TOWARD AN ECONOMIC THEORY OF LIABILITY

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RECENTLY there have appeared a number of important articles by both lawyers and economists analyzing the economic effects of liability rules.[1] This paper formalizes the analysis of these effects. When two parties, the injurer and the victim, can both take measures to reduce the likelihood of accidents, and the measures are costly for both, the standard theory of production with two inputs and one output yields the conditions for the socially optimal amount of each accident avoidance measure. The effects of decentralizing the problem and using only liability rules to solve it can then be analyzed in terms of a two-person noncooperative game. I first show that there is a complete symmetry within each of the following pairs of liability rules: no liability and strict liability; the negligence rule and strict liability with contributory negligence; and the negligence rule with contributory negligence and strict liability with what I call dual contributory negligence. An analysis of the legal standards for negligence follows. I show that there is an important ambiguity in the so-called "Learned Hand Rule" for determining the level of avoidance effort below which a party is adjudged negligent. Two of the formulations of the rule lead to inefficient results.

Next the main results of the paper are presented. First, given the negligence rule with contributory negligence and what I call the Incremental Standard of care, the social optimum is shown to be the unique equilibrium for the noncooperative game. A corollary is that the same results hold when the negligence standard is replaced by strict liability with contributory negligence. When the Incremental Standard is replaced by the Limited Information

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[1] See Guido Calabresi, The Costs of Accidents: A Legal and Economic Analysis (1970), and (with Jon T. Hirschoff), Toward a Test for Strict Liability in Torts, 81 Yale L.J. 1055 (1972); R. H. Coase, The Problem of Social Cost, 3 J. Law & Econ. 1 (1960); Harold Demsetz, When Does the Rule of Liability Matter?, 1 J. Leg. Studies 13 (1972); Roland N. McKean, Products Liability: Trends and Implications, in Symposium on Products Liability, 38 U. Chi. L. Rev. 3 (1970); Richard A. Posner, A Theory of Negligence, 1 J. Leg. Studies 29 (1972), and Strict Liability: A Comment, 2 J. Leg. Studies 205 (1973).

Incremental Standard the identity between equilibrium and optimality is destroyed. In an appendix the Coase Theorem is restated in our framework and compared with our results.

I. The Model

Consider a small device, a black box, which is attached to some otherwise useful object such as a railway crossing, an airplane, or a sidewalk. The only function of the device is to emit a bill for a large amount of money from time to time, so we shall call it a liability generator. That large amount of money is fixed at A.

On the liability generator are two controls, X and Y. Each of these controls is continuously variable. Increasing either or both increases the probability that the accident will be avoided, but at a decreasing rate. The probability that an accident is avoided in a given time interval is denoted $P(X, Y)$, so the probability that an accident occurs in that interval is $1 - P(X, Y)$.[2] Figure 1 shows the relationship between X, Y and $P(X, Y)$. Examples of what will be meant here by controls are built-in safety devices and careful driving in the railway crossing case, defect-free radar and careful flying in the airplane case, and shoveling snow and careful walking in the sidewalk case.

By this description of accident avoidance we have constructed an almost exact analogue of the neoclassical production function, familiar to economists, where there are two inputs, X and Y, producing one output, $P(X, Y)$, which in our case is the probability that an accident will be avoided.[3] Just as the inputs to production are costly and should be economized on, so the accident avoidance controls X and Y are costly. Let the cost per unit of X be W_x and the cost per unit of Y be W_y. The total cost for a given level of X and Y is then

[2] The time interval is chosen small enough that the probability of more than one accident in the interval is negligible.

[3] I say an almost exact analogue because we must take explicit account of the possibility that the accident can be completely avoided and further amounts of accident avoidance contribute nothing. Formally, we assume (1) that $P(X, Y)$ is continuous and twice differentiable in X and Y, (2) that the marginal products of both inputs are nonnegative, and (3) marginal products do not increase. The possibility of complete avoidance admits the possibility that marginal products and their rate of change are zero. Our final assumption about $P(X, Y)$ is (4) that $P_{xy}(X, Y) \equiv \frac{\partial P^2(X, Y)}{\partial X \partial Y} < 0$. We shall discuss this assumption later in the analysis.

I have chosen to have two controls to concentrate on the necessity in most accident contexts of having more than one party take costly measures to avoid accidents. Formally, it is a trivial extension to have more than two parties, but the legal implications of explicitly recognizing that there are more than two parties with responsibilities to avoid accidents should be explored carefully. The occasional cases where only one party can avoid the accident are special cases of this analysis where the equiprobability curves of Figure 1 would be either horizontal or vertical straight lines.

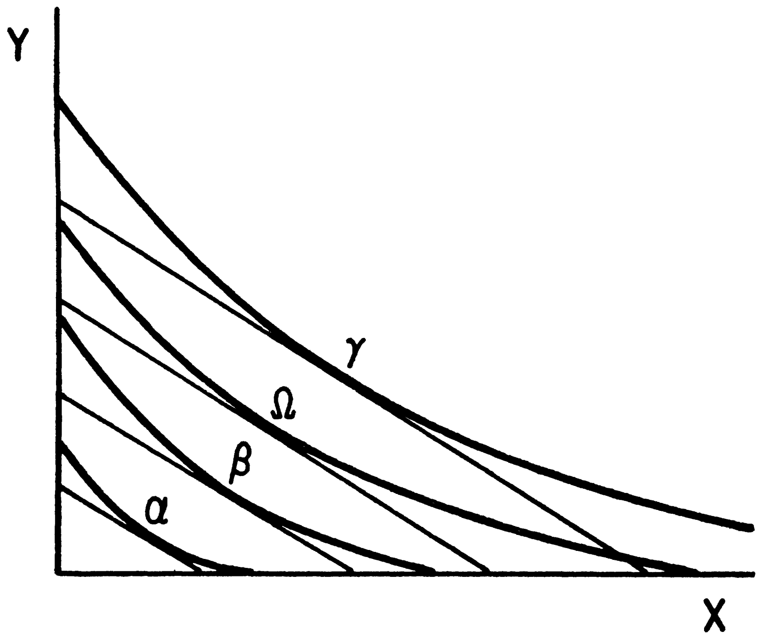


FIGURE 1

$W_x X + W_y Y$. The straight lines in Figure 1 are all of the combinations of X and Y having the same total cost. This completes the transformation of the accident avoidance problem into the standard production theory format. To find the least cost combination to X and Y which will yield a given probability of accident avoidance one need only find the point on the equiprobability curve just tangent to a cost line. Examples are drawn in Figure 1 at α, β, γ and Ω.

Which combination of avoidance measures X and Y and the resulting probability of accident P(X, Y) is the most preferred? To answer that question we consider the case where both X and Y are under the control of the same party. In the simple case first before us, all quantities are measured in terms of money, so it makes no difference who makes the decision. The outcome we shall call the social optimum, and we shall use it to evaluate outcomes when X and Y are controlled by different people, which is of course the most interesting problem.

The social optimum we shall define as that combination of avoidance measures which minimizes the sum of the costs of the controls and the expected cost of the accident. With the apparatus that we have developed it is now a simple matter to write down the problem and to characterize the solution.

Formally, the socially optimal values of X and Y are those that minimize social cost, $C_S(X, Y)$. Thus

$$\min_{X, Y} C_S(X, Y) = W_x X + W_y Y + A[1 - P(X, Y)]. \tag{1}$$

The first two terms are the costs of the two controls, for example the costs of the crossing gates and of slower, more cautious driving in the railway crossing case. The last term is the expected cost of the accident and is the product of the cost of the accident and the probability that it will occur.

