Technology choice and environmental regulation under asymmetric information

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\textbf{A B S T R A C T}

We focus on the incentives of an industry with a continuum of small firms to invest in a cleaner technology under two environmental policy instruments: tradable emission permits and emission taxation. We assume asymmetric information, in that the firms’ abatement costs with the new technology are either high or low. Environmental policy is set either before the firms invest (commitment) or after (time consistency). Under commitment, the welfare comparison follows a modified Weitzman rule, featuring reverse probability weighting for the slope of the marginal abatement cost curve. Both instruments can lead to under- or overinvestment ex post. Tradable permits lead to less than optimal expected new technology adoption. Under time consistency, the regulator infers the cost realization and implements the full-information social optimum.

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1. Introduction

Regulators often have to set environmental policy without being completely informed about the costs of new technology. The range of technologies that can be adopted in reaction to environmental...
(e.g. climate change) policy, might differ significantly, ranging from fuel substitution, to solar, wind or nuclear energy, up to carbon capture and storage (Krysiak, 2008). We can therefore expect that, at least for the less mature technologies, the regulator is able to only imperfectly observe their features and, therefore, to infer how good regulated firms are in using them. As Nentjes et al. (2007) point out, when environmental policy is made stricter (e.g. through stricter environmental standards) regulators might indeed face uncertainty concerning the capability of a regulated industry to develop and install the needed technology. Existing and past environmental policy choices have in several cases been based on imperfect anticipations of the related costs and impacts, most notably so when setting feed-in tariffs (FITs) for renewable energy generation.¹ On the basis of these considerations, we can expect the regulator’s informational burden related to new technology deployment to be significant. We can also expect that the firms themselves know more about the cost of new technologies than the regulator.

We then focus on the incentives of an industry with a continuum of small firms to invest in a cleaner technology under two environmental policy instruments: tradable emission permits and emission taxation. We are thus comparing price and quantity instruments under uncertainty about abatement costs (that we model à la Weitzman, 1974) in a model of technology adoption and policy timing. Environmental policy is set either before the firms invest (commitment) or after (time consistency). We assume asymmetric information, in that the firms’ abatement costs with the new technology are either high or low. In addition to this “aggregate” informational asymmetry, there is an idiosyncratic (i.e. firm-level) informational asymmetry about fixed adoption costs. This is close to several real-life policy problems where entire sectors are subject to regulation and are expected to implement the same clean technology (e.g. renewable energy).

The comparison of incentives towards cleaner technology adoption has been the subject of a substantial amount of literature, starting from the seminal papers by Downing and White (1986) and Milliman and Prince (1989).² This literature had the merit to bring to scholars’ attention the need to explicitly include technological change in instruments comparisons. Requate and Unold (2001, 2003) build and comment upon earlier papers on the relative merits of different environmental policy instruments in terms of technology adoption. Through the lens of general models featuring heterogeneous (Requate and Unold, 2001) and homogeneous (Requate and Unold, 2003) firms, the authors compare emission taxes or abatement subsidies and tradable emission permits endogenizing the number of adopters of the new technology. They focus on two extreme cases: one in which the regulator sets the policy that was optimal without the new technology, and another, which is the most relevant for our purposes, where the regulator knows about the new technology. In the latter case, with commitment as well as with time consistency, the regulator can implement the social optimum.

Even with perfect information, commitment and time consistency do not usually implement the first best if there are additional market failures (other than pollution). If the number of firms is small, they can affect environmental policy under time consistency, which typically precludes attainment of the first best. However, this does not mean that commitment leads to higher welfare than time consistency. Amacher and Malik (2002) demonstrate these findings for emission taxation of a single firm choosing whether or not to adopt a new abatement technology, so that technology adoption is a discrete variable.

Our model is more specific than Requate and Unold’s settings in order to keep it manageable with the added complexity of asymmetric information. In our setting, firms in the industry are symmetric in terms of abatement costs, as in Requate and Unold (2003), but asymmetric in terms of fixed adoption

¹ In the case of the UK FITs scheme to support photovoltaic (PV) electricity, due to complexities in the monitoring process and unexpected reduction in PV panels cost, installed plants overshot significantly with respect to forecasts, leading to the need for an early review of tariffs (UK National Audit Office, 2011). In other countries, such as Italy, the significant costs related to the FIT system have shown “...the inability of the regulator to directly control how much new capacity investors install in a given year, and the consequent inability to control costs.” (OECD, 2013, p. 165). Similarly, in Germany, the costs related to FITs have increased far above government expectations (OECD, 2012). Finally, focusing on the diffusion impact of the first EU ETS phase in Italy, Borghesi et al. (2015) underline how specific sector-level features might lead to counterintuitive (and unexpected) outcomes.

² This literature has since been surveyed by Jaffe et al. (2003) and Requate (2005).
costs. We will see that with asymmetric information, the regulator can implement the first best under time consistency, but not under commitment. More specifically, we show that under commitment both instruments can lead to over- or underinvestment ex post, depending on the realization of the cost parameter. Asymmetric information, coupled with the assumption that the regulator sets the policy taking into account the knowledge she has of the distribution of cost parameters, implies therefore that the clear cut link between taxes (permits) and over (under) investment obtained in Requate and Unold (2001, 2003) does not hold. However, underinvestment with permits is confirmed in expected terms. Finally, again like Requate and Unold (2001, 2003) we find that the regulator can implement the welfare optimum under time consistency. This is because the regulator can perfectly infer the cost realization of the new technology.

Weitzman (1974) was the first to systematically address the relative performance of price and quantity regulation under uncertainty in environmental policy or indeed any area of policy. Whereas Weitzman (1974) concentrates mainly on uncertainty about the intercept of the industry’s Aggregate Marginal Abatement Cost (AMAC) curve, we consider uncertainty about its slope.

As in Weitzman (1974), we find that the comparison of price and quantity instruments depends on the (average) slopes of the AMAC and Marginal Environmental Damage curves. However, in our paper the average slope of the AMAC curve is determined by a counterintuitive rule of reverse probability weighting: the steep slope is weighted by the probability that the slope is flat and vice versa.

Recent papers in the Weitzman (1974) vein include Mandell (2008), who considers regulating part of the polluters by tradable permits and the other part by emission taxation. Krysiak and Oberauner (2010) let the firms choose between the two instruments. Stranlund (2015) compares an emission tax to a hybrid tradable permit scheme with a price floor and a price ceiling (as introduced by Roberts and Spence, 1976) when uncertainty about abatement costs and environmental damage is correlated (as analyzed by Stavins, 1996) and finds that when the correlation is negative, taxes can still be preferable to the hybrid instrument. Ambec and Coria (2013) compare price and quantity instruments for the control of two polluters with asymmetric information about their interdependent abatement costs. Yates (2012) deviates from the usual comparison of constant permit supply with a constant tax rate, showing that the optimal permit supply function is better at dealing with abatement cost uncertainty than the optimal pollution tax function.

Combining asymmetric information and innovation, Mendelsohn (1984), Krysiak (2008) and Storrøsten (2014) examine how endogenous technical change affects the choice between price and quantity instruments under commitment. In all three papers, technology choice is continuous: A firm can invest to reduce the intercept and (in Krysiak, 2008 and Storrøsten, 2014) the slope of its MAC curve. Mendelsohn (1984) considers a single firm, with asymmetric information about marginal abatement costs and investment costs. Krysiak (2008) considers an industry with many ex-ante identical small firms who discover their marginal abatement costs after they have made their investment decision. Storrøsten (2014) adds product demand uncertainty to Krysiak’s (2008) model. All three papers find (as we do in our model) that endogenous technical change reduces the slope of the long-run MAC curve, making quantity regulation more attractive.

The rest of the paper is organized as follows. We set out the model in Section 2. In Section 3 we derive the social optimum (or first best) for the full-information benchmark. Section 4 discusses how the firms make their emission and technology adoption decisions. In Section 5 (6) we analyze the regulator’s behaviour and we derive the subgame perfect equilibria under commitment (time consistency). In both sections we first determine the full-information equilibrium as a benchmark, confirming Requate and Unold’s (2001) finding that this implements the first best. Section 7 concludes.

2. The model

There is a continuum of firms with mass 1, currently using abatement technology 1. A firm’s total and marginal abatement costs with the current technology are:

\[ C_1(e_1) = \frac{1}{2}(1 - e_1)^2, \quad MAC_1(e_1) = 1 - e_1 \]  

(1)
with $e_1$ the emission level. Note that with the current technology, there are no fixed costs and all firms have the same cost function. Note also that, as is standard, the cost function is decreasing and convex in emissions.

