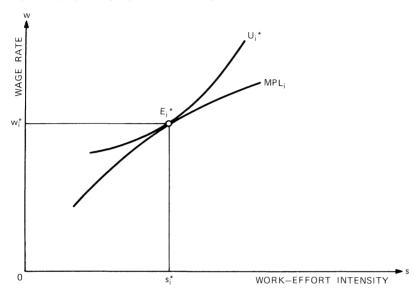
The third type of equilibrium entails a pooling contract in which the firm employs both types of workers, but assigns them to the same job. The firm treats all workers equally and breaks even by paying a wage rate equal to the weighted average of the high and low marginal productivities. Thus, pooling can be considered as one particular instance of cross subsidization among the worker types. However, we would expect that pooling is an inefficient form of subsidization; in general, there is a separating wage structure that will be more efficient than a given pooling contract. For this reason the pooling equilibrium is an unlikely occurrence. In fact, we shall later demonstrate in Section 5 that the equilibrium never entails a pooling contract under a specific assumption we already have in the model.

To be fully emphatic, the neoclassical equilibrium is attainable by a process in which all firms are allowed to offer only singleton wage structures. Feasible neoclassical wage structures are in this sense isomorphic to our notion of the neoclassical mechanism wherein each contract is marketable by separate firms. For this reason, if an internal labor market exists in equilibrium, then it must be superior in efficiency to the neoclassical market mechanism itself.

■ As a standard of comparison, it is useful to characterize the equilibrium which obtains under the regime of full information, wherein any given individual's worker type is known to all. In the absence of informational asymmetry, competitive equilibrium entails the neoclassical separating set ( $[E^*_1]$ ,  $[E^*_2]$ ), which is depicted in Figure 2. At  $E^*_i$ , the *i*th worker type's indifference curve  $U^*_i$  is

4. Competitive wage structures

FIGURE 2
THE OPTIMAL CONTRACT OF THE "i"TH WORKER TYPE



tangent to its own  $MPL_i$  curve; the worker of the *i*th type receives his own marginal productivity wage  $w^*_i$  on the job  $s^*_i$ . Hereafter we shall call  $E^*_i$  the *i*th worker type's *optimal contract*, and ( $[E^*_1]$ ,  $[E^*_2]$ ) the *full information equilibrium*.

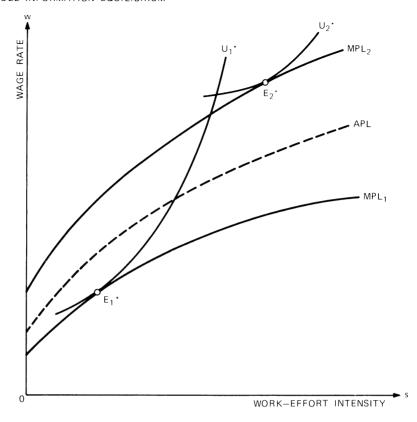
Returning to the case of asymmetric information, we may first point out that there exists a situation in which lack of information about the worker types may have no effect on the market's performance in achieving the optimal outcome.

*Proposition 1:* A Wilson equilibrium coincides with the full information equilibrium ( $[E^*_1]$ ,  $[E^*_2]$ ) if and only if  $U_1(E^*_1) \ge U_1(E^*_2)$ .

Such a case is depicted in Figure 3, in which Wilson competition leads to a trivial neoclassical separating equilibrium, i. e.,  $([E^*_1], [E^*_2])$ .

More interesting, and perhaps also more relevant, is the case in which the Wilson equilibrium never results in the optimal contracts. This case is shown in Figure 4, and corresponds to a situation where  $U_1(E^*_2) > U_1(E^*_1)$ . If  $(E^*_1, E^*_2)$  is considered as a candidate for equilibrium, then  $E^*_2$  will always be pooling because  $U_i(E^*_2) > U_i(E^*_1)$  for i = 1, 2. As a result,  $APL(s^*_2)$  at  $E^*_2$  falls below  $MPL_2(s^*_2)$  which is equal to  $w^*_2$ . Consequently, the firm offering  $E^*_2$  loses, be it  $[E^*_2]$  or  $[E^*_1, E^*_2]$ . Hence  $(E^*_1, E^*_2)$  cannot be an equilibrium.