For the social cost to be at a minimum it must be true that the marginal cost of each input be equal to the expected value of its marginal product. That is

$$W_x = A\,P_x(X, Y) \qquad \text{and} \tag{2}$$

$$W_y = A\,P_y(X, Y) \tag{3}$$

where $P_x(X, Y)$ denotes the marginal product of X and $P_y(X, Y)$ denotes the marginal product of Y. The optimal solution can be found by solving equations (2) and (3) simultaneously for X and Y. Call the values which minimize social cost X_Ω and Y_Ω. Then it is true for all values of X and Y that

$$C_S(X_\Omega, Y_\Omega) \leqslant C_S(X, Y) \qquad \text{for all X and Y.} \tag{4}$$

The optimal values are those that would be chosen if both X and Y were under common control. The problem that interests us here is where X and Y are controlled by different people.

Before proceeding further it will be helpful to deal with two misleading concepts, one used by lawyers and the other used by economists. The first is the notion that liability ought to be placed on whoever caused the accident. In the scheme we have described, who caused the accident, the victim or the injurer? They both did. The problem is exactly analogous to the futile arguments among economists in the nineteenth century about whether capital or labor causes production. They both do. We must go on to further considerations to decide how the fruits of production should be divided; similarly we have to decide who should bear the liability for accidents when they are jointly caused. The only role for discussions of causation is to exonerate a party completely by proving that he is a stranger to the accident.

A similar issue arises with the notion of "least cost avoider." A number of writers, among them Calabresi and Demsetz, have recommended placing liability on the best cost avoider. For example, Demsetz states: "It is difficult to suggest any criterion for deciding liability other than placing it on the party able to avoid the costly interaction most easily."[4] Looking back at

[4] Harold Demsetz, *supra* note 1, at 28.

Figure 1, how can we identify the best cost avoider? One possible meaning would be to compare the cost of the victim's completely avoiding the accident (*i.e.*, setting $P(0, Y) = 0$, with the cost of the injurer's completely avoiding the accident $P(X, 0) = 0$). The party with the lowest cost would be the cheapest cost avoider. Another possible interpretation of the phrase "least cost avoider" is that it is a local notion depending on (X, Y), that is, on what the parties in fact chose to do. This leaves unanswered how liability should be assigned in order to induce the parties to take the optimal amount of avoidance action. The notion of least cost avoider is thus likely to confuse rather than to clarify matters.

II. Decentralization Using a Liability Rule

When the two avoidance mechanisms are in the hands of different people, for example the manufacturer and the driver when the liability generator is attached to an automobile, or the manufacturer and patient in the case of a drug, the question arises as to how the bill shall be paid when an accident takes place. The question is answered by the court. Since this paper is concerned with how injurer and victims can be expected to respond under different liability rules I have chosen to provide only the simplest abstraction of the court and its functioning. For our purposes the only function of the legal system is the impeccable administration of whatever liability rule is in force. In order to concentrate on what is our major concern, the parties' behavior, we ignore the distinction between judge and jury, the problems of proof and burden of proof, the uncertainty of the outcome, the mistakes that the court can make, and the expense of operating the legal system.

To fix ideas for the rest of the discussion, let X be controlled by Xavier the Injurer, and Y by Yvonne the Victim. The rule that determines which party must pay the bill emitted by the liability generator is called a liability rule. The liability rule will in general be a function of the level of avoidance chosen by each party and will be expressed as the fraction of the bill, A, owed by each party. Let $L_x(X, Y)$ and $L_y(X, Y)$ be the liabilities of Xavier and Yvonne, respectively. Then

$$L_x(X, Y) \geqslant 0,\ L_y(X, Y) \geqslant 0 \qquad \text{for all } X, Y \tag{5}$$

$$L_x(X, Y) + L_y(X, Y) = 1 \qquad \text{for all } X, Y. \tag{6}$$

The amount that must be paid by Xavier when an accident takes place is $A\,L_x(X, Y)$ and the amount to be paid by Yvonne is $A\,L_y(X, Y)$.

A number of different liability rules have been discussed in the recent literature.[5] They are simple to put into the framework we propose but for some

[5] See Calabresi, Calabresi and Hirschoff, and Posner works cited in note 1 *supra*.

of them we require one further notion, that of a legal standard of negligence. Let (X^*, Y^*) be the legal standard of negligence. If X is less than X^* then the injurer will be found legally negligent. Similarly, if Y is less than Y^*, the victim will be found negligent. Now we are ready to describe the liability rules.

1. *No liability*. The victim is liable under all circumstances.

$$L_x(X, Y) = 0 \qquad L_y(X, Y) = 1 \qquad \text{for all } X, Y. \tag{7}$$

2. *Strict liability*. The injurer is liable under all circumstances.

$$L_x(X, Y) = 1 \qquad L_y(X, Y) = 0 \qquad \text{for all } X, Y. \tag{8}$$

3. *The negligence rule*. The victim is liable unless the injurer is found negligent.

$$L_x(X, Y) = \begin{cases} 0 \\ 1 \end{cases} \quad L_y(X, Y) = \begin{cases} 1 & \text{if } X \geqslant X^* \\ 0 & \text{if } X < X^* \end{cases}. \tag{9}$$

4. *Strict liability with contributory negligence*. The injurer is liable unless the victim is found negligent.

$$L_x(X, Y) = \begin{cases} 1 \\ 0 \end{cases} \quad L_y(X, Y) = \begin{cases} 0 & \text{if } Y \geqslant Y^* \\ 1 & \text{if } Y < Y^* \end{cases}. \tag{10}$$

5. *The negligence rule with contributory negligence*. The injurer is liable if he is negligent *and* the victim is not. The victim is liable otherwise.

$$L_x(X, Y) = \begin{cases} 1 \\ 0 \end{cases} \quad L_y(X, Y) = \begin{cases} 0 & \text{if } X < X^* \text{ and } Y \geqslant Y^* \\ 1 & \text{otherwise} \end{cases}. \tag{11}$$

The negligence rule with contributory negligence has been the dominant rule in tort law since *Brown v. Kendall*.[6] In that landmark case the defendant (the injurer) took up a stick to separate fighting dogs belonging to the plaintiff (the victim) and the defendant. In the course of beating the dogs the defendant accidentally hit the plaintiff in the eye, injuring him severely. Chief Justice Shaw laid down the rule: ". . . if both plaintiff and defendant at the time of the blow were using ordinary care, or if at that time the defendant was using ordinary care, and the plaintiff was not, or if at that time, both the defendant and the plaintiff were not using ordinary care, then the plaintiff could not recover."[7]

Notice that there is complete symmetry in our model between no liability and strict liability on the one hand and between the negligence rule on the

[6] 60 Mass. (6 Cush.) 292 (1850).

[7] *Id.* at 296.

one hand and strict liability with contributory negligence on the other. The strict liability rule that would be symmetrical to the negligence rule with contributory negligence has been mentioned only by Calabresi and Hirschoff in the recent literature. It might be called:

6. *Strict liability with dual contributory negligence.* The victim is liable if he is negligent *and* the injurer is not. The injurer is liable otherwise.

$$L_x(X, Y) = \begin{cases} 0 \\ 1 \end{cases} \quad L_y(X, Y = \begin{cases} 1 & \text{if } Y < Y^* \text{ and } X \geqslant X^* \\ 0 & \text{otherwise} \end{cases} . \tag{12}$$

The six liability rules are displayed graphically in Figure 2.

There are two further liability rules that are somewhat outside the tradition encompassed in the six rules presented here in that the standard of care for one party is the level of care exercised by the other party. They are relative negligence and comparative negligence.

7. *Relative negligence.* The injurer is liable if the increment to accident avoidance per dollar of avoidance by him is greater than that per dollar of avoidance by the victim, *i.e.*, if a dollar spent by the injurer could have bought more avoidance than a dollar spent by the victim.