The firms must choose whether or not to invest in a cleaner technology. Firm $i$’s variable abatement cost with the new technology is $^3$

$$VC_i(e_\theta) = \frac{1}{2} (\theta - e_\theta)^2 \tag{2}$$

with $e_\theta$ the emission level. Two sources of asymmetric information are present in our model: on variable costs and on fixed costs. First of all, asymmetric information is assumed concerning the cost parameter $\theta$, which is known by the firms but not by the regulator; the latter has an a priori distribution on $\theta$ according to which it takes the value $L$ with probability $\nu$, and the value $H$ with probability $1 - \nu$, where $(1/2) < L < H < 1$. Note that the cost parameter is the same for all firms, i.e. $\theta$ is an aggregate asymmetric information parameter, linked to factors such as the speed at which the cost of the new technology falls over time, or difficulties by firms to get the needed financial resources.

The assumption $L > (1/2)$ is intended to limit our attention to the more plausible case of incremental innovation, and not of technologies that drastically reduce abatement costs.$^{4}$

Adoption also implies fixed cost $F$ of switching from the current to the new technology. Each firm $i$ knows its own $F$. The regulator knows that fixed cost $F$ is uniformly distributed between 0 and $F$, but she does not know the fixed cost of any individual firm. As a result, fixed cost $F$ is a source of idiosyncratic (i.e. firm-specific) asymmetric information.

From (2), firm $i$’s total and marginal abatement costs with the new technology are:

$$C_i(e_\theta) = VC_i(e_\theta) + F = \frac{1}{2} (\theta - e_\theta)^2 + F, \quad MAC_i(e_\theta) = \theta - e_\theta \tag{3}$$

Total emissions $E$ are:

$$E = \pi_\theta e_\theta + (1 - \pi_\theta) e_1 \tag{4}$$

with $\pi_\theta$ the share of firms adopting the new technology. Total and marginal environmental damage is, respectively:

$$D(E) = \frac{1}{2} dE^2, \quad MD(E) = dE \tag{5}$$

with $d > 0$.

In order to rule out corner solutions where $\pi_\theta = 1$ and/or $e_\theta = 0$, we shall assume$^{5}$:

$$F > L(1 - L) \tag{6}$$

$$L > \frac{dF}{d [F + (1-H)^2] + F} \tag{7}$$

$$H > \frac{d}{d + 1} \tag{8}$$

Note that (6) together with $H > L > (1/2)$ implies:

$$F > (1 - \theta)^2 \tag{9}$$

$^3$ For simplicity, we ignore the possibility that technological change reduces MAC for low levels of abatement, but increases it for high levels (Amir et al., 2008; Bauman et al., 2008; Bréchet and Jouvet, 2008). Perino and Requate (2012) explore the implications in a model with perfect information.

$^4$ The consequence of dramatic technological change (i.e. $\theta < (1/2)$) is left for further research.

$^5$ In Appendix B we show that these conditions are necessary and sufficient to ensure interior solutions.
We analyze two environmental policy instruments in two policy regimes. The two environmental policy instruments are emission taxation and auctioned tradable emission permits. Until recently, tradable permit schemes were mostly based on grandfathering. Currently there is a movement toward auctioning of permits, especially in Phase 3 (2013–2020) of the EU Emissions Trading System, the largest tradable permit scheme in existence. We will assume that the tradable permits are fully auctioned. This makes for the clearest comparison with emission taxation. With both instruments, the firms have to pay the government for all their emissions. More importantly, with either instrument the regulator can only set the value of a single variable (the tax rate or the total amount of permits). In the conclusion, we will discuss how grandfathering of permits would affect our results.

The two policy regimes we consider are commitment and time consistency, the difference between them occurring in stages one and two of the game between the regulator and the industry. In stage zero of each game, nature draws the cost realization \( \theta \) and each firm \( i \)'s fixed cost \( F \). As already mentioned, these costs are revealed to the firms, but not to the regulator. All other parameters are common knowledge.

Under commitment, the regulator sets the total amount of emission permits or the emission tax rate in stage one.\(^6\) In stage two, the firms choose a technology. This order is reversed under time consistency. Finally, in stage three the firms choose their emission level.

3. Full-information social optimum

In this section we derive the social optimum for the case where the regulator knows the realization of \( \theta \) and each firm \( i \)'s fixed cost \( F \) in (3). Fig. 1 illustrates the outcome for \( d = 1.2, \theta = 0.6, F = 0.32 \).

Given \( \theta \), the regulator sets the share \( \tau_\theta \) of investing firms and emissions \( e_\theta (e_1) \) by firms with the new (current) technology. It is easily seen that the regulator would like for each firm with the same technology to emit the same amount, and for the firms with the lowest fixed costs to invest in the new technology. Social costs are:

\[
SC_\theta = (1 - \pi_\theta) \frac{1}{2} (1 - e_1)^2 + \pi_\theta \frac{1}{2} (\theta - e_\theta)^2 + \frac{1}{2} \pi_\theta^2 F + \frac{1}{2} d|\pi_\theta e_\theta + (1 - \pi_\theta)e_1|^2
\]  

---

\(^6\) In line with the literature, we assume that for the time horizon of our model under commitment, the regulator cannot adjust environmental policy after inferring the cost realization \( \theta \) from the firms' behaviour. This does not imply that the regulator can never learn \( \theta \) or she can never adjust policy. Our point is rather that learning \( \theta \) and adjusting policy takes time. Our model is only valid for the period until the regulator can adjust her policy. See Costello and Karp (2004) for a model of learning when regulating a single non-strategic firm with emission taxation or an emission quota.
The first term on the RHS is total abatement cost for the firms with the current technology, from (1). The second (third) term is total variable (fixed) cost for the firms with the new technology, by (3). There are \( \pi_0 \) firms investing in the new technology, with costs uniformly distributed between 0 and \( \pi_0F \), so that average cost is \( (1/2)\pi_0F \) and total fixed cost is \( (1/2)\pi_0^2F \). Lastly, the fourth term is environmental damage, from (4) and (5).

Minimizing (10) with respect to \( e_1 \) and \( e_0 \) yields:

\[
\tau = 1 - e_1 = \theta - e_0 = dE
\]

This is the standard condition that marginal abatement costs of all firms should be equal to each other and to marginal environmental damage. We denote the level at which MACs are equalized by \( \tau \), which may be interpreted as the shadow cost of emissions.

In Fig. 1, the curves MAC\(_1\) and MAC\(_0\) show the marginal abatement costs for the current and the new technology, respectively. Interpreted as functions of \( e_1 \) and \( e_0 \), they show a single firm’s MAC with the current and the new technology respectively. Interpreted as functions of \( E \), they show the industry’s MAC if all firms used the same technology. In Fig. 1, when MAC equals \( \tau \), for instance, a firm with the new (current) technology emits \( e'_1(e'_0) \) in the social optimum according to (11).\(^7\)

Minimizing (10) with respect to \( \pi_0 \) yields:

\[
\frac{1}{2}(\theta - e_0)^2 - \frac{1}{2}(1 - e_1)^2 + \pi_0F = (e_1 - e_0)dE
\]

This is the equivalent of equation (7) in Requate and Unold (2001, p. 544). It says that for the marginal firm that adopts the new technology (the adopting firm with the highest fixed cost \( F \)), its increase in abatement costs (the LHS of (12)) should equal the decrease in environmental damage that it causes (the RHS of (12)).

By (11), the first two terms on the LHS of (12) cancel out and:

\[
\pi_0F = \tau(e_1 - e_0) = \tau(1 - \theta)
\]

Fig. 1 illustrates Eq. (13) for the optimal share of adopting firms. With the shadow cost of emissions equal to \( \tau \), a firm that switches to the new technology reduces environmental cost by the area \( B_0B_1J_1J_0 \) which is equal to \( \tau(e'_1 - e'_0) = \tau(1 - \theta) \). This means that it is socially optimal for all firms with fixed cost of less than \( \tau(1 - \theta) \) to switch to the new technology. The switching firm with the highest fixed cost thus has fixed cost \( \pi_0F \) exactly equal to area \( B_0B_1J_1J_0 \) in Fig. 1. Note that the area \( BB_1J_1 \) equals \( (1/2)\tau\pi_0(1 - \theta) \), which by (13) equals aggregate fixed costs \( (1/2)(\pi_0)^2(1 - \theta) \) of switching to the new technology.

Substituting (11) and (13) into (4), aggregate emissions at \( \tau \) are:

\[
E_\theta = e_1 - \pi_0(1 - \theta) = e_1 - \frac{\tau(1 - \theta)^2}{F}
\]

From (11), (13) and (14), we can define the aggregate marginal abatement cost curve:

\[
1 - e_1 = c_\theta(1 - E_\theta) = AMAC_\theta
\]

where \( c_\theta \) is the negative of the slope of the AMAC\(_\theta\) curve with\(^8\):

\[
c_\theta = \frac{F}{F + (1 - \theta)^2} > \theta, \quad c_L < c_H
\]

\(^7\) In Appendix B we verify that \( \tau < \theta \), so that \( c_\theta > 0 \) in the social optimum.