In the remainder of this section, we shall focus mainly on this latter case, where the informational asymmetry influences the outcome. First, we make the following observation. Competition among firms guarantees that the lowest productivity worker obtains at least



 $U_1(E^*_1)$  in equilibrium regardless of the presence or otherwise of an informational asymmetry. In Figure 4, an equilibrium contract  $C_1$  for the type 1 worker must lie above the  $U^*_1$  indifference curve. Note that any such contract specifies a wage-job combination  $(w_1, s_1)$  such that

$$w_1 \geq MPL_1(s_1),$$

with the equality holding only at  $E_1^*$ . Since a wage structure must be feasible, this implies that an equilibrium contract  $C_2$  for the type 2 worker must lie below the  $MPL_2$  curve, so that

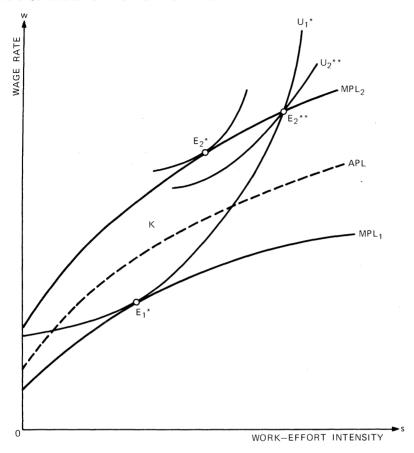
$$w_2 \leq MPL_2(s_2)$$
.

These two inequalities indicate that, in a subsidizing equilibrium, the type 2 worker loses by  $MPL_2(s_2) - w_2$ , whereas the type 1 worker gains by  $w_1 - MPL_1(s_1)$ . Hence, we establish:

Proposition 2: In an internal labor market equilibrium, the higher productivity worker subsidizes the lower productivity worker.

Furthermore, we can assert that an equilibrium contract  $C_2$  for the type 2 worker must satisfy  $U_2(C_2) \ge U_2(E^{**}_2)$ , given that  $U_1(C_1) \ge$ 

<sup>&</sup>lt;sup>12</sup> If the equilibrium contract  $C^*_1$  provides  $U_1(C^*_1) < U_1(E_1)$ , then a new firm can enter to offer  $[C_1]$  such that  $U_1(C_1) > U_1(C^*_1)$  and  $MPL_1(s_1) > w_1$ . The pooling at  $C_1$  is no problem, for it can only raise the productivity at  $s_1$  to  $APL(s_1)$ . Thus, the firm with  $[C_1]$  can make strictly positive profits.



 $U_1(E^*_1)$ .<sup>13</sup> In Figure 4, this benchmark contract  $E^{**}_2$  is located at the intersection of the  $MPL_2$  and  $U^*_1$  curves, and the type 2 worker's indifference curve passing through  $E^{**}_2$  is labelled  $U^{**}_2$ .

We may succinctly summarize the above observations in terms of the area K in Figure 4; an equilibrium contract  $C_i$  for the ith worker type must belong to K, bounded by the concave  $MPL_2$ , convex  $U^*_1$ , and the positive quadrant. Then, an equilibrium pair of contracts, say  $(C_1, C_2)$ , can be considered as a point in  $K \times K$ . It will be useful later for the analysis of equilibrium to remember that  $K \times K$  is compact.

We now present a simple but useful fact.

Proposition 3: Under  $U_1(E^*_2) > U_1(E^*_1)$ , the equilibrium is a neoclassical separating equilibrium if and only if it entails  $([E^*_1], [E^{**}_2])$ .

<sup>&</sup>lt;sup>13</sup> The assertion is trivial if  $U_1(E^*_1) > U_1(E^*_2)$ . We thus consider the case where  $U_1(E^*_2) \ge U_1(E^*_1)$ . Let  $[C_2]$  be a singleton wage structure such that  $U_1(E^*_1) > U_1(C_2)$  and  $U_2(C_2) > U_2(E^*_1)$ . Then there will be a feasible singleton wage structure  $[C'_2]$  which dominates  $[C_1, C_2]$  under the proviso that  $U_1(C_1) \ge U_1(E^*_1)$ . Entry by more dominant singleton wage structures such as  $[C'_2]$  will continue at least until  $E^{**}_2$  is provided.

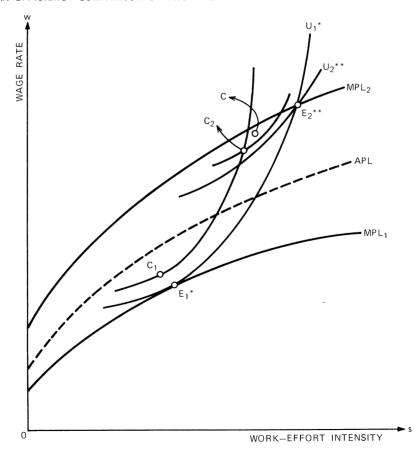
<sup>&</sup>lt;sup>14</sup> To be more precise, an equilibrium  $C_2$  belongs to the convex set  $K_2$  which is bounded by  $MPL_2$ ,  $U^{**}_2$  and the positive quadrant. Since  $K_2$  is a subset of K, it suffices to refer only to K for the purpose of equilibrium analysis.