In our notation the liability rule for the injurer is

$$L_x(X, Y) = \begin{cases} 1 & \text{if} \quad \dfrac{P_x(X, Y)}{W_x} > \dfrac{P_y(X, Y)}{W_y} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$L_y(X, Y)$ is, as usual, the complement of $L_x(X, Y)$, that is,

$$L_y(X, Y) = 1 - L_x(X, Y) \tag{14}$$

Graphically, there is a very simple interpretation of the relative negligence rule. The slope of the equiprobability lines (isoquants) is $-\dfrac{P_x(X, Y)}{P_y(X, Y)}$ and the slope of the equal cost lines is $-\dfrac{W_x}{W_y}$. Therefore the injurer is negligent at (X, Y) if the equiprobability line is steeper than the equal cost line at (X, Y). Notice that at any (X, Y) either the injurer or the victim is relatively negligent. In case of ties we have arbitrarily assumed that the victim is liable. The negligence boundary is the locus of least cost combinations for different probabilities. See Figure 3.

8. *Comparative negligence.* The doctrine of comparative negligence apportions the liability according to the relative liability of the two parties. This doctrine is established in admiralty law, but otherwise it is not recognized

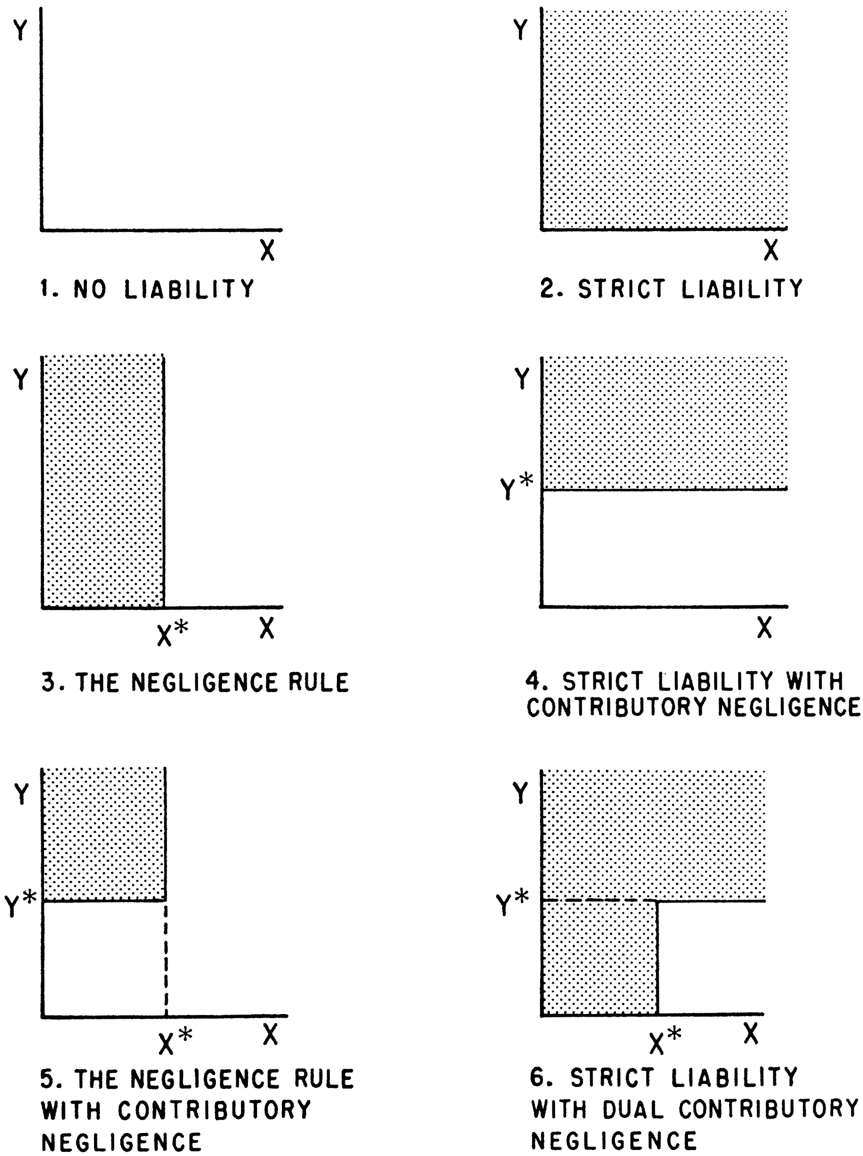


FIGURE 2 (INJURER LIABLE IN SHADED AREA)

in any state except where it is established by statute, *e.g.*, in Wisconsin, Georgia, and Mississippi.[8] It is not clear whether negligence is defined as a

[8] See Charles O. Gregory & Harry Kalven, Jr., Cases and Materials on Torts 250-54 (2d ed. 1969).

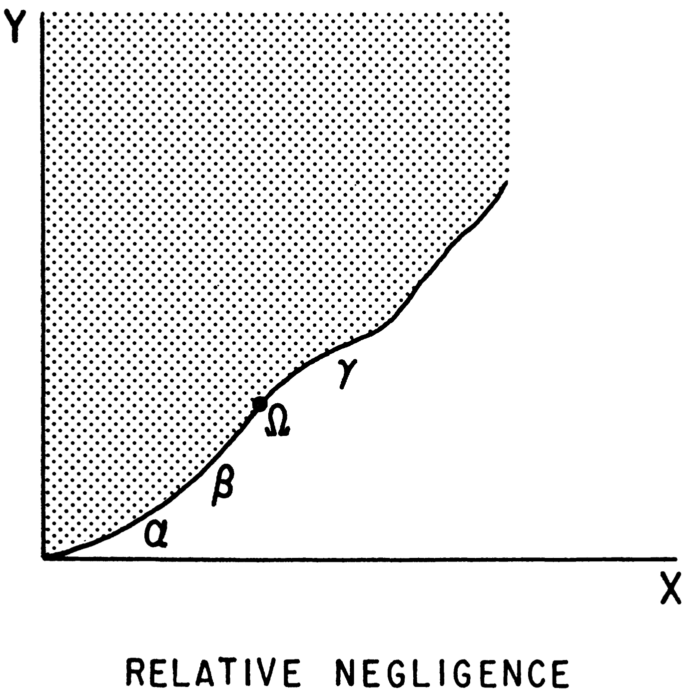


FIGURE 3 (INJURER LIABLE IN SHADED AREA)

marginal concept or an average one. If a marginal negligence concept is used, then, if n_x is the negligence of the injurer and n_y the negligence of the victim,

$$n_x = \frac{P_x(X, Y)}{W_x} \quad \text{and} \quad n_y = \frac{P_y(X, Y)}{W_y} \tag{15}$$

Negligence is the incremental reduction in accident probability per dollar spent, and the liability of the injurer is his negligence divided by the negligence of both parties:

$$L_x(X, Y) = \frac{n_x}{n_x + n_y} \tag{16}$$

The liability of the victim is $1 - L_x(X, Y)$.

III. THE LEGAL STANDARD OF NEGLIGENCE

We noted above that some of the liability rules require a legal determination of negligence by one party or the other. Call the legal standard of negligence for the injurer X* and for the victim Y*. In legal discussions this is often called the standard of care. If X is found to be less than X* then the injurer will be judged negligent. Similarly, if Y is found to be less than Y* the victim will be judged negligent.

The negligence standard most often cited in legal discussions of the problem is the so-called Learned Hand Rule,[9] which can be paraphrased as follows. The duty of a party is a function of three variables: (1) the probability of an accident; (2) the gravity of the accident; and (3) the burden of adequate precaution. Liability depends on whether the product of the first two is greater than the third. In our notation, the probability of an accident is $1 - P(X, Y)$, the gravity of the accident is A, and the burden of precaution is $W_x X$ for the injurer and $W_y Y$ for the victim. Thus, the Learned Hand Rule compares $A(1 - P(X, Y))$ with $W_x X$ for the injurer and with $W_y Y$ for the victim.

The Learned Hand Rule is ambiguous in important ways, as we shall show. In fact there are three closely related standards of negligence that can be derived from Judge Hand's formulation. The first we shall call the Literal Learned Hand Standard, or the Literal Standard for short. If we take literally the standard as given by Judge Hand, the injurer would be found negligent if

$$W_x X < A(1 - P(X, Y)) \tag{17}$$

and the victim would be found negligent if

$$W_y Y < A(1 - P(X, Y)). \tag{18}$$

This standard compares the total cost to a party with the expected cost of the accident.