\(^8\) The inequality \( c_\theta > 0 \) follows from (6). Like Mendelsohn (1984) and Krysiak (2008), we find in our model that endogenous technical change reduces the slope of the AMAC curve (\( c_\theta < 1 \)).
The AMACθ function gives the industry’s aggregate marginal abatement costs for a given level of $E_θ$, with $e_1$, $e_θ$ and $π_θ$ set optimally according to (11) and (13). AMACθ is a weighted average of MACθ and MAC1. The higher is $π_θ$, the closer AMACθ is to MAC1.

We can now solve (11) and (15) for the optimal level of total emissions and the corresponding level of marginal damages and aggregate marginal abatement costs:

$$E_θ^* = \frac{c_θ}{c_θ + d}, \quad \tau_θ^* = \frac{c_θ d}{c_θ + d}$$  \hspace{1cm} (17)

Substituting from (17) into (13) yields the optimal fraction of adopting firms:

$$\pi_θ^* = \frac{(1 - \theta) c_θ d}{F (c_θ + d)} = \frac{d (1 - \theta) (F (d + 1) + d (1 - \theta))^2}{d + \theta} < 1$$  \hspace{1cm} (18)

The first inequality follows from (6) and the fact that $π_θ^*$ is decreasing in $F$.

Note that $π_θ^* > π_θ^*$ since by (9) and (18):

$$\frac{d}{d\theta} \left[ \frac{F (d + 1) + d (\theta - 1)^2}{d + \theta} \right]^2 < 0$$

Aggregate total abatement costs ATACθ for a given level of $E$ with $e_1$, $e_θ$ and $π_θ$ set optimally according to (11) and (13) follow from integrating AMACθ in (15) with respect to $E$ and noting that aggregate fixed costs are zero for $E = 1$:

$$ATAC_θ = \frac{1}{2} c_θ (1 - E)^2$$  \hspace{1cm} (19)

Fig. 1 shows the optimal emission levels $e_1^*$, $e_θ^*$ and $E_θ^*$, as well as the optimal level $τ_θ^*$ of marginal abatement costs. Given that the optimal share $π_θ^*$ of firms have switched to the new technology, the industry’s marginal abatement costs (net of fixed costs) are MACθ. Aggregate variable abatement costs in the optimum are then given by the area $E^* θ KJ^*$ below the MACθ curve. Following our discussion above for $τ^*$, aggregate fixed costs are $K^* KJ$. This area is equal to $J^* KJ_1$. Thus, aggregate total abatement costs $ATAC_θ$ in the social optimum, consisting of fixed plus variable abatement costs, are $J^* KJ_1 + E^* θ KJ^* = E^* θ KJ_1$, the area below the AMACθ curve.

Minimized social costs are:

$$SC_θ = ATAC_θ(E_θ^*) + D(E_θ^*) = \frac{1}{2} c_θ (1 - E_θ^*)^2 + \frac{1}{2} dE_θ^* \frac{dE_θ^*}{2 (c_θ + d)}$$  \hspace{1cm} (20)

The second equality follows from (5) and (19). The third equality follows from (17). In Fig. 1, aggregate total abatement costs are $E^* θ KJ_1$ and total environmental damage is $OK_θ$, so that total social cost in the optimum is $OK_1$.

4. Firms’ choices

After deriving our first best benchmark, we now turn to the regulated firms’ choices concerning emission levels (Section 4.1) and technology adoption (Section 4.2) under emission taxes and tradable permits. The firms know the cost realization $θ$ and take environmental policy as well as the share of adopting firms ($π$) as given. Let $x$ denote the tax rate $t$ under emission taxation and the permit price $p$ under tradable permits.

4.1. Emissions

In stage 3 of the commitment as well as of the time consistency regime, each firm chooses its emission level, having already made its technology choice. A firm minimizes the sum of tax payment (under emission taxation) or permit purchases (under auctioned permits) and variable abatement costs. Under emission taxation, each firm takes the tax rate as given, because it has already been set by
the regulator. Under tradable permits, each firm takes the permit price as given, because the permit market is perfectly competitive since there is a continuum of firms.

A firm with the current technology minimizes \( C(e_1) + x e_1 \), so that by (1) it sets:

\[
e_1 = 1 - x
\]

4.2. Technology choice

Technology choice is stage 2 under commitment and stage 1 under time consistency. In both policy regimes and with both instruments, each firm takes \( x \) (the tax rate or the permit price) as given. With commitment to emission taxation, the tax rate has been set in stage 1. With time-consistent taxation, the firms realize that the tax rate (to be set in stage 2) depends on the adoption rate \( \pi \), but each firm considers itself too small to affect \( \pi \). Under tradable permits, the firms realize that the stage-3 permit price depends on \( \pi \), but again each firm considers itself too small to affect \( \pi \).

In the technology choice stage, each firm thus compares its overall costs (including tax payment or permit purchase) with and without adoption of the new technology, taking \( x \) as given. Overall costs of each firm without adopting are, from (1) and (21):

\[
K_1 = \frac{1}{2} (1 - e_1)^2 + x e_1 = \frac{x(2 - x)}{2} \tag{24}
\]

while firm i’s overall costs with the new technology are, from (3) and (22):

\[
K^i_\theta = \frac{1}{2} (\theta - e_\theta)^2 + x e_\theta + F^i = \frac{x(2\theta - x)}{2} + F^i \tag{25}
\]

With new technology cost realization \( \theta \), firm i adopts if \( K^i_\theta < K_1 \) or, by (24) and (25):

\[
F^i < F_\theta \equiv x (1 - \theta) \tag{26}
\]

As a consequence, the proportion of firms investing will be given by the proportion of firms featuring \( F < F_\theta \). Since fixed costs are uniformly distributed between 0 and \( F \), the share of adopting firms is, from (26):

\[
\pi_\theta = \frac{F_\theta}{F} = \frac{x(1 - \theta)}{F} \tag{27}
\]

Note that (27) is the same as (13) with \( x = \tau \), and (21) and (22) imply the first equality of (11). Thus we can use \( AMAC_\theta \) as defined in (15) and \( ATAC_\theta \) from (19) for the policy scenarios as well. The reason is that firms make the socially optimal investment decision, given \( x \).

Total emissions for a given level of \( x \) follow from setting \( AMAC_\theta \) equal to \( x \) in (15):

\[
E_\theta = \frac{c_\theta - x}{c_\theta} \tag{28}
\]

With tradable permits, we solve (27) and (28) simultaneously to find the equilibrium values of technology adoption \( \pi_\theta \) and permit price \( p_\theta \) given \( E \) and \( \theta \):\(^\text{10}\)

\[
p_\theta(E) = c_\theta(1 - E), \quad \pi_\theta(E) = \frac{c_\theta}{F} \frac{(1 - \theta)(1 - E)}{F + (1 - \theta)^2} \tag{29}
\]

\(^9\) The permit price depends directly on \( \pi \) through the permit market, according to (23). With time consistency, \( \pi \) also affects the permit price indirectly through its effect on the emission level \( E \) that the regulator chooses in stage two.

\(^{10}\) The final equality follows from (16).
5. Commitment

In this section, we derive the subgame perfect equilibrium for commitment. In stage one of the commitment game, the regulator sets the emission tax rate or the total amount of permits to be auctioned. In stage two, firms make their technology choices, as analyzed in Section 4.2. In stage three, firms decide on their emission level and (under tradable permits) permit purchases, as analyzed in Section 4.1.

In Section 5.1, we first establish the full-information benchmark. We analyze emission taxation (tradable permits) under asymmetric information in Section 5.2 (5.3). Finally, we compare the instruments with each other in Section 5.4.

5.1. Full-information benchmark

If the regulator knows the cost realization $\theta$, she can implement the full-information social optimum with tradable permits as well as with emission taxation. With tradable permits, the regulator will set the number of permits at $E^*_\theta$ given by (17). Substituting this into (29) shows that the share of adopting firms in stage two will then be the optimal amount $\pi^*_\theta$ given by (18). With emission taxation, the regulator would set the tax rate equal to $\tau^*_\theta$ given by (17). Substituting this into (27) shows that in stage two, the share of adopting firms will be $\pi^*_\theta$ as given by (18), and substituting it into (28) shows that in stage 3 total emissions will be $E^*_\theta$ given by (17).

We thus confirm Requate and Unold’s (2001) finding that under perfect information, commitment implements the first best with tradable permits and emission taxation when regulating pollution and technology adoption by a continuum of heterogeneous firms.