It is worth noting that this neoclassical separating equilibrium coincides with the Nash equilibrium obtained in the original A-R-S model. The above proposition also tells us that the type 2 worker can never attain its own optimal contract  $E^*_2$  when the informational asymmetry is nonnegligible. It can be said that the lower productivity workers impose an externality on the higher productivity workers, and thus drive the higher productivity workers out of their optimum position when quality uncertainty is present. It is the goal of our analysis to show that an internal labor market could make a Pareto-improvement to this neoclassical outcome ( $[E^*_1]$ ,  $[E^{**}_2]$ ).

□ Efficient wage structures. Determination of the equilibrium wage structure implies a comparison among alternative wage structures, and the consequent need for a criterion whereby the structures can be ordered in a manner appropriate to Wilson competition. Since information is asymmetric in the model, any such ranking criterion can be expected to diverge from the standard Paretian criterion. For example, as in Figure 5, suppose that all firms are providing the internal wage structure  $[C_1, C_2]$ . Then, let a singleton wage structure [C] such that  $U_2(C) > U_2(C_2)$  be introduced. Since C deprives  $C_2$  of its type 2 workers, the  $[C_1, C_2]$  firm retains only type 1 workers. Because  $MPL_1(s_1) < w_1$  at  $C_1$ , this  $[C_1, C_2]$  wage structure loses. As it will be

FIGURE 5

AN EFFICIENCY COMPARISON OF TWO WAGE STRUCTURES



dropped from the market, the type 1 workers are discharged and will join the type 2 workers at C. Consequently, the pooled average productivity at C will fall below the promised wage rate w of C. C eventually loses. Hence, the firm gifted with Wilson foresight will not introduce [C] as against  $[C_1, C_2]$ .

This type of comparison test can be made on any two wage structures. Intuitively, from the viewpoint of a profit maximizing firm. efficiency of a wage structure requires that it be both competitive and feasible under Wilson foresight. We say that  $[C_1, C_2]$  dominates  $[C'_1, C_2]$  $(C'_2]$  if  $U_2(C_2) > U_2(C'_2)$ . This definition of Wilson-dominance implies that  $[C_1, C_2]$  can render  $[C'_1, C'_2]$  unprofitable. If  $U_2(C_2) > U_2(C'_2)$ , then all type 2 workers will reject  $C'_2$ , leaving  $[C'_1, C'_2]$  riddled with the type 1 workers. Unless  $[C'_1, C'_2]$  constitutes a neoclassical separating pair, it will now make negative profits. A firm under Wilson foresight will not, however, offer one wage structure  $[C_1, C_2]$ against another  $[C'_1, C'_2]$ , unless the former at least breaks even after its introduction has possibly rendered  $[C'_1, C'_2]$  unprofitable. If this condition holds, then we say that  $[C_1, C_2]$  is Wilson-feasible relative to  $[C'_1, C'_2]$ . These two key notions, dominance and feasibility, can be used to determine a profit maximizing wage structure in our competitive environment. This leads us to the following definition.

Definition: A wage structure  $[C_1, C_2]$  is said to be *efficient* if it is feasible and if there exists no other wage structure that is both dominant and Wilson-feasible vis- $\dot{a}$ -vis  $[C_1, C_2]$ .

Under the informational asymmetry, an efficient mode of production sustainable by competitive profit maximization must embody an effective means of screening. For this reason, a Pareto-superior wage structure<sup>15</sup> that is marketable under full information may no longer be feasible when the informational externality is present. Competition will ensure that firms can gain a viable work force only by offering contracts which are sufficient to attract type 2 workers. An efficient wage structure is one which maximizes  $U_2(C_2)$  subject to well-defined constraints. Whenever a wage structure  $[C_1, C_2]$  makes strictly positive profits, there will be another separating wage structure  $[C_1, C_2]$  that is Wilson-feasible and for which  $U_2(C_2) > U_2(C_2)$ . In equilibrium, all firms adopt only efficient wage structures, earning zero profit.