There is one question for which the Literal Standard unambiguously gives the correct answer, but it is typically the wrong question, and might well suggest the wrong answer to the right question. Consider Figure 4. As drawn, the expected benefits are greater than the costs to the injurer for every level of X. The question that the Literal Standard answers correctly is this: Is it better to provide complete protection, $X = T$, rather than no protection at all, $X = 0$? Then we need only compare the product of the probability of an accident and the gravity of the accident, TC, with the burden of adequate precaution, TB. In the case described in the figure, because TC is greater than TB, it is better to provide complete protection than no protection at all. Should it be the duty of the injurer to provide complete protection in this case? Clearly the answer is no. Although it is better to provide full protection than none at all, the optimal amount of protection from an overall point of view is Ω, where the marginal cost of protection is equal to the marginal expected benefit from the protection. By moving from

[9] United States v. Carroll Towing Co., 159 F.2d 169 (2d Cir. 1947)

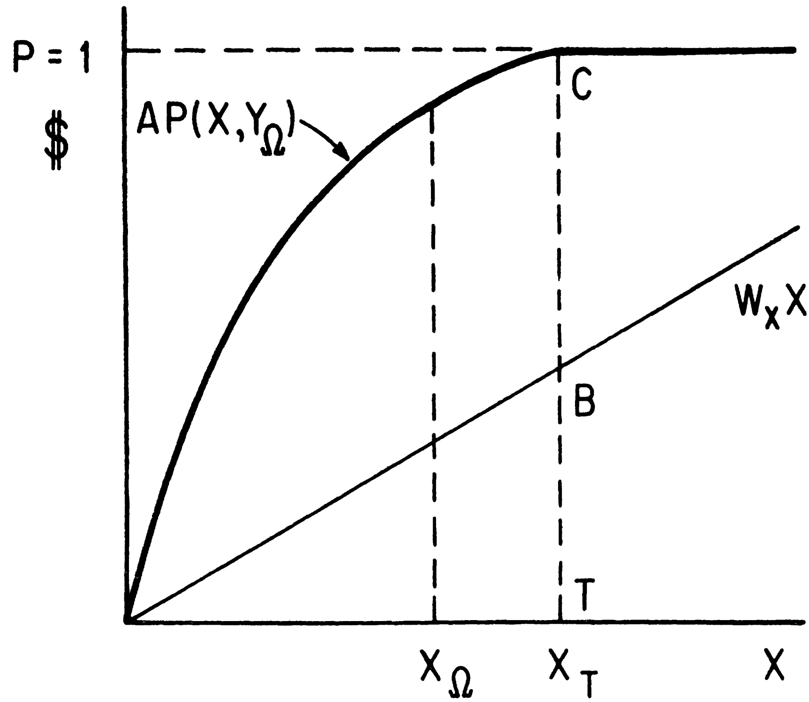


FIGURE 4

X_T to X_Ω costs would decrease more than benefits. Clearly, we need a better negligence standard.[10]

The second standard derivable from Judge Hand's formulation is the Incremental Standard. Here we assume that the judicial system can ferret out complete information about the underlying technology of accident prevention so that negligence for one party is determined on the *assumption that the other party is already acting in an optimal manner.* Thus the negligence standard for the injurer, X^*, is that value of X such that

$$W_x = A\,P_x(X^*, Y_\Omega) \tag{19}$$

and for the victim is Y^*, that value of Y such that

$$W_y = A\,P_y(X_\Omega, Y^*). \tag{20}$$

[10] Calabresi and Hirschoff seem to have been misled by a literal reading of the Learned Hand rule. They say, for example (*supra* note 1, at 1057), "If the cost to the defendant of avoiding the accident would have been less than the cost of the accident, discounted by the probability of its occurrence, the defendant's failure to avoid the accident is termed negligence." This is the Learned Hand Literal Standard, in our terminology.

Under the Incremental Standard the negligence standard is identical to the conditions for social cost minimization independently for both the injurer and the victim. Thus $X^* = X_\Omega$ and $Y^* = Y_\Omega$. Solving for the Incremental Standard is equivalent to identifying the social cost minimizing solution. The court uses the Incremental Standard to find the social optimum and defines any avoidance less than optimal as negligent. Below we shall investigate the effect of using the Incremental Standard with the various liability rules.

The third standard we shall call the Limited Information Incremental Standard. Rather than assuming that the court knows the entire technology of accident prevention as we did in the full information standard, we assume here that the court is able to investigate only the effects on the probability of an accident of small changes in the immediate neighborhood of the (X, Y) pair actually chosen by the two parties. In mathematical terms this means that the court is only able to determine the partial derivatives of P(X, Y) at (X, Y). In effect, if the court uses the Limited Information Incremental Standard it is saying that an avoidance level is negligent if it is less than that which would be optimal *given what the other party has in fact done*. The injurer will be held negligent at some point (X, Y) if that X is less than would be required to minimize social cost, treating Y as fixed. Thus the lowest value of X for which the injurer will not be judged negligent is the minimum with respect to X of

$$C_S(X, Y) = W_xX + W_yY + A(1 - P(X, Y))$$

The necessary marginal conditions for a minimum imply that the injurer will be found negligent at (X, Y) if

$$W_x < A\,P_x(X, Y) \tag{21}$$

and the victim will be found negligent at (X, Y) if

$$W_y < A\,P_y(X, Y). \tag{22}$$

To emphasize that the level of X that just allows the injurer to avoid being found negligent, X^*, depends on the avoidance actions taken by the victim, we shall use the functional notation $X^*(Y)$. Similarly, since the level of Y that just allows the victim to avoid being found negligent depends on the actions taken by the injurer, we write $Y^*(X)$.

The Incremental Standards are a good approximation, I think, of the way that courts actually proceed. The attorney for the plaintiff will try to find some act which, if the defendant had taken it, would have significantly reduced the probability of the accident at low cost. But that is precisely the statement that the increment in the expected loss was greater than the cost of avoidance, which is the definition of the Incremental Standards of negligence. The defendant will try to respond that the expected benefits of the proposed act were, in fact, less than the costs of undertaking it. When the court is asked to decide between the two points of view it is being asked to compare the incremental expected benefits with the incremental costs. Thus it is not peculiar that the outcome of large, important cases often seems to turn on the value of small changes in the behavior of one party or the other.

IV. A Noncooperative Game Played According to Liability Rules

What are the equilibrium values of X and Y when they are chosen by different people independently? How will Xavier and Yvonne behave when their only communication with each other will be in court after an accident occurs? All other transactions are assumed to be so costly that they do not take place.

We shall assume that the goal of Xavier is to choose X to minimize his expected private costs, that is:

$$\min_{X} C_x(X, Y) = W_x X + A\,L_x(X, Y)(1 - P(X, Y)) \tag{23}$$

and Yvonne minimizes her private costs:

$$\min_{Y} C_y(X, Y) = W_y Y + A\,L_y(X, Y)(1 - P(X, Y)). \tag{24}$$

These are simply the sum of the cost of their own preventive efforts and their expected liability when an accident occurs.

Both parties are assumed to know A, W_x, W_y, and $P(X, Y)$ as well as the liability rule in force and that the other party is interested only in private costs. Our problem is to try to find equilibria for the games and determine whether they are unique. An equilibrium for a game is a pair of values (X, Y) chosen by the two players such that, given the choice by the opponent, neither party has an incentive to change his own choice.

Before analyzing the various rules it will be useful to describe two response functions for each party. They are called response functions because, like the response functions in Cournot's analysis of duopoly, they show what response a party would choose, given the choice of the opponent.