5.2. Asymmetric information, emission taxation

In stage 1, the regulator sets the tax rate $t$ that minimizes expected social cost. For cost realization $\theta$, social costs are, substituting (28) into (5) and (19) with $x = t$:

$$SC_{\theta} = D(E_{\theta}) + ATAC_{\theta}(E_{\theta}) = \frac{d}{2} \left( \frac{c_{\theta} - t}{c_{\theta}} \right)^2 + \frac{t^2}{2c_{\theta}}$$

(30)

Accounting for the probability that the firms are efficient (inefficient), given by $\nu (1 - \nu)$, we can write expected social costs using (30) as:

$$\tilde{SC}(t) = \nu \left[ \frac{d}{2} \left( \frac{c_{L} - t}{c_{L}} \right)^2 + \frac{t^2}{2c_{L}} \right] + (1 - \nu) \left[ \frac{d}{2} \left( \frac{c_{H} - t}{c_{H}} \right)^2 + \frac{t^2}{2c_{H}} \right]$$

The first order condition for $t$ requires:

$$(t - d)c_{L}c_{\tilde{\theta}} + dt \left[ (1 - \nu)c_{L}^2 + \nu c_{H}^2 \right] = 0$$

with $c_{\tilde{\theta}}$ defined as the reverse probability-weighted average of $c_{\theta}:

$$c_{\tilde{\theta}} = (1 - \nu)c_{L} + \nu c_{H}$$

(31)

Solving for the optimal tax rate yields:

$$t^{CO} = \Omega d c_{H} c_{L} c_{\tilde{\theta}}$$

(32)

with

$$\Omega = \frac{1}{c_{H} c_{L} c_{\tilde{\theta}} + d \left[ (1 - \nu)c_{L}^2 + \nu c_{H}^2 \right]}$$

(33)

Substituting (32) into (27), the share of adopting firms under cost realization $\theta$ is then:

$$\pi_{\theta}(t^{CO}) = \frac{(1 - \theta)}{F} \Omega d c_{H} c_{L} c_{\tilde{\theta}}$$

(34)
We define excess social costs \( XSC_\theta \) as the social costs in excess of the minimized full-information social costs given by \( SC_\theta \) in (20) for a given \( \theta \). From (30) and (32):

\[
XSC_\theta(t^{CO}) = \frac{d}{2} \left( \frac{c_\theta - t^{CO}}{c_\theta} \right)^2 + \frac{(t^{CO})^2}{2c_\theta} - \frac{c_\theta d}{2(c_\theta + d)}
\]

\[
= \frac{\Omega^2 d^4 [(c_L - c_\theta)vc_L^2 + (c_H - c_\theta)(1 - v)c_L^2]^2}{2c_\theta^2(d + c_\theta)}
\]

Expected excess social costs \( XSC \) are then, under taxation:

\[
XSC(t^{CO}) = vXSC_L + (1 - v)XSC_H = \frac{\Omega d^4 Var(c_\theta)}{2(d + c_\theta)(d + c_L)}
\]

with the variance of \( c_\theta \) given by:

\[
Var(c_\theta) = v(1 - v)(c_H - c_L)^2
\]

5.3. Asymmetric information, tradable permits

Expected social costs with tradable permits are, from (5), (19) and (29):

\[
\hat{SC}(E) = \frac{1}{2} dE^2 + \frac{1}{2} vC_L (1 - E)^2 + \frac{1}{2} (1 - v)c_H (1 - E)^2
\]

The first order condition with respect to \( E \) is:

\[
(d + \bar{c}_\theta)E = \bar{c}_\theta
\]

with the expected value of \( c_\theta \) given by:

\[
\bar{c}_\theta = vC_L + (1 - v)c_H
\]

Solving for \( E \) yields:

\[
E^{CO} = \frac{\bar{c}_\theta}{d + \bar{c}_\theta}
\]

Substituting (39) into (29), we find the equilibrium permit price \( p_\theta \) and adoption share \( \pi_\theta \) for cost realization \( \theta \):

\[
p_\theta(E^{CO}) = \frac{dc_\theta}{d + \bar{c}_\theta}, \quad \pi_\theta(E^{CO}) = \frac{c_\theta d(1 - \theta)}{F(d + \bar{c}_\theta)}
\]

Following the same reasoning as in the case of taxation, we substitute from (39) into (29) and subtract expected first best social costs obtained from (20) to calculate expected excess social costs under emissions trading and commitment:

\[
XSC(E^{CO}) = \hat{SC}(E^{CO}) - \frac{vC_L d}{2(c_L + d)} - \frac{(1 - v)c_H d}{2(c_H + d)} = \frac{d^2 Var(c_\theta)}{2(d + c_H)(d + c_L)(d + \bar{c}_\theta)}
\]

5.4. Comparison

In this section we compare emission permits and emission taxes under commitment, both with first best levels and with each other, in terms of welfare and technology adoption.

5.4.1. Technology adoption

Let us first analyze ex post technology adoption, i.e. for a given realization of \( \theta \).\(^{11}\)

\(^{11}\) See Appendix A for the proofs of Propositions 1, 2 and 5.
Proposition 1. Under commitment, taxation (emissions trading) leads ex post to over(under) investment for a favourable cost realization, and to under(over) investment for an unfavourable cost realization.

Intuitively, under emission taxation, the second best tax rate must be between the ex post optimal level for $\theta = L$ and the ex post optimal level for $\theta = H$. Then, as the share of adopting firms is increasing in the tax rate for a given firms’ type, we can easily conclude that ex post overinvestment (underinvestment) takes place when $\theta = L$ (when $\theta = H$). With tradable permits, it is straightforward to show from (17) and (39) that the total amount of permits must be between the ex post optimal amount for $\theta = L$ and the ex post optimal amount for $\theta = H$. This clearly implies that the second best permits price is lower (higher) than in the social optimum when $\theta = L$ (when $\theta = H$). Since, for a given firms’ type, the share of adopting firms is increasing in the permit price by (27), then tradable permits lead to ex post underinvestment when $\theta = L$ and overinvestment when $\theta = H$, the opposite of what happens under emission taxation.

Let us now turn to the ex ante expected share of adopters $\bar{\pi}$.

Proposition 2. Expected adoption with tradable permits under commitment is lower than under first best.

Fig. 2 illustrates the intuition behind Proposition 2 for $\nu = (1/2)$. Define $E(\bar{\pi})$ as the emission level that would lead to first best expected adoption $\bar{\pi}^{*} = v\pi_{L}^{*} + (1 - v)\pi_{H}^{*}$, and $\bar{E}$ as the probability-weighted average of the full-information optimal emission levels. Under $\nu = (1/2)$, $\bar{E}$ is halfway between the first best levels $E_{L}^{*}$ and $E_{H}^{*}$. Thus the increase in emissions from $E_{L}^{*}$ to $\bar{E}$ for $\theta = L$ is the same as the decrease in emissions from $E_{H}^{*}$ to $\bar{E}$ for $\theta = H$. However, this change decreases technology adoption more for $\theta = L$ than it increases it for $\theta = H$, because firms are more sensitive to a change in total allowed emissions when the cost of the new technology is low, since by (29)\textsuperscript{12}:

$$\frac{\partial \pi_{L}}{\partial E} - \frac{\partial \pi_{H}}{\partial E} = \frac{(1 - H)c_{H} - (1 - L)c_{L}}{F} < 0$$

As a result, $E(\bar{\pi}^{*}) < \bar{E}$. It can also be shown that $E^{CO} > \bar{E}$. At $\bar{E}$, the marginal social cost related to emissions increase from $E_{L}^{*}$ to $\bar{E}$ under type $L$, i.e. distance $BJ$ in Fig. 2, is smaller than the marginal social cost of reducing emissions from $E_{H}^{*}$ to $\bar{E}$ under type $H$, distance $GB$.\textsuperscript{13} As optimality requires

\textsuperscript{12} We prove this inequality in Appendix A.2.

\textsuperscript{13} The proof is as follows. Distance $GB$ equals $AMAC_{H}(\bar{E}) - MD(\bar{E})$, while distance $BJ$ equals $MD(\bar{E}) - AMAC_{L}(\bar{E})$. From (5) and
that the two marginal social costs be equal to each other, then \( \tilde{E} \) is too low to be optimal and thus \( E^C > \tilde{E} \). This in turn implies that \( E(\pi^*) < E^C \), so that, as adoption is decreasing in the aggregate cap, expected adoption under tradable permits is smaller than first best. The intuition is as follows. Because AMAC\(_H\) is steeper than AMAC\(_L\), the regulator is more worried about emissions turning out too low in hindsight when \( \theta = H \) than about emissions turning out too high when \( \theta = L \). As a result, the regulator sets allowed emissions relatively closer to \( E_H^* \) rather than \( E_L^* \), and expected adoption will be lower than in the optimum.