Under the Wilson concept of competition, the market offers a pair of contracts  $(C_1, C_2)$ , and behaves as if it maximizes the utility of higher productivity workers. To make this implicit maximization tractable, it will be mathematically convenient to treat all firms as if they offer only doubleton wage structures  $[C_1, C_2]$  as vectors in  $R_+^2 \times R_+^2$ . Of course, it is important to keep in mind that these doubleton wage structures all differ in economic features, because some of them are neoclassical and others internal. In other words,  $(C_1, C_2)$  will be treated mathematically as  $[C_1, C_2]$  even though the appropriate economic interpretation could have been  $[C_1]$  and  $[C_2]$ . In the same vein, a pooling wage structure [C] can be represented as [C, C]. The

# 5. Equilibrium analysis

<sup>15</sup> [
$$C_1$$
,  $C_2$ ] is Pareto-superior to [ $C'_1$ ,  $C'_2$ ] if  $\begin{pmatrix} U_1(C_1) \\ U_2(C_2) \end{pmatrix} > \begin{pmatrix} U_1(C'_1) \\ U_2(C'_2) \end{pmatrix}$ .

underlying maximization program will be useful in establishing the existence of Wilson equilibrium. However, its primary function is to characterize fully the properties of efficient wage structures.

We introduce the following lemma to define the constrained maximization program.

Lemma 1: Let  $[C_1, C_2]$  be an efficient and separating wage structure that belongs to  $K \times K$ . Then,  $U_1(C_1) = U_1(C_2)$  and  $U_2(C_2) > U_2(C_1)$ .

Next, we define the average per worker profit,  $\pi$ , attributed to  $[C_1, C_2]$  as

$$\pi(C_1, C_2) = \lambda [MPL_1(s_1) - w_1] + (1 - \lambda)[MPL_2(s_2) - w_2].$$

Lemma 2 below indicates that this definition of the profit rate is innocuous *vis-à-vis* the outcome of our equilibrium analysis.

Lemma 2: In equilibrium, each efficient wage structure  $[C_1, C_2]$  attracts type 1 workers  $(L_1)$  and type 2 workers  $(L_2)$  in the ratio  $L_1/L_2 \le \lambda/1 - \lambda$ .

The feasibility constraint for a wage structure can then be written as  $\pi(C_1, C_2) \ge 0$ .

The preceding lemmata and discussion on competitive wage structures imply the following proposition.

Proposition 4: If  $[C^*_1, C^*_2]$  is an efficient wage structure, then it is a solution to the programming problem:

$$\underset{C_1, C_2}{\text{Max}} U_2(C_2) \tag{P}$$

subject to

$$U_1(C_1) \ge U_1(E^*_1) \tag{1}$$

$$U_1(C_1) \ge U_1(C_2) \tag{2}$$

$$\pi(C_1, C_2) \ge 0. \tag{3}$$

$$C_1 \ge 0, C_2 \ge 0. \tag{4}$$

The constraints (1) and (3) say that an efficient wage structure must contain  $C_1$  in K because competition guarantees  $E^*_1$  to the type 1 worker. The constraint (2) is indispensable in capturing the process leading to a Wilson-efficient wage structure. Finally, without loss of generality, the constraint (3) can be replaced by the equality constraint  $\pi(C_1, C_2) = 0.16$ 

 $\square$  Existence and uniqueness of equilibrium. Let us inspect the constraint set M defined by conditions (1)-(4) of the maximization program. M is nonempty because  $[E^*_1, E^{**}_2]$  is a member. M is closed, but unbounded. However, the nature of the constraints and the logic of efficient wage structures indicate that  $U_2$  is to be maximized either by  $[E^*_1, E^*_2]$  or by some other wage structure that belongs to  $K \times K$ . Without loss of generality, we can redefine the maximization program on the virtual constraint set  $M^*$  defined by

$$M^* = [M \cap (K \times K)] \cup \{[E^*_1, E^*_2]\}.$$

<sup>&</sup>lt;sup>16</sup> I am grateful to A. M. Spence for pointing out the correct specifications of this maximization problem.

<sup>&</sup>lt;sup>17</sup> M includes  $K \times K$  and the unbounded set  $\{(E^*_1, C_2)|C_2 = (w_2, s_2) \text{ and } w_2 = MPL_2(s_2)\}$ .

Since  $K \times K$  is compact, so is  $M^*$ . Since the objective function  $U_2$  is continuous, the maximization program has a solution.