The first response function we call $X^f(Y)$. The superscript f denotes full cost. It is defined as the injurer's preferred value of X given Y, assuming that the injurer must pay in full for any damages. That is, $X^f(Y)$ is that X for which $W_x X + A(1 - P(X, Y))$ is at a minimum. Alternatively, $X^f(Y)$ is the value of X which minimizes social cost when Y is fixed, since the only difference between private and social costs is the constant $W_y Y$, since Y is considered fixed. It is a necessary condition of minimization that at each value of $X^f(Y)$

$$W_x = A\,P_x(X^f, Y). \tag{25}$$

We take the total differential of this condition

$$A\,P_{xx}\,dX^f + A\,P_{xy}\,dY = 0 \tag{26}$$

and solve it for $\frac{dX^f}{dY}$

$$\frac{dX^f}{dY} = -\frac{P_{xy}}{P_{xx}}. \tag{27}$$

This gives us the change in the optimal X for a change in Y. Convexity implies that $P_{xx} < 0$. Therefore if $P_{xy} < 0$ as we assume, then

$$\frac{dX^f}{dY} < 0 \tag{28}$$

that is, a decrease in Y will call forth an increase in X.

Throughout the rest of our analysis we shall employ one slightly restrictive assumption about the technology of accident avoidance, namely that $P_{xy} < 0$. The technical effect of the assumption is to insure that the full cost response functions X^f and Y^f are both downward sloping. If they were upward sloping there could be a second intersection on one axis or the other and hence a second equilibrium. Assuming $P_{xy} < 0$ is equivalent to assuming that a decrease in accident avoidance by one party ought to call forth an increase in accident avoidance by the other. For example, if cars drive faster on one residential street than on another we would expect mothers to take greater care that their children do not stray onto the first street than onto the second, or as trains approach a crossing more rapidly we would expect cars to approach more slowly. Only in the bizarre case where the technology of accident prevention were such that the appropriate response to faster trains would be *less* caution on the part of drivers would the assumption of $P_{xy} < 0$ be inappropriate.

Correspondingly, there is, of course, the response function for the victim, $Y^f(X)$, which is the victim's preferred value of Y given X, assuming that the victim must pay in full for any damages. Therefore $Y^f(X)$ is the value of X which minimizes $W_yY + A[1 - P(X, Y)]$. Figure 5 shows $X^f(Y)$ and $Y^f(X)$.

The second response function we shall consider gives the party's preferred strategy, given the opponent's strategy, for the actual expected private costs, $C_x(X, Y)$ or $C_y(X, Y)$, which depend on the liability rule in force. This response function is denoted $X^e(Y)$ for the injurer and $Y^e(X)$ for the victim. The superscript e signifies equilibrium. $X^e(Y)$ is that value of X for each Y

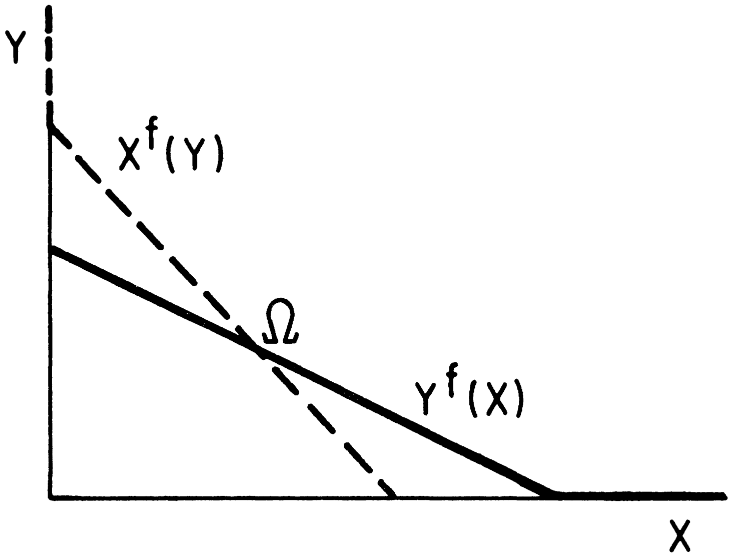


FIGURE 5

that minimizes $C_x(X, Y)$. For example, if the liability rule is the negligence rule with contributory negligence and the negligence standard is the Incremental Standard,

$$C_x(X, Y) = \begin{cases} W_xX + A[1 - P(X, Y)] & \text{if } X < X_\Omega \text{ and } Y \geqslant Y_\Omega \\ W_xX & \text{otherwise.} \end{cases} \tag{29}$$

In the same case $Y^e(X)$ is that value of Y which minimizes the victim's expected private costs, which are

$$C_y(X, Y) = \begin{cases} W_yY & \text{if } X < X_\Omega \text{ and } Y \geqslant Y_\Omega \\ W_yY + A[1 - P(X, Y)] & \text{otherwise.} \end{cases}$$

$X^e(Y)$ and $Y^e(X)$ are plotted in Figure 6. The shaded area is the area where the injurer is liable.

V. Equilibrium Under the Various Liability Rules

1. *No Liability*

First consider the rule of no liability. All bills emitted by the liability generator are to be paid by the victim, Yvonne, without regard to any measures either she or the injurer took. Then the costs paid by the injurer are only the costs of his own avoidance measures, so his problem is to

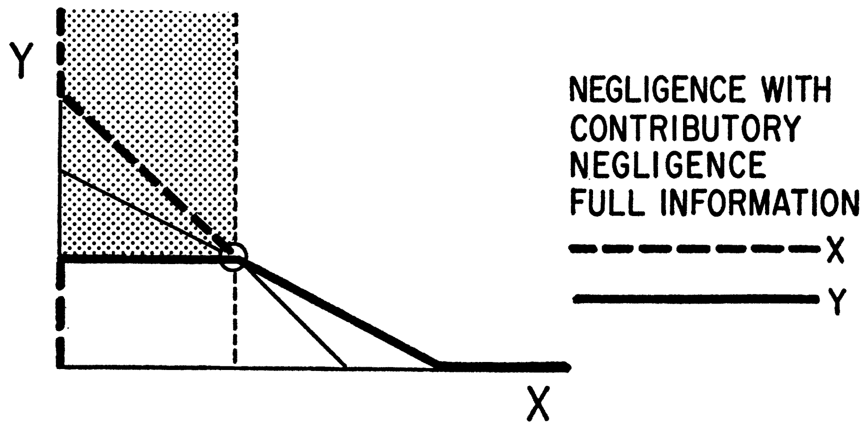


FIGURE 6

$$\min_{X} C_x(X, Y) = W_x X \tag{30}$$

It is obvious that he will choose $X = 0$; there are no benefits to him from avoiding accidents. In other words $x^e(Y) = 0$ for all Y.

For the victim the task is to

$$\min_{Y} C_y(0, Y) = W_y Y + A(1 - P(0, Y)) \tag{31}$$

that is, minimize the sum of his avoidance costs and expected losses given that the injurer will do nothing. This sum will be at a minimum when

$$W_y = A\, P_y(0, Y) \tag{32}$$

that is, at $Y^f(0)$. The equilibrium values of the game are $X = 0$ and $Y = Y^f(0)$, rather than the social optimum (X_Ω, Y_Ω). It is obvious that the equilibrium is unique. See Figure 7a.

2. *Strict Liability*

Next consider the rule of strict liability. The injurer is responsible for all accidents. The argument is identical to that for the rule of no liability except that the roles of injurer and victim, X and Y, are reversed. The victim will have no incentive to take any precautions: the injurer will choose the best level of precaution given that the victim takes none. The equilibrium is $(X^f(0), 0)$. Again, it is obvious that the equilibrium is unique. See Figure 7b.

The situation changes radically when we move on to liability rules that apply negligence standards to one or both parties.