Turning to ex ante expected adoption under taxation, we get a less straightforward result. Expected adoption under taxation is more likely to be higher than optimal (and than expected adoption under tradable permits) when the fixed cost of adoption is high, technology improvement is relatively insignificant, and the damage parameter is small.\(^{14}\) The intuition is that all these factors make the AMAC\(_L\) curves, and especially the AMAC\(_H\) curve, relatively steep compared to the MD curve. This means that the regulator is more worried about the welfare loss from underinvestment for \( \theta = H \) than about overinvestment for \( \theta = L \). As a result, she will set a relatively higher tax rate, closer to \( \tau_H^* \) and further from \( \tau_L^* \). This higher tax rate results in higher than optimal expected adoption. Clearly, when expected adoption exceeds first best, it also exceeds expected adoption under tradable permits, as the latter always falls short of first best expected adoption from Proposition 2.

5.4.2. Welfare

Comparing excess social costs between instruments yields, from (35) and (41):

\[
\Delta = XSC(t^C) - XSC(E^C) = \frac{2\Omega d^2 \text{Var}(c_\theta)}{2(d + \tilde{e}_\theta)} [d - \tilde{e}_\theta]
\]

(42)

where \( \tilde{e}_\theta \) is the expected value of the slope of the AMAC\(_L\) curve given by (38) and \( \text{Var}(c_\theta) \) is the corresponding variance, given by (36). Crucially, \( \tilde{e}_\theta \) as defined in (31) is the average slope of the AMAC curve, featuring reverse probability weights: the slope of the AMAC\(_L\) curve when \( \theta = L \) is weighted with the probability that \( \theta = H \), and vice versa. This leads us to the following proposition:

**Proposition 3.** Under commitment, taxation is preferred to tradable permits if and only if the slope \( d \) of the Marginal Damage (MD) curve is smaller than the weighted average of the slopes of the Marginal Abatement Cost curves AMAC\(_L\) and AMAC\(_H\) in (15), with reverse probability weights.

Eq. (42) is reminiscent of Weitzman (1974) in that taxation is preferred if and only if the weighted average slope of the Marginal Abatement Cost (MAC) curve is larger than the slope of the Marginal Damage (MD) curve. However, as we have seen, the probability weighting is reversed in determining the average slope \( \tilde{e}_\theta \). This is a new result in the “prices vs. quantities” literature which, following Weitzman (1974), has mainly concentrated on additive uncertainty (i.e. about the intercept of the MAC curve). In our setting, the cost parameter enters the individual firm’s marginal abatement cost function in an additive way in (3). However, when we take endogenous technology choice into account, the cost parameter alters the slope of the industry’s aggregate marginal abatement cost curve by (15) and (16). There is thus multiplicative uncertainty about the AMAC curve (i.e. uncertainty about its slope). Weitzman (1974, p. 486; 1978) and Malcomson (1978) derive expressions for the comparative advantage of prices over quantities under multiplicative uncertainty. However, the role of reverse probability weighting is not apparent from these expressions nor is it discussed by the authors.\(^{15}\)

We can explain the reverse probability weighting with the aid of Fig. 3. Suppose the regulator is practically certain that the cost realization is \( H \). She would then issue \( E_H^* \) permits and set the tax rate at

\[ AMAC_H(\tilde{E}) + AMAC_L(\tilde{E}) - 2MD(\tilde{E}) = \frac{d(c_L - c_H)^2}{2(d + c_H)(d + c_L)} > 0 \]

\(15\) The formal proof is available from the corresponding author upon request.

τ∗H, both given by (17). If, against all expectations, the cost realization is L, the welfare loss is RSN with emission permits and ZRF with taxation. The welfare loss is larger with taxes, because AMAC_L is flatter than MD. This result is similar to Weitzman (1974), however the new element is that it is the slope of the AMAC curve in the unlikely scenario (that the cost realization is L in this example) that is relevant for the comparison between tradable permits and taxes. In general, the slope of the AMAC curve in the less likely scenario receives the larger weight. This explains the reverse probability weighting of the AMAC slopes in (42).

6. Time consistency

In this section, we derive the subgame perfect equilibrium for time consistency. In stage one of the time consistency game, firms choose whether to invest in the new technology, as analyzed in Section 4.2. In stage two, having observed the proportion of investing firms, the regulator sets the emission tax rate or the total amount of permits to be auctioned. In stage three, firms decide on their emission level and (under tradable permits) permit purchases, as analyzed in Section 4.1.

In Section 6.1, we first establish the full-information benchmark. We analyze emission taxation (tradable permits) under asymmetric information in Section 6.2 (6.3).

6.1. Full information benchmark

In this subsection we establish the subgame perfect equilibrium under time consistency, given that the regulator knows the cost realization θ. In stage two of the game, the regulator knows the proportion π of firms that have invested in the new technology in stage one, and she knows that in stage three the firms will set their emissions according to (21) and (22). The regulator minimizes the sum of variable abatement cost and environmental damage which, from (1), (3), (5), (21) and (22) is:

\[ SC^{TC}_θ = (1 - π) \frac{1}{2} (1 - e_1)^2 + π \frac{1}{2} (θ - e_0)^2 + \frac{1}{2} \partial E_θ^2 = \frac{1}{2} \Lambda_θ^2 + \frac{1}{2} \partial E_θ^2 \]  \hspace{1cm} (43)

16 If the regulator is practically certain that the cost realization is L, she would issue E_L^π permits and set the tax rate at τ∗L, both given by (17). If the cost realization is H, the welfare loss is RSR with tradable permits and SGK with taxation. The welfare loss is larger with permits, because AMAC_H is steeper than MD.
With emission taxation, substituting (23) into (43) yields:

$$ SC^{TC}_{\theta}(t_\theta) = \frac{1}{2} t_\theta^2 + \frac{1}{2} d \left[ 1 - t_\theta - \pi(1 - \theta) \right]^2 $$

(44)

Solving the first order condition for $t_\theta$ yields:

$$ t^{TC}_\theta = \frac{d \left[ 1 - \pi(1 - \theta) \right]}{d + 1} $$

(45)

In stage one, the share of adopting firms as a function of $x = t^{TC}_\theta$ is given by (27). Solving (27) and (45) for $\pi_\theta$ and $t_\theta$, we find the full-information first best values of $\pi^*_\theta$ from (18) and $t^*_\theta$ from (17), respectively.

With auctioned permits, substituting (23) into (43) yields:

$$ SC^{TC}_{\theta}(E_\theta) = \frac{1}{2} \left[ 1 - E_\theta - \pi(1 - \theta) \right]^2 + \frac{1}{2} dE^2_\theta $$

(46)

Solving the first order condition for $E_\theta$ yields:

$$ E^{TC}_\theta = \frac{1 - \pi \left( 1 - \theta \right)}{d + 1} $$

(47)

In stage one, the share of adopting firms is given by (29). Solving (29) and (47) for $\pi_\theta$ and $E_\theta$, we find the full-information first best values of $\pi^*_\theta$ from (18) and $E^*_\theta$ from (17).

We thus confirm Requate and Unold’s (2001) finding that under perfect information, time consistency implements the first best with tradable permits and emission taxation when regulating pollution and technology adoption by a continuum of heterogeneous firms. Now we turn to compare emission taxation and emission permits under time consistency and asymmetric information.

6.2. Asymmetric information, emission taxation

In this subsection we establish the subgame perfect equilibrium under emission taxation, given that the regulator does not know the cost realization $\theta$. In stage two of the game, the regulator knows the proportion $\pi$ of firms that have invested in the new technology in stage one. Having observed $\pi$, the regulator updates her beliefs on the cost realization. Let us denote the regulator’s stage-2 probability that $\theta = L$ by $q$. The regulator also knows that in stage three the firms will set their emissions according to (21) and (22) with $x = t$. The regulator minimizes the sum of variable abatement cost and environmental damage according to her updated beliefs:

$$ SC^{TC}(t) = qSC^{TC}_L(t) + (1 - q)SC^{TC}_H(t) $$

where, analogous to (44):

$$ SC^{TC}_{\theta}(t) = \frac{1}{2} t^2 + \frac{1}{2} d \left[ 1 - t - \pi(1 - \theta) \right]^2 $$

Solving the first order condition for $t$ yields:

$$ t^{TC} = \frac{d \left[ 1 - \pi (1 - qL - (1 - q)H) \right]}{d + 1} $$

(48)

We wish to specify beliefs in such a way that they generate the full-information first best as the unique solution given $\theta$. Furthermore, $q$ should be nondecreasing in $\pi$ and beliefs should be consistent.
The latter condition means that in any candidate equilibrium, \( q > 0 \) for \( \theta = L \) and \( q < 1 \) for \( \theta = H \). A \( q(\pi) \) function that satisfies these conditions (for emission taxation as well as for tradable permits) is:

\[
q(\pi) = \begin{cases} 
0 & \text{for } \pi \in [0, \pi_H^*] \\
\pi_H^*(\pi - \pi_H^*) / (\pi_L^* - \pi_H^*) & \text{for } \pi \in (\pi_H^*, \pi_L^*) \\
1 & \text{for } \pi \in [\pi_L^*, 1] 
\end{cases}
\] (49)

Fig. 4 illustrates the \( q(\pi) \) function for \( H = 0.8, L = 0.6, F = 0.32, d = 2 \), so that by (18), \( \pi_H^* = (5/13) \) and \( \pi_L^* = (5/8) \).