Such a solution would indeed constitute an efficient wage structure in a Wilson equilibrium if the maximization program has a unique solution. However, neither M nor  $M^*$  need be convex. 18 and the maximization program may have multiple solutions, some of which are not obtainable as an outcome of Wilson competition. To illustrate, suppose that solutions of the maximization program are  $[C_1, C_2]$  and  $[C'_1, C'_2]$ , such that  $U_2(C_2) = U_2(C'_2)$  but  $U_1(C_1) < U_1(C'_1)$ . This means that all type 1 workers will choose  $C_1$  over  $C_1$ , while the type 2 workers are indifferent between  $C_2$  and  $C'_2$ . Under normal circumstances, the type 2 workers will be distributed, if not evenly, at least in such a way that some will be at  $C_2$  while others are at  $C_2$ . Thus,  $[C'_1, C'_2]$  will have an internal ratio of type 1/type 2 much above the population ratio  $\lambda/1 - \lambda$ . Consequently, when  $[C'_1, C'_2]$  is designed to break even on the  $\lambda/1 - \lambda$  ratio, it will now lose.<sup>19</sup> This and other similar considerations suggest that  $[C'_1, C'_2]$  cannot remain feasible vis-à-vis  $[C_1, C_2]$  as the competitive process converges to an equilibrium.20

To identify an efficient wage structure that obtains in the Wilson equilibrium, we must discover a solution  $[C^*_1, C^*_2]$  such that  $U_1(C_1)$  is the minimum value of  $U_1$  among all the solutions to the maximization program. It is a straightforward matter to demonstrate that an efficient wage structure exists in  $M^*$ , but we further claim that the efficient wage structure is unique. To prove the latter half of our claim, we need a lemma.

Lemma 3: Let  $[C_1, C_2]$  and  $[C'_1, C'_2]$  be two distinct solutions to the maximization program (1), so that  $U_2(C_2) = U_2(C'_2)$ . Then  $C_2 = C'_2$  if and only if  $U_1(C_1) = U_1(C'_1)$ .

The consequence of this lemma will be used to establish the uniqueness of an efficient wage structure. We now state the following theorem.

Theorem: An equilibrium exists and is unique.

☐ **Properties of equilibrium.** Having resolved the problem of existence and uniqueness, we now turn to the characterization of equilibrium properties. We may first establish that the equilibrium in our model is always separating, be it a neoclassical or an internal labor market.

<sup>&</sup>lt;sup>18</sup> The constraints (2), (4), and (5) all define closed convex sets. However, the closed set defined by the constraint (3) need not be convex.

<sup>&</sup>lt;sup>19</sup> If  $[C'_1, C'_2]$  survives in Wilson competition, then in view of Lemma 2 it contradicts the assumption that  $U_2(C'_2)$  is a maximum subject to  $\pi(C'_1, C'_2) \ge 0$ .

 $<sup>^{20}</sup>$  The infeasibility of  $[C'_1, C'_2]$  vis- $\hat{a}$ -vis  $[C_1, C_2]$  can also be argued from the viewpoint of stability. Suppose that all firms offer only  $[C_1, C_2]$ . Then  $[C'_1, C'_2]$  will not be introduced, for it surely attracts all the type 1 workers but fails to induce actively the transfer of type 2 workers. On the other hand, if all firms are offering  $[C'_1, C'_2]$ , then some firms will find an incentive to switch to  $[C_1, C_2]$  to earn a higher profit rate so long as  $[C'_1, C'_2]$  remains in the market. The foregoing argument may suggest that Wilson-dominance should be redefined as either (i)  $U_2(C_2) > U_2(C'_2)$  or (ii)  $U_2(C_2) = U_2(C'_2)$  and  $U_1(C_1) < U_1(C'_1)$ . However, we consider that such a redefinition is unnecessarily strong, because only in the case of multiple solutions to the "static" maximization program do the dynamic aspects of competition necessitate breaking the tie.

Proposition 5: Under the assumption that  $MPL'_2 > MPL'_1$ , the equilibrium never entails a pooling contract.<sup>21</sup>

Now, three different types of separating equilibria are possible, each of which can be distinguished by different constraints binding in the solution to the above maximization program. In view of Propositions 2 and 3 and Lemma 1, we may state the following.

- (1) Equilibrium will entail the full information contracts, as  $([E^*_1], [E^*_2])$ , in which the first constraint is binding but the second is not.
- (2) Equilibrium will entail the neoclassical separating wage structures ( $[E^*_1]$ ,  $[E^{**}_2]$ ) in which both the first and second constraints are binding.
- (3) Equilibrium will entail the internal labor market in which the first constraint is nonbinding but the second constraint is binding.