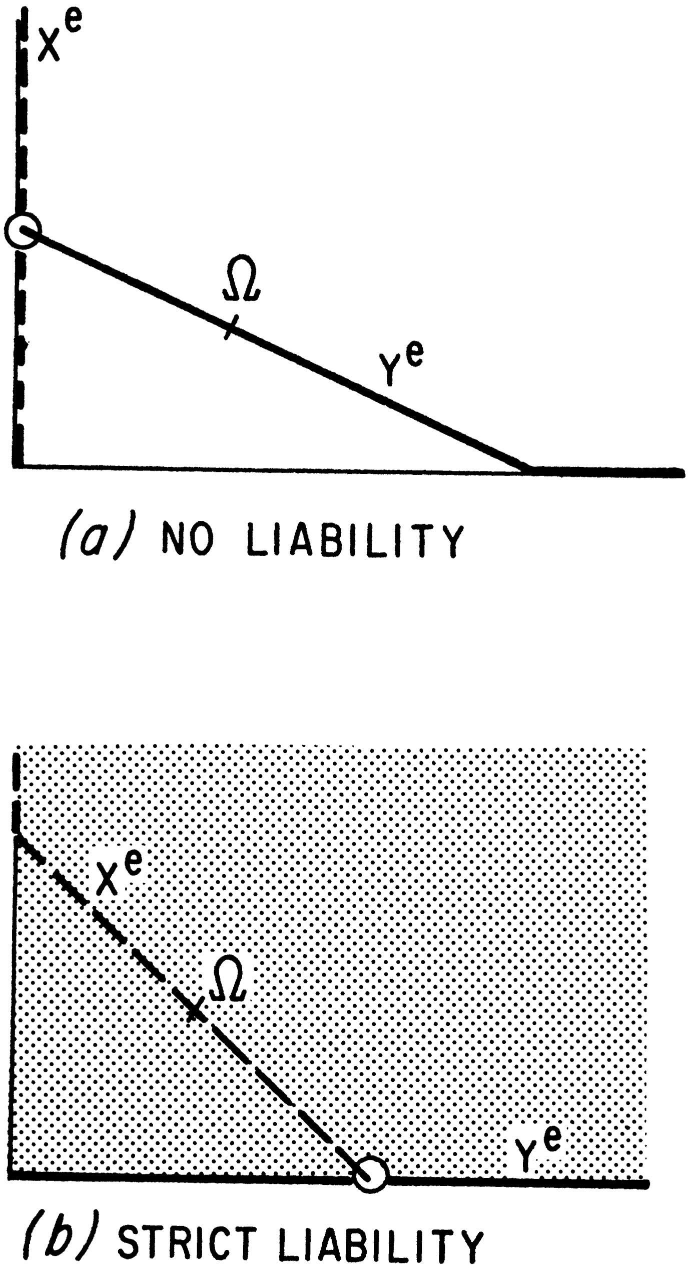


FIGURE 7

3. *Negligence Rule with Contributory Negligence, Incremental Standard*

We can now state the first of the major results of the paper.

Theorem 1: If the social optimum (X_Ω, Y_Ω) is a unique minimum of the social cost function $C_S(X, Y)$, the liability rule in force is the Negligence Rule with Contributory Negligence, the negligence standard is the Incremental Standard, and $P_{xy} < 0$, then the social optimum is a unique noncooperative equilibrium.

Proof of Theorem 1: First we show that (X_Ω, Y_Ω) is a non-cooperative equilibrium. For the injurer we have to show that his costs, given Y_Ω, are less at X_Ω than at any other value of X. That is, we have to show that

$$C_x(X_\Omega, Y_\Omega) \leqslant C_x(X, Y_\Omega) \qquad \text{for all X.} \tag{33}$$

Recall that

$$C_x(X, Y_\Omega) = \begin{cases} W_x X + A[1 - P(X, Y_\Omega)] & \text{if } X < X_\Omega \\ W_x X & \text{if } X \geqslant X_\Omega \end{cases} \tag{34}$$

This cost function is plotted in Figure 8a. It is clear that it is a minimum at X_Ω.

In analogous fashion we must show that given the injurer's choice of X_Ω, the victim can do no better than Y_Ω. See Figure 8b. That is, we must show that

$$C_y(X_\Omega, Y_\Omega) \leqslant C_y(X_\Omega, Y) \qquad \text{for all Y.} \tag{35}$$

This we do by writing out $C_y(X_\Omega, Y)$

$$C_y(X, Y) = W_y Y + A[1 - P(X_\Omega, Y)] \tag{36}$$

and minimizing it. The condition for its being a minimum is

$$W_y = A\,P_y(X_\Omega, Y) \tag{37}$$

which is identically a condition for the minimization of social cost at (X_Ω, Y_Ω). Therefore $C_y(X_\Omega, Y)$ is minimized at Y_Ω, which completes the proof that (X_Ω, Y_Ω) is a noncooperative equilibrium. What has been shown is that $X^e(Y_\Omega) = X_\Omega$ and $Y^e(X_\Omega) = Y_\Omega$.

To show that (X_Ω, Y_Ω) is a unique equilibrium we show that X^e and Y^e intersect only once, at (X_Ω, Y_Ω). See Figure 9a. A party will never choose a point where he is liable other than on the full cost equilibrium, X^f or Y^f. Also a party will never choose any point where he is not liable other than the one which requires him to do the least. Therefore, each party's equilibrium will be either on the full cost equilibrium X^f or Y^f or the lowest cost point where he is not liable. Thus in the figure X^e crosses the shaded area on X^f and then drops to zero as soon as the injurer is not liable. Similarly, Y^e stays

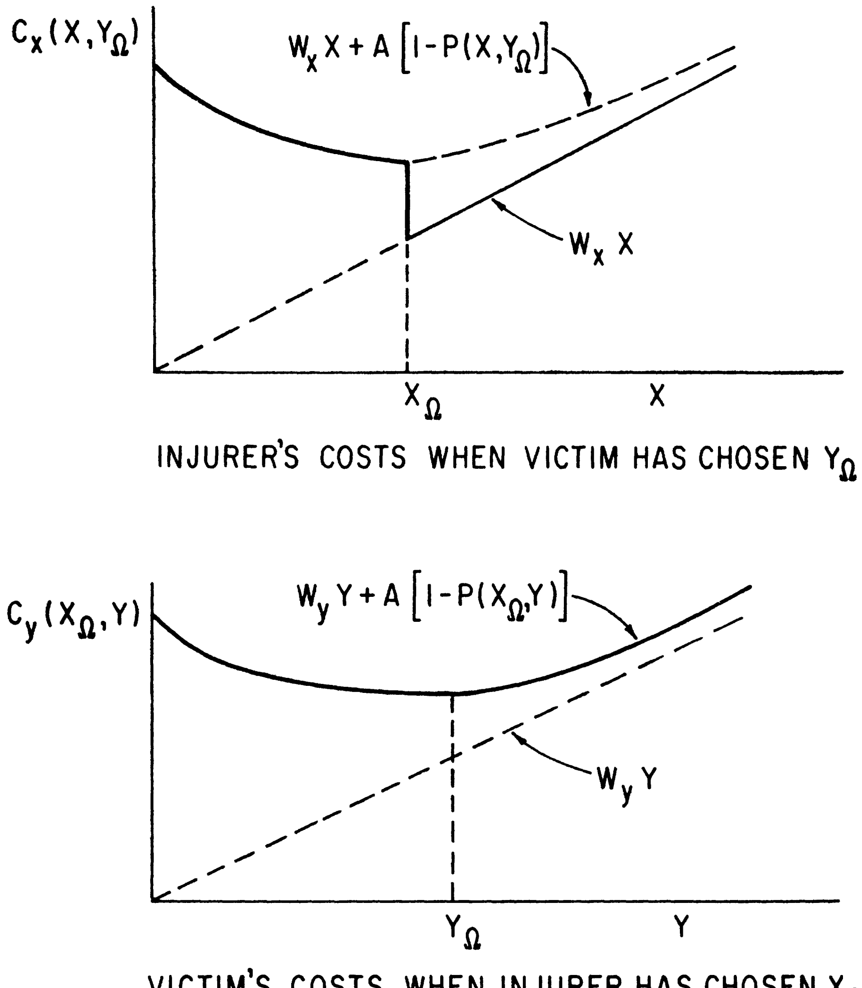


FIGURE 8

at the lowest part of the shaded area, where the victim is not liable, and then crosses the unshaded area, where she is liable, on Y^f. The only intersection of X^e and Y^e is at (X_Ω, Y_Ω); hence (X_Ω, Y_Ω) is a unique non-cooperative equilibrium, because both parties are in equilibrium there and only there.