Substituting (49) into (48) yields:

\[
t_{TC} = \begin{cases} 
t_H^{TC} & \text{for } \pi \in [0, \pi_H^*] \\
t_q^{TC} & \text{for } \pi \in (\pi_H^*, \pi_L^*) \\
t_L^{TC} & \text{for } \pi \in [\pi_L^*, 1] 
\end{cases}
\] (50)

with \( t_q^{TC} \) given by (45) and:

\[
t_q^{TC} = \frac{d \left[ (1 - \pi H)(\pi_L^* - \pi_H^*) - (H - L)\pi_L^*(\pi - \pi_H^*) \right]}{(d + 1)(\pi_L^* - \pi_H^*)}
\]

We see that \( t_{TC} \) is a continuous, decreasing and piecewise linear function of \( \pi \). Fig. 5 illustrates the outcome for \( d = 2, H = 0.8, L = 0.6, F = 0.32 \).

Moving to stage one, we know from Section 4.2 that the share of adopting firms as a function of \( x = t \) is given by (27) with:

\[
\frac{\partial x(t)}{\partial t} = \frac{1 - \theta}{F} > 0, \quad \frac{\partial x(t)}{\partial \theta} = -\frac{t}{F} < 0
\] (51)
Fig. 5 shows $\pi_L$ and $\pi_H$ in ($\pi$, $t$) space for the specific parameter values. By (51), $\pi_{\theta}$ is increasing in $t$ and the $\pi_H$ curve is to the left (and above) the $\pi_L$ curve.

Solving (27) and (50) for $\pi_{\theta}$ and $t^{TC}_{\theta}$, we find that the full-information first best combination of $(\pi^*_\theta, t^*_\theta)$ from (18) and (17) respectively, is a solution. Indeed, as illustrated in Fig. 5, this is the unique solution because $\pi_{\theta}$ is increasing in $t$ and $t^{TC}$ is continuous and decreasing in $\pi$. We can therefore conclude the following:

**Proposition 4.** In the subgame-perfect equilibrium under time-consistent emissions taxation, and under regulator’s beliefs as specified in (49), the regulator correctly infers the cost realization from observing the share of adopting firms. As a result, the full-information first best is the only equilibrium.

6.3. Asymmetric information, tradable emission permits

In this subsection we establish the subgame perfect equilibrium under tradable permits, given that the regulator does not know the cost realization $\theta$. In stage two of the game, the regulator knows the proportion $\pi$ of firms that have invested in the new technology in stage one. Having observed $\pi$, the regulator updates her beliefs on the cost realization. As with emission taxation, we assume that $q$, the regulator’s stage-2 probability that $\theta = L$, is given by (49). The regulator also knows that in stage three the firms will set their emissions according to (21) and (22) with $x = p_{\theta}$. The regulator minimizes the sum of variable abatement cost and environmental damage according to her updated beliefs:

$$SC^{TC}(E) = qSC^{TC}_L(E) + (1 - q)SC^{TC}_H(E)$$

where, analogous to (46):

$$SC^{TC}_{\theta}(E) = \frac{1}{2} [1 - (1 - \pi(1 - \theta))]^2 + \frac{1}{2} dE^2$$

The first order condition is:

$$q[1 - E - \pi(1 - L)] + (1 - q)[1 - E - \pi(1 - H)] = dE$$
Solving for $E$ yields:

$$E = \frac{1 - \pi [1 - qL - (1 - q)H]}{1 + d}$$  \hfill (52)

Substituting (49) into (52), $E^{TC}$ becomes:

$$E^{TC} = \begin{cases} 
  E_{H}^{TC} & \text{for } \pi \in [0, \pi_{H}^{*}] \\
  E_{q}^{TC} & \text{for } \pi \in (\pi_{H}^{*}, \pi_{L}^{*}) \\
  E_{L}^{TC} & \text{for } \pi \in [\pi_{L}^{*}, 1]
\end{cases}$$

with $E_{H}^{TC}$ given by (47) and:

$$E_{q}^{TC} = \frac{(1 - \pi [1 - H])(\pi_{L}^{*} - \pi_{H}^{*}) - (H - L)\pi_{H}^{*}(\pi - \pi_{H}^{*})}{(d + 1)(\pi_{L}^{*} - \pi_{H}^{*})}$$  \hfill (54)

Note that $E^{TC}$ is a continuous, piecewise linear and decreasing function of $\pi$. Fig. 6 illustrates $E^{TC}$ for the same parameter values as used in Fig. 5.

Moving on to stage one, we know from Section 4.2 that the share of adopting firms is given by (29) with:

$$\frac{\partial \pi_{q}(E)}{\partial E} = - \frac{1 - \theta}{F + (1 - \theta)^{2}} < 0, \quad \frac{\partial \pi_{q}(E)}{\partial \theta} = \frac{[(1 - \theta)^{2} - F](1 - E)}{[F + (1 - \theta)^{2}]^{2}} < 0$$  \hfill (55)

The second inequality follows from (9). Fig. 6 shows the $\pi_{L}$ and $\pi_{H}$ curves in $(\pi, E)$ space for specific parameter values. Note that the $\pi_{L}$ curve is to the right of the $\pi_{H}$ curve by $\partial \pi_{q}/\partial \theta < 0$ in (55).

It is easily seen that $(\pi_{H}^{*}, E_{H}^{*})$ is a solution to (29) and (53) for cost realization $\theta$. The following proposition shows that this is the unique solution, as Fig. 6 illustrates for the specific parameter values.
Proposition 5. **Under time consistency and tradable permits, and under regulator’s beliefs as specified in** (49), **the regulator correctly infers the cost realization from observing the share of adopting firms. As a result, the full-information first best is the unique equilibrium.**

7. Conclusion

Asymmetric information is an important reason why regulators struggle to set appropriate environmental policy. We have modelled the environmental regulation of an industry consisting of a continuum of small firms, with asymmetric information about the (fixed and variable) cost of a new abatement technology. Under commitment (time consistency), the regulator sets environmental policy before (after) the firms make their technology adoption decision. With commitment, the regulator cannot implement the first best (unlike with full information, as in Requate and Unold, 2001, 2003). Tradable permits lead to higher welfare than emission taxation if and only if the slope of the marginal damage curve is steeper than the probability-weighted slopes of the marginal abatement cost curves. While this result is similar to the Weitzman (1974) rule, the probability weighting is such that the slope of the high-cost curve is weighted by the probability that cost is low, and vice versa. We further found that time consistency allows the regulator to infer the cost of the new technology. The outcome is thus the same as under full information. We know from Requate and Unold (2001) that the first best is implemented in this case.

Since time consistency leads to a better outcome than commitment, one may wonder why we need to analyze commitment in any detail. Why doesn’t the regulator follow the time-consistent route of setting policy after the firms have made their investment decisions and uncertainty has been resolved? One reason may be that policy can only be set at certain fixed intervals. A regulator may not always have the opportunity to wait until uncertainty has been resolved, especially if there are several sources of uncertainty, emerging and resolving themselves at different points in time. Moreover, a regulator may not be able to respond swiftly once uncertainty has been resolved.

Similarly, one may wonder why the regulator can only commit to a single tax rate or amount of tradable permits. If the regulator could commit to a policy menu, with the tax rate or amount of permits dependent on the firms’ adoption decisions, she would be able to implement the first best. For instance, the regulator could commit to the time-consistent policy. One difficulty with this is that, again, the regulator would need to be able to swiftly implement the policy once the relevant information is available. In addition, this kind of policy menu may be difficult to design, to explain and to agree upon. Finally, the different situations (here: the adoption rate of the new technology) that determine the policy to be implemented must be defined on the basis of verifiable information. It is perhaps for these reasons that such policy menus are hardly observed in practice.

We therefore consider our time consistency scenario to be a stylized benchmark, as it is difficult to imagine an immediate adjustment of environmental policies to adoption choices by regulated firms. The type of asymmetric information modelled in our paper has therefore to be interpreted as one of the many possible bricks in a general comparison among environmental policy tools under commitment and time consistency, suggesting circumstances when, ceteris paribus, asymmetric information on abatement costs may shift the balance in favour of time consistency.