We may therefore assert that the internal labor market equilibrium will entail a separating wage structure  $[C^*_1, C^*_2]$  such that  $U_1(C^*_1) = U_1(C^*_2)$  and  $U_2(C^*_2) > U_2(C^*_1)$ . Since  $[C^*_1, C^*_2]$  lies strictly in the interior of  $K \times K$ , we have

$$\begin{pmatrix} U_1(C^*_1) \\ U_2(C^*_2) \end{pmatrix} \gg \begin{pmatrix} U_1(E^*_1) \\ U_2(E^{**}_2) \end{pmatrix}.$$

Thus,  $C^*_2$  internally subsidizes  $C^*_1$  and the firm breaks even on  $[C^*_1, C^*_2]$  as a whole. Nevertheless,  $[C^*_1, C^*_2]$  is more efficient than, and is fact Pareto superior to, the neoclassical separating pair ( $[E^*_1]$ ,  $[E^{**}_2]$ ). This efficiency property of the internal labor market can be interpreted as a partial attenuation of the informational asymmetry by means of internal screening. A particular example of such an internal labor market is depicted in Figure 6 in Section 7.

We have completed investigation of our model which is designed to analyze efficiency/inefficiency of internal labor markets within the specification of workers' abilities and attitudes toward jobs. Before proceeding to a general statement of our conclusions, we give a numerical example using Akerlof's parametric specification in the A-R-S rat race.

### 6. An example

■ Akerlof characterized the two types of workers in terms of the following linear marginal productivity functions and quadratic utility functions:

$$MPL_1(s) = 1 + s$$
  
 $MPL_2(s) = 2 + s$ 

 $<sup>^{21}</sup>$  In the absence of the slope condition that  $MPL'_2 > MPL'_1$ , an example can be constructed where pooling happens to be the most efficient wage structure. However, we hasten to add that this slope condition is not crucial to our model. As long as the weaker condition that  $MPL_2 > MPL_1$  holds, the admission of the pooling equilibrium only makes the equilibrium taxonomy more complex, but does little to alter the logic of the rat race. In this regard, the stipulation of the slope condition is fairly innocuous; additionally, such a slope condition is a natural and certainly a plausible interpretation of the meaning of the high and low productivity workers.

It is to be noted that equilibrium may entail a pooling contract under the provisos that the firm is restricted to offer *only* singleton wage structures and that the neoclassical equilibrium does not obtain. This pooling equilibrium was Wilson's original remedy to the nonexistence of Nash neoclassical separating equilibrium in the Rothschild-Stiglitz model.

$$U_1(w, s) = (w - s) - \theta(s - 1)^2$$
  

$$U_2(w, s) = (w - s) - \theta(s - 2)^2.$$

Average productivity is defined to be

$$APL(s) = (2 - \lambda) + s.$$

These relations satisfy our assumptions that<sup>22</sup>

$$MPL_2(s) > MPL_1(s)$$
 for all  $s \ge 0$ ,

and

$$MRS_1(w, s) = 1 + 2 \theta(s - 1)$$
  
> 1 + 2 \theta(s - 2)  
=  $MRS_2(w, s)$ .

We then solve the maximization problem corresponding to (P) by using Kuhn-Tucker conditions on the saddle point. Depending upon the parametric value of  $\theta$  in the utility functions, we obtain the following different types of equilibria.

(1) If  $\theta \ge 1$ , then the full information equilibrium obtains as

$$E^*_1 = (1, 2),$$
  
 $E^*_2 = (2, 4).$ 

It can be verified that the first constraint is binding, but the second is not.

(2) If  $1 > \theta \ge (1 - \lambda)^2$ , then the equilibrium entails the neoclassical separating contracts:

$$E^*_1 = (1, 2)$$
  
 $E^{**}_2 = (1 + 1/\sqrt{\theta}, 3 + 1/\sqrt{\theta}).$ 

In this solution both the first and second constraints are binding.

(3) If  $(1 - \lambda)^2 > \theta > 0$ , then the equilibrium entails an internal labor market with the wage structure:

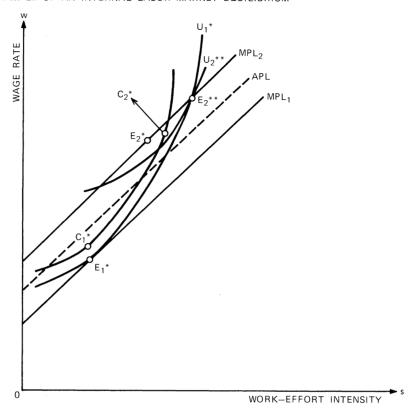
$$C^*_1 = (1, 3 - \lambda - \theta/1 - \lambda), C^*_2 = (2 + \lambda/1 - \lambda, 4 + \lambda^2/1 - \lambda + \theta\lambda/(1 - \lambda)^2).$$

In this case the first constraint is nonbinding, but the second is binding. This internal labor market equilibrium is depicted in Figure 6.