4. *The Negligence Rule, Incremental Standard*

Corollary 1: The results of Theorem 1 hold if the liability rule is changed from the negligence rule with contributory negligence to the negligence rule.

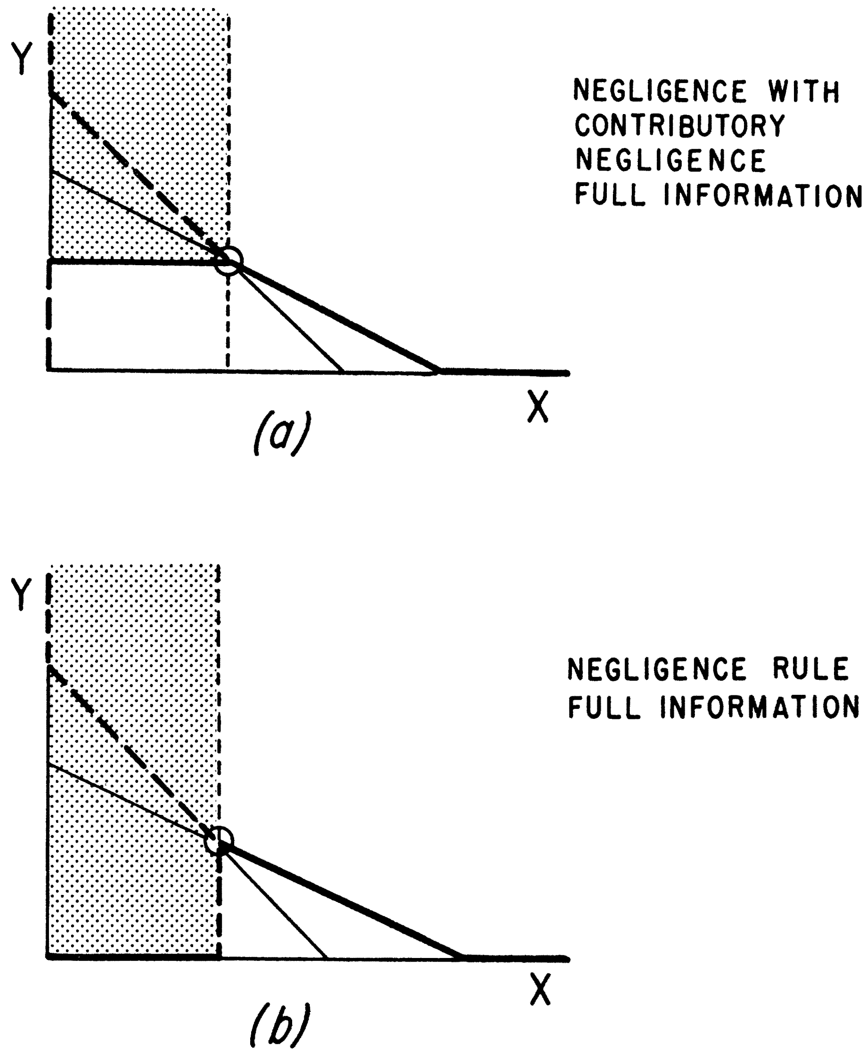


FIGURE 9

Proof: The equilibrium reaction functions $X^e(Y)$ and $Y^e(X)$ are plotted in Figure 9b for the case of the negligence rule and the Incremental Standard. The changes in the functions resulting from the change in liability rule are inconsequential from the point of view of the proof of the theorem. The reaction functions still only intersect at the social optimum, (X_Ω, Y_Ω).

5. *Strict Liability with Contributory Negligence and with Dual Contributory Negligence*

Corollary 2: The results of Theorem 1 hold if the liability rule is changed to either strict liability with contributory negligence or strict liability with dual contributory negligence.

Proof: To obtain the results for strict liability with contributory negligence we merely exchange the roles of X and Y in the proof of Corollary 1. To obtain the results for strict liability with dual contributory negligence we exchange X and Y in the argument of the theorem.

One can summarize the results on decentralized games with liability rules so far as follows: with either no liability or strict liability rules in effect the incentives on the party without liability are perverse so the resulting equilibrium will not be the optimal one. For all four remaining rules, which are based on negligence of one or both parties, given the Incremental Standard the game will have a unique equilibrium and that equilibrium is the social optimum. Within the framework of our analysis there is no preferred choice among any of the four rules.

Perhaps it would be useful to point out here what the analysis does not say. It does not say that there is no basis for choice among the four rules; only that the factors that we have chosen to analyze do not lead us to choose any one of the negligence rules. If the cost of using the courts to transfer liability is very high, then the negligence rule with contributory negligence becomes more attractive because it transfers liability least often. Within the scope of our analysis any of the negligence rules is preferable to either the no liability rule or strict liability. That preference could, of course, be overthrown by factors outside the analysis. One could construct an argument for no liability (as in the case of "no-fault" insurance schemes) by showing that the costs of administering the negligence system far outweigh the losses caused by the incorrect incentives which are inherent in no-fault schemes. Also our analysis assumes a completely informed court system that does not have difficulty in producing accurate evidence. There are no evidentiary problems and who has the burden of proof is irrelevant.

VI. The Limited Information Incremental Standard

Now we turn to the case where the court no longer knows what the social optimum is and only judges each party's behavior on the basis of small changes from where the parties actually were when the accident occurred. That is, we shall assume that the courts use the Limited Information Incremental Standard as the standard of care. The boundary between the injurer being negligent and not, $X^*(Y)$, is no longer a vertical line at X_Ω but rather is identical with $X^f(Y)$ passing through (X_Ω, X_Ω). Similarly, $Y^*(X) = Y^f(X)$.

1. *The Negligence Rule with Contributory Negligence*

Theorem 2: The results of Theorem 1 are destroyed when the Incremental Standard of negligence is replaced by the Limited Information Incremental Standard. The social optimum is not an equilibrium; in fact there is no equilibrium.

Proof: First we shall show that the social optimum, (X_Ω, Y_Ω), is not an equilibrium. To do so we need only consider the cost function of the injurer when the victim is at the social optimum $C_x(X, Y_\Omega)$.

$$C_x(X, Y_\Omega) = W_x X \tag{38}$$

That is, at no value of X is the injurer liable when $Y = Y_\Omega$; therefore the injurer will have no incentive to provide any accident avoidance measures at all. His costs will be at a minimum if $X = 0$. Therefore (X_Ω, Y_Ω) is not an equilibrium.

X^e and Y^e are plotted in Figure 10a for the case of the Limited Information Incremental Standard. The argument for the shape of the curves is the same as in Theorem I and does not need to be repeated. What appears to be an intersection at $(0, Y^f(0))$ is not. $X^e(Y^f(0))$ is not zero. (Only for $Y^f(0) - \epsilon$, where ϵ is arbitrarily small, is X^e zero.) Hence there is no equilibrium and the parties will cycle about as shown in Figure 10a.

Corollary 1: When the liability rule is changed from the negligence rule with contributory negligence to the negligence rule, the social optimum is again not an equilibrium.

Proof: Refer to Figure 10b. The logic of the proof is to confirm that X^e and Y^e are as drawn.

Corollary 2: Symmetrical results hold for strict liability with contributory negligence when the standard of care is the Limited Information Incremental Standard.

Proof: Merely exchange the roles of X and Y in Corollary 1.

Corollary 3: Symmetrical results hold for strict liability with dual contributory negligence when the standard of care is the Limited Information Incremental Standard.

Proof: Exchange the roles of X and Y in Theorem 2.

2. *Relative Negligence*

Theorem 3: If the liability rule in force is the relative negligence rule, then the social optimum is a unique noncooperative equilibrium.

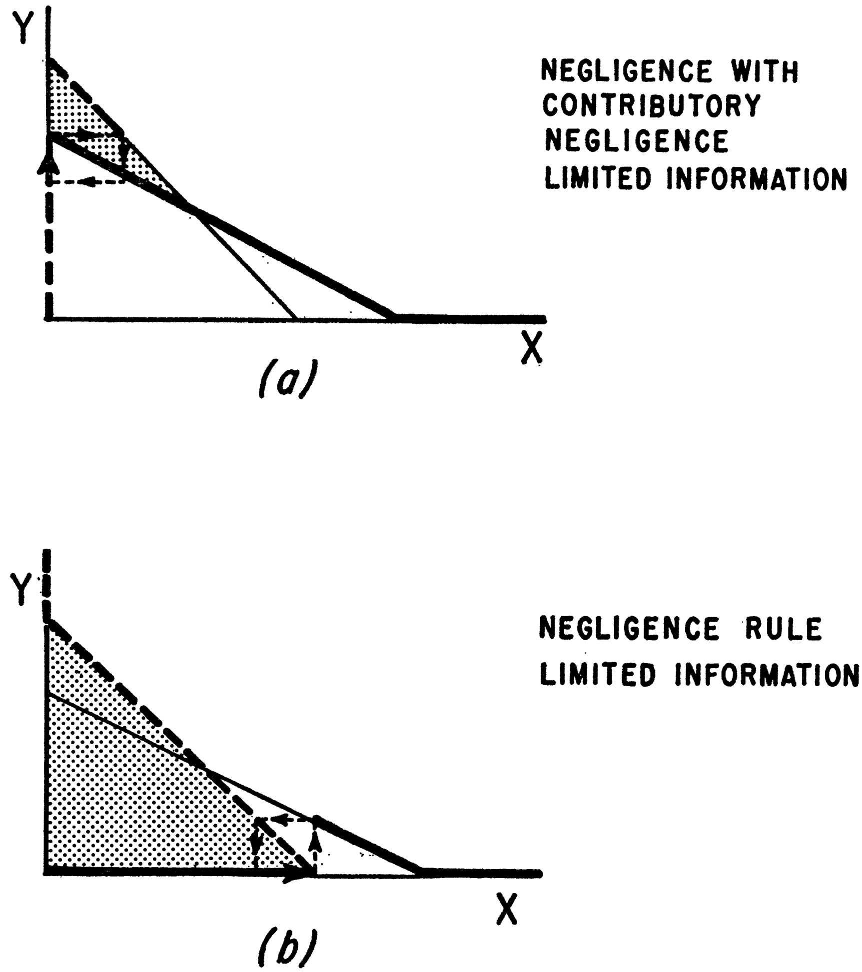
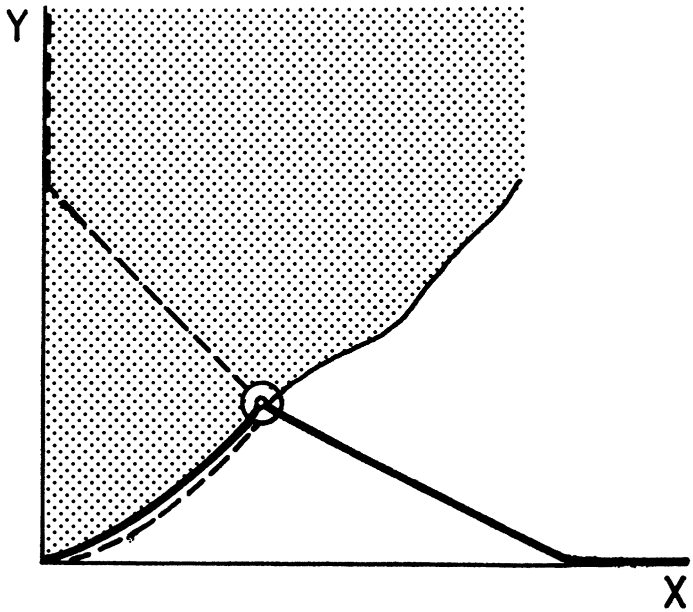


FIGURE 10

Proof: X^e and Y^e are plotted in Figure 11. The shaded area is the area where the injurer is liable. In the unshaded area, the victim is liable. As before, the injurer will choose the least costly place where he is not liable unless X^f lies to the left, *i.e.*, is less expensive. Then he will choose X^f. Similarly, the victim will choose a strategy of the least costly place where she is not liable unless Y^f lies below that point, in which case she will choose Y^f. X^e and Y^e intersect at (X_Ω, Y_Ω). The two equilibria are adjacent from zero to (X_Ω, Y_Ω). In that range the injurer's strategy will be to follow exactly the relative negligence boundary (the expansion path in conventional economic production theory parlance) because by our convention the victim is



RELATIVE NEGLIGENCE

FIGURE 11

liable exactly on the boundary. The victim's strategy here is, in principle, undefined. There does not exist an optimal strategy in this area, only a limit to the strategy. What she wants to do is to get as close to the boundary as possible without touching it. For any given strategy not on the boundary one can in principle get closer to the boundary, *e.g.*, half as far, but this technical difficulty is not critical since (X_Ω, Y_Ω) is a well-defined equilibrium in any case.

3. *Comparative Negligence*

Rather than analyze completely the comparative negligence rule, I shall simply show that the social optimum is *not* an equilibrium under the comparative negligence rule. At (X_Ω, Y_Ω), $L_x(X, Y) = \frac{1}{2}$ since both parties are equally "negligent." Let Y be fixed at Y_Ω. The injurer will choose X_Ω only if his expected cost is at a minimum there. Now

$$C_x(X_\Omega, Y_\Omega) = W_x X + \frac{1}{2} A[1 - P(X, Y)] \qquad (39)$$

which will be the minimum over X only if

$$W_x = \frac{1}{2} A\, P_x(X_\Omega, Y_\Omega). \tag{40}$$

But by the definition of (X_Ω, Y_Ω),

$$W_x = A\, P_x(X_\Omega, Y_\Omega) \tag{41}$$

so (X_Ω, Y_Ω) cannot be an equilibrium.[11]

Conclusions

In this paper economic analysis has been used to subject a variety of liability rules and standards of care to scrutiny in an abstract setting. It is my contention, which I hope the paper supports, that economic analysis is a useful method of organizing legal questions. It brings out similarities among apparently different legal doctrines, and it allows one to bring a large body of analysis to bear.

The paper first described accident avoidance by two parties in such a way that we could exploit the analogy with the economic theory of production. We used the model thus developed to characterize eight different liability rules discussed in the legal literature, and three variants of the Learned Hand formulation of the standard of care. The literal variant of Judge Hand's formulation we rejected immediately because it was not an incremental standard. Then we analyzed each of the liability rules, using (where appropriate) both of the remaining standards of care, the Incremental Standard and the Limited Information Incremental Standard. The analysis consisted of considering the victim and the injurer as noncommunicating parties in a game played according to liability rules. We identified the noncooperative equilibrium point as the solution for the game and in each case compared it with the social optimum.

To summarize the results, equilibrium for the simple no-liability rule as well as for the simple strict-liability rule did not coincide with the social optimum, nor did equilibrium for the comparative-negligence rule. When the standard of care was the Incremental Standard the social optimum was identical with the equilibrium for the relative-negligence rule, as well as with the equilibrium for all four of the common law negligence-based rules. The standard of care is critical, for, when it was changed to the Limited Information Incremental Standard, the identity between equilibrium and optimality was destroyed.

[11] This argument is not strictly true. We have left out the effect of changing X on the liability rule. It is possible, for particular values of the first and second derivatives, that the neglected effect will exactly cancel out the difference we point to in the text. Though it is possible, in general we cannot expect it to happen.

APPENDIX

The Case of Free Transactions: The Coase Theorem

In the body of the paper I analyzed the polar case where the only transactions that occurred between parties took place in court after an accident occurred. The parties played a game according to whatever liability rules were in force at the time. Here I explore briefly the opposite case, where transactions