We have assumed that firms are heterogeneous only in their cost of adopting the new technology. This assumption is not, in itself, expected to affect our main results.

We have used quadratic functional forms for the abatement cost and damage functions in order to obtain definite results. This has allowed us to derive the modified Weitzman rule with reverse probability weighting. This result, like the original Weitzman rule, only holds if abatement cost and environmental damage are (or can be approximated by) quadratic functions of emissions.

We anticipate that time consistency would also implement the full-information first best with more general functional forms. However, care must be taken to specify the regulator’s beliefs such that this is the unique subgame perfect equilibrium.

The main reason why we found that time consistency implements the first best is our assumption that the industry consists of many small firms. This means that an individual firm considers itself too small to affect environmental policy. Our model is therefore most applicable to the regulation of a large number of polluters, such as the EU Emission Trading System or the now practically defunct Sulfur
Allowance Trading programme in the US (Schmalensee and Stavins, 2013). The small firms assumption is the main driving force behind our conclusion that firms make the socially optimal adoption decision, for a given tax rate or permits price, which in turn implies that the asymmetric information about fixed costs does not matter in equilibrium. As a result, the regulator, in facing informational asymmetries, only has to infer a single parameter concerning regulated firms’ costs. If, on the other hand, the industry contained a few large firms, these firms would generally not take the socially optimal adoption choice under any instrument or policy timing; for example, they would be able to affect time-consistent policy, so that it would typically not implement the first best anymore and might even be worse than commitment (Amacher and Malik, 2002). However, when there are large firms in the industry, this also means that the tradable permit market is not perfectly competitive anymore. Tradable permits would not lead to equalization of marginal abatement costs across firms, thereby introducing another difference between this instrument and emission taxation. Needless to say, tradable permits might still be second best and better for welfare than emission taxation, because of the other market failures and opportunities for firms’ strategic behaviour.

We have assumed that the emission permits are auctioned to the firms. This makes for the clearest comparison with emission taxation. With both instruments, the firms have to pay the government for all their emissions. More importantly, with either instrument the regulator can only set the value of a single variable (the tax rate or the total amount of permits). Grandfathering instead of auctioning permits gives the regulator another variable to set: the number of grandfathered permits. If the number of grandfathered permits is fixed, i.e. it does not change with the firm’s adoption decision or with the total adoption rate, the outcome will be the same in terms of adoption rate and permit price (as in Requate and Unold, 2001). Indeed, keeping the number of grandfathered permits fixed is the regulator’s best strategy. This is immediately clear for time consistency, where auctioned permits already implement the full-information first best, so that any deviation can only reduce welfare.

With commitment, we have seen that auctioned tradable permits result in too little expected adoption of the new technology. Thus it might seem that the regulator could increase welfare by specifying the grandfathering rules in a way that stimulates adoption. However, it should be borne in mind that given the total amount of permits issued, small firms in our setting will make the socially optimal adoption decisions when the number of grandfathered permits is fixed. Varying the number of grandfathered permits will only increase aggregate abatement costs (and thereby social costs) for a given level of permits. It does not help the regulator with the main problem that she has to set the total amount of permits before she knows (or can infer) the cost realization.

Finally, an interesting extension of this paper could be in the direction of more complex informational structures. For example, less straightforward conclusions in the time consistency case could be obtained assuming that the firms only learn their marginal abatement cost parameter after investing in the new technology, as, for example, in Krysiak (2008). We leave this issue for future research.

Appendix A. Proofs

A.1. Proposition 1

Under emission taxation, from (17) and (32):

$$\tau_L^* < t^<CO < \tau_H^*$$

(A.1)

As the share of adopting firms is increasing in $\tau$ by (13) and in $t$ by (27):

$$\pi_L(t^<CO) > \pi_L^*, \quad \pi_H(t^<CO) < \pi_H^*$$

with $\pi_L^*$ given by (18). Thus with taxation, there is ex post over-(under-)investment when $\theta = I(H)$. With tradable permits, from (17) and (39):

$$E_L^* < E^<CO < E_H^*$$
As total emissions are decreasing in the optimal AMAC by (15) and in \( p \) by (28), these inequalities imply:
\[
 p_L(E^{CO}) < \tau_L^\ast, \quad p_H(E^{CO}) > \tau_H^\ast
\]  
(A.2)
with \( \tau_H^\ast \) given by (17). Then, since the share of adopting firms is increasing in \( \tau \) by (13) and in \( p \) by (27):
\[
\pi_L(E^{CO}) < \pi_L^\ast, \quad \pi_H(E^{CO}) > \pi_H^\ast
\]
with \( \pi_H^\ast \) given by (18). Thus with tradable permits, there is ex post under- (over-) investment when \( \theta = L(H) \).

A.2. Proposition 2

We first compare the expected share of adopters under tradable permits to that with the optimal policy for each cost realization. From (18):
\[
\tilde{\pi}^\ast = \nu \pi_L^\ast + (1 - \nu) \pi_H^\ast = \frac{1}{F} \left( \nu(1 - L) \frac{dc_L}{d + c_L} + (1 - \nu)(1 - H) \frac{dc_H}{d + c_H} \right)
\]  
(A.3)
Define \( E(\tilde{\pi}^\ast) \) as the emission level (the same for both cost realizations) that would, under tradable permits, lead to \( \tilde{\pi}^\ast \). From (29) and (A.3):
\[
E(\tilde{\pi}^\ast) = \frac{(1 - \nu)(1 - H)c_H^2(c_L + d) + \nu(1 - L)c_L^2(c_H + d)}{(d + c_H)(d + c_L)(1 - \nu)(1 - H)c_H + \nu(1 - L)c_L}
\]  
(A.4)
Let us now compare \( E(\tilde{\pi}^\ast) \) to \( \bar{E} \), defined as the probability-weighted average of the full-information optimal emission levels given by (17):
\[
\bar{E} = \nu E_L^\ast + (1 - \nu)E_H^\ast = \nu \frac{c_L}{d + c_L} + (1 - \nu) \frac{c_H}{d + c_H}
\]  
(A.5)
From (A.4) and (A.5):
\[
E(\tilde{\pi}^\ast) - \bar{E} = \frac{dv(1 - \nu)(c_H - c_L) \Phi_E}{(d + c_H)(d + c_L)(\nu(1 - L)c_L + (1 - \nu)(1 - H)c_H)} < 0
\]  
(A.6)
The inequality follows from (16) and:
\[
\Phi_E \equiv (1 - L)c_L - (1 - H)c_H > 0
\]  
(A.7)
where \( \Phi_E > 0 \) follows from applying (9) to:
\[
\frac{\partial}{\partial \theta} \left[ c_\theta (1 - \theta) \right] = \frac{F \left[ F - (1 - \theta) \right]}{(F + (1 - \theta)^2)^2} < 0
\]
To conclude the proof, we need to compare \( \bar{E} \) to \( E^{CO} \). From (39) and (A.5):
\[
E^{CO} - \bar{E} = \frac{dv(1 - \nu)(c_H - c_L)^2}{(d + c_H)(d + c_L)(d + vc_L + (1 - v)c_H)} > 0
\]  
(A.8)
Putting (A.6) and (A.8) together, we see that \( E(\tilde{\pi}^\ast) < E^{CO} \). Since \( \pi \) is decreasing in \( E \) by (29), this means that expected adoption with tradable permits is lower than \( \tilde{\pi}^\ast \).
A.3. Proposition 5

We first note that there is no solution where \( \pi \in (\pi_H^\alpha, \pi_H^\gamma) \), because the only point of intersection between the \( E_H^{TC} \) curve in (54) and the \( \pi_H \) curve in (29) (both being linear) is \( (\pi_H^\alpha, E_H^\alpha) \).

Moreover, there is no candidate equilibrium where beliefs are inconsistent. There is no solution where \( \theta = L \) and \( q = 0 \), because the unique solution \( \pi_{LH}^{TC} \) to (29) with \( \theta = L \) and (47) with \( \theta = H \) lies outside the range where \( q = 0 \):

\[
\pi_{LH}^{TC} = \frac{d(1-L)}{F + (1-L)^2} \frac{(1+d) - (1-H)(1-L)}{F(d+1) + d(1-H)^2} = \pi_H^\gamma
\]

where the second equality follows from (18) and the inequality follows from:

\[
\pi_{LH}^{TC} - \pi_H^\gamma = \frac{\psi}{\left[ F + (1-L)^2 \right] (1+d) - (1-H)(1-L) \left[ F(d+1) + d(1-H)^2 \right]} > 0
\]

where by (9):

\[
\psi = d(1+d)(H-L)[F - (1-H)(1-L)] > 0 \tag{A.9}
\]

Neither is there a solution where \( \theta = H \) and \( q = 1 \), because the solution \( \pi_{HL}^{TC} \) to (29) with \( \theta = H \) and (47) with \( \theta = L \) lies outside the range where \( q = 1 \):

\[
\pi_{HL}^{TC} = \frac{d(1-H)}{F + (1-H)^2} \frac{(1+d) - (1-H)(1-L)}{F(d+1) + d(1-L)^2} = \pi_L^\gamma
\]

The inequality follows from:

\[
\pi_L^\gamma - \pi_{HL}^{TC} = \frac{\psi}{\left[ F + (1-H)^2 \right] (1+d) - (1-H)(1-L) \left[ F(d+1) + d(1-L)^2 \right]} > 0
\]

with \( \psi > 0 \) given by (A.9).

### Appendix B. Conditions for an interior solution

B.1. Positive emission levels by adopting firms

In this section we verify that all possible scenarios feature \( e_0 > 0 \) for any probability \( v \) of \( \theta = L \). From (22) this implies that the optimal MAC, the tax rate and the permit price must all be below \( \theta \).

B.1.1. Commitment and full-information social optimum

For \( \theta = L \), it follows from (A.1) and (A.2) that:

\[
p_L(E^CO) < \tau_H^* < t^CO \tag{B.1}
\]

Thus we need to make sure that \( t^CO < L \). From (A.1) we know that \( t^CO < \tau_H^* \). As a result:

\[
t^CO < \tau_H^* = \frac{d c_H}{d + c_H} < L \tag{B.2}
\]

The final inequality follows from (7) and (16).

For \( \theta = H \), it follows from (A.1) and (A.2) that:

\[
t^CO < \tau_H^* < p_H(E^CO) \tag{B.3}
\]

Thus we need to make sure that \( p_H(E^CO) < H \). From (40) this implies:

\[
p_H(E^CO) = \frac{d c_H}{d + v c_L + (1-v)c_H} < \frac{d c_H}{d + c_L} = p_H(E^CO)_{v=1} < H \tag{B.4}
\]
The first inequality follows from the fact that the LHS is increasing in \( v \). Noting that \( c_H > H \) by (6) and (16), the second inequality can be rewritten as:
\[
d < \frac{Hc_L}{c_H - H} \tag{B.5}
\]

Condition (7) is a sufficient condition for (B.5) to hold, because using (16), the former can be rewritten as:
\[
d < \frac{Lc_H}{c_H - L}
\]
and
\[
\frac{Lc_H}{c_H - L} < \frac{Hc_L}{c_H - H} \Leftrightarrow \frac{L}{c_L(c_H - L)} < \frac{H}{c_H(c_H - H)}
\]
where the second inequality holds because, from (16):
\[
\frac{\partial (\theta/(c_0(c_H - L)))}{\partial \theta} = \frac{F \left[ (1 - 2\theta) (\theta - 1)^2 + F \right] + (1 - H)^2 2\theta^2 (1 - \theta)}{c_H \left[ \theta(1 - H)^2 - F(1 - \theta) \right]^2} > 0
\]

The inequality follows from \((1/2) < L < \theta < H < 1\) and (6).

### B.1.2. Time consistency

Under time consistency, \( e_0 > 0 \) should hold in any subgame where the regulator believes there is a positive probability that \( \theta \) occurs. From (22) and (49), this means that \( t^{TC}_\theta, p^{TC}_H < H \) for \( q < 1 \), i.e. for \( \pi \in [0, \pi\_H^*] \) and \( t^{TC}_\theta, p^{TC}_L < L \) for \( q > 0 \), i.e. for \( \pi \in (\pi\_H^*, 1] \).

For taxation and \( q < 1 \), we find from (50):
\[
t^{TC}(\pi) \leq t^{TC}(0) = \frac{d}{d + 1} < H \tag{B.6}
\]
The first inequality follows from the fact that \( t^{TC} \) is decreasing in \( \pi \). The second inequality follows from (8).

For taxation and \( q > 0 \), so that \( \pi \in (\pi\_H^*, 1] \), we find from (17) and (50):
\[
t^{TC}(\pi) \leq t^{TC}(\pi\_H^*) = \tau\_H < L \tag{B.7}
\]
The first inequality follows from the fact that \( t^{TC} \) is decreasing in \( \pi \). The second inequality follows from (B.2).

For tradable permits and \( q < 1 \), note first that for \( \pi \in [0, \pi\_H^*] \) where \( q = 0 \), we have \( p_H(E^{TC}_q, \pi) = t^{TC}_q(\pi) < H \) by (23), (50), (53) and (B.6). From (23) and (53), for \( \pi \in (\pi\_q^*, \pi\_L^*) \) and \( \theta = L, H \), the total differential of \( p_\theta(E^{TC}_q, \pi) \) with respect to \( \pi \) is constant and may be positive:
\[
\frac{dp_\theta(E^{TC}_q, \pi)}{d\pi} = \frac{\partial p_\theta(E^{TC}_q, \pi)}{\partial E^{TC}_q} \frac{dE^{TC}_q}{d\pi} + \frac{\partial p_\theta(E^{TC}_q, \pi)}{\partial \pi} = \frac{(1 - L)\pi\_L^* - (1 - H)\pi\_H^*}{(1 + d)(\pi\_L^* - \pi\_H^*)} - (1 - \theta) \tag{B.8}
\]
We thus have to verify that \( p_H(E^{TC}_q, \pi\_L^*) = p_H(E^*_q, \pi\_L^*) < H \). From (29) and (B.4) we see that:
\[
p_H(E^*_q, \pi\_L^*) < p_H(E^*_q, \pi\_H(E^*_q)) = p_H(E^{CO})_{v=1} < H
\]
The first inequality follows from \( \partial p_H/\partial \pi < 0 \) by (23) and \( \pi_H(E^*_q) < \pi_L(E^*_q) = \pi\_L^* \) by (17), (29) and (55). The second inequality follows from (B.4).

For tradable permits and \( q > 0 \), note first that for \( \pi \in (\pi\_L^*, 1] \) where \( q = 1 \), we have \( p_\theta(E^{TC}_q, \pi) = t^{TC}_q(\pi) < L \) by (23), (50), (53) and (B.7). We know from (B.8) that for \( \pi \in (\pi\_H^*, \pi\_L^*) \), the total
differential of \( p_l(E_q^C, \pi) \) with respect to \( \pi \) is constant and may be negative. We thus have to verify that \( p_l(E_q^C, \pi^*_H) = p_l(E_q^H, \pi^*_H) < L \). From (17) and (29) we see that:

\[
p_l(E_q^H, \pi^*_H) < p_H(E_q^H, \pi^*_H) = \tau^*_H < L
\]

The first inequality follows from \( \partial p_l/\partial \theta > 0 \) in (23). The second inequality follows from (B.2).

### B.2. Less than complete adoption

In this section we verify that \( \pi_\theta < 1 \) in all scenarios. We know from Section 6 that time consistency implements the full-information social optimum. Thus we only need to check that \( \pi_\theta < 1 \) in the full-information social optimum and with commitment.

From (27), \( \pi_\theta \) is increasing in \( x \). Thus we need to make sure that \( \pi_\theta < 1 \) holds for the highest possible \( x \), which from (B.3) and (B.1) implies:

\[
\pi_l(t^{CO}) < 1, \quad \pi_H(E^{CO}) < 1
\]  
(B.9)

From (27), (51), (55), (B.2) and (B.4), we know that:

\[
\pi_l(t^{CO}) < \pi_l(\tau^*_H) = \frac{dc_H}{d + c_H} \frac{1 - L}{F}
\]  
(B.10)

\[
\pi_H(E^{CO}) < \pi_H(E^*_L) = \frac{dc_H}{d + c_L} \frac{1 - H}{F}
\]  
(B.11)

From (B.9) to (B.11) we thus need to make sure that:

\[
\frac{dc_H}{d + c_H} \frac{1 - L}{F} < 1 \]  
(B.12)

\[
\frac{dc_H}{d + c_L} \frac{1 - H}{F} < 1 \]  
(B.13)

Inequality (B.12) holds because:

\[
\frac{dc_H}{d + c_H} \frac{1 - L}{F} < \frac{L(1 - L)}{F} < 1
\]

The first inequality follows from (7) and (16). The second inequality follows from (6). Since (B.12) holds, (B.13) also holds, because:

\[
\frac{1 - L}{d + c_H} > \frac{1 - H}{d + c_L}
\]

The inequality follows from applying (9) to:

\[
\frac{d}{d\theta} \left\{ (1 - \theta)(d + c_\theta) \right\} = -d - \frac{F}{d + c_l} \frac{F - (1 - \theta)^2}{1 - \theta^2} < 0
\]

**References**