■ The aim of our model has been to provide analytic conditions sufficient to necessitate the primitive emergence of a hierarchical organization. That is, to answer the question, under what conditions does a competitive firm create an internal wage structure? Our intuition was that efficient screening might require subsidy of one group of workers by another within the firm. The fundamental logic of such a competitive screening model is that firms compete in terms of quality in wage structures as well as in promised wage rates. In the Wilson regime of the rat race there always exists an equilibrium, even when a firm offers a pair of contracts as its wage structure. We then

# 7. Conclusion and qualifications

 $<sup>^{22}</sup>$  This MPL condition is weaker than that stated in Section 3. However, see the previous note for a fuller discussion on the conditions concerning the MPL assumptions.



categorized the set of equilibria as separating neoclassical and (separating) internal labor market equilibria, and provided analytical conditions that give rise to each of them.

Our model has no training cost, ignoring as it does even the pecuniary cost of the screening mechanism. Nonetheless, screening theory predicts that if an internal labor market is the efficient solution, then the lower (higher) productivity workers tend to receive wages above (below) their marginal productivities. Thus, efficiency in the system may require that higher productivity workers subsidize the lower productivity workers. This can be viewed as lump-sum transfer solution to internalize the negative informational externality exerted by the lower productivity workers upon the high productivity workers. Hence, the internal labor market solution, if feasible, is a Pareto superior system to the neoclassical separating contract set in which everyone still receives his own marginal productivity but suffers the full impact of the externality.<sup>23</sup>

The exposition has been limited to an economy consisting of only two types of workers. If our model were extended to any finite

<sup>&</sup>lt;sup>23</sup> However, the lump-sum transfer alone is not sufficient to eliminate the externality completely. Such elimination would require a sort of progressive taxation on the grades of job positions. Since the higher grade jobs correspond to the higher wage rates in equilibrium, the enactment of a progressive income tax might be effective in further implementing the corrective merit of the lump-sum subsidies. Also see Mirrlees (1974, 1976) for motivation and theory of the optimal income tax under uncertainty, and its relation to the screening problem.

n-types of workers, the analysis would become much more complicated in its attempts at characterizing, and even proving the existence of an equilibrium. However, it is unlikely that we would lose the essential feature that follows from the specification of our model, to wit, an efficient wage structure may involve an internal allocation with subsidization among different job positions.<sup>24</sup>

Of particular interest to our study is a recent contribution by Williamson, Wachter, and Harris (1975) (hereafter W-W-H) in which the internal labor market has been interpreted in terms of economic efficiency. We interpret W-W-H as a study of comparative contracts. They attempt to answer the question: What mode of labor contracts is operative in a complex and uncertain world? W-W-H propose an approach utilizing an "organizational failures framework," in which a bounded rationality-information impactedness pair is coupled with opportunism under small number exchange conditions. In this situation, the essential concern of labor management is to minimize opportunism and to economize on bounded rationality. It is then argued that within the limits of human rationality, an internal labor market may provide a "transactionally" more efficient method of labor management than other known forms of labor contracts.

The important qualitative feature underlying their analysis is the idiosyncratic nature of task-specific "learning-by-doing" which takes place within the firm. The immediate consequence of this is that the potentially qualified, but inexperienced, worker can never be regarded as equivalent to a veteran on the job. Only through observing the process of learning-by-doing can the employer ascertain the employee's quality. In short, on-the-job training becomes the process of screening the worker's quality on the job. W-W-H are mainly concerned with classes of "informational nonseparability" which involve workers' attitudes and adaptations in an intertemporally uncertain process of on-the-job training.

It is with regard to these points that we have introduced the rat race model, hoping thereby to formalize and justify W-W-H's notion of the efficiency of internal labor markets. We have elaborated on the following three features, which are well expressed by W-W-H (1975, pp. 274–275):

The practice of restricting entry to lower level jobs and promoting from within has interesting experience-rating implications. It permits firms to protect themselves against low productivity types, who might otherwise successfully represent themselves to be high productivity applicants, by bringing employees in at low level positions and then upgrading them as experience warrants. . . . Reliance on internal promotion has affirmative incentive properties in that workers can anticipate that different talent and degrees of cooperativeness will be rewarded.

Were it, however, that markets could equally well perform these experience-rating functions, the port of entry restrictions described would be unnecessary. The (comparative) limitations of markets in experience-rating aspects accordingly warrant attention.

Although the attachment of wages to jobs rather than to individuals may result in an imperfect correspondence between wages and marginal productivity at ports of entry, productivity differentials will be recognized over time and a more perfect correspondence can be expected for higher level assignments in the internal labor market job hierarchy.

Nevertheless, the rat race version remains as an essentially static

<sup>&</sup>lt;sup>24</sup> A theorem in Wilson (1976) also suggests the possible validity of such a claim.

model, whereas W-W-H are concerned with a dynamic intertemporal process of internal screening.<sup>25</sup> The atemporal nature of our model explains its incompleteness in dealing with the transactional rationality which W-W-H consider crucial for the existence of an internal labor market.

A satisfactory model calls for reinterpretation of the wage structure as an internal promotion ladder with a port of entry, and for an explicit incorporation of the intertemporal process of on-the-job screening.<sup>26</sup> Interestingly, such modifications will likely be of theoretical necessity if nonmyopic equilibrium notions such as Wilson's are to be justified.

As the maximization program has shown, the Wilson firms in equilibrium behave *in toto* as if they tacitly agree to provide only doubleton wage structures, and their foresight discourages them from offering a singleton wage structure such as *C* in Figure 5 for a "short-run" gain.<sup>27</sup> However, as a logical parable, if a firm decides to be a myopic Nash firm, then it can raid the market with this *C* to gain short-run profits, and quickly exit the market before such a strategy becomes eventually self-defeating. The real issue here concerns the identification of institutional features or technological requirements which might operate to discourage the firm from adopting such short-run strategies.

One possible answer may be found in task idiosyncrasies and on-the-job training. Task idiosyncrasies imply that it takes a considerable amount of time and effort for a worker to learn the specific skill required for a given job. It could be argued that it is this long-run process of on-the-job training and screening that provides one key to an understanding of labor markets. Thus, if a worker develops his skill and reveals his true value only by progressing sequentially from lower to higher ladder jobs, 28 then both the employer and employee recognize the advantage of implicit long-term labor contracts. If screening evolves over the worker's life-cycle, and if the firm faces a steady turnover of its work force, then there may be a case for arguing that it is necessary for the firm to stay in the market for some duration. In skilled labor markets, promotion opportunities and efficiency in on-the-job training attract better workers. Under these circumstances, firms are likely to value the long-run efficiency of their wage structures, and thus are unlikely to opt for policies leading merely to short-run gains. These considerations lead us to adopt an

<sup>&</sup>lt;sup>25</sup> The Wilson equilibrium is a static concept; it does not explicate how it could be reached, but only pertains to a stationary state in which no one has an active incentive to break away from his current strategy.

<sup>&</sup>lt;sup>26</sup> Riley (1976) develops a model which synthesizes human capital theory and screening theory. His model suggests the equality in present discounted values of wage and productivity streams. However, in his model, the firm is restricted to offer only one contract, i. e., a singleton wage structure.

<sup>&</sup>lt;sup>27</sup> This observation suggests that the Wilson equilibrium could be regarded as a solution to a supergame with an appropriately constrained strategy space. Also see Heal (1976) and Akerlof (1976) in this regard.

<sup>&</sup>lt;sup>28</sup> Note, however, that this kind of idiosyncrasy is not synonymous with technological nonseparability. We concur with W-W-H that most production processes are separable, and that each idiosyncratic task could be separately established and manned by independent specialists.

equilibrium notion less myopic than Nash as the concept proper to analysis of labor markets.<sup>29</sup>

Finally, it should be noted that the Wilson concept of equilibrium can also be interpreted in the light of bounded rationality. The Wilson concept, as stated, anticipates only the losing consequence of new offers. A more rational model would indeed consider more active reactions that are designed to retaliate against new offers. Thus, the Wilson notion of equilibrium should be seen as one aspect of the bounded rationality that we have tacitly inherited from W-W-H.

A central underpinning of this paper is that Wilson equilibrium is a relevant concept for the analysis of internal labor markets, if wage structures are designed to reflect long-term job values. Our rat race story is an attempt to model one aspect of an efficiency rationale which could justify the economic existence of internal labor markets. It does not pretend to be a comprehensive model capable of accounting for the great many key attributes that collectively make up the institutionally rich context of the internal labor market.

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<sup>&</sup>lt;sup>29</sup> Riley (1976) also expresses a view similar to ours when he adopts a concept of "reactive" equilibrium rather than Nash equilibrium. See also the introductory and concluding sections of Spence (1973).

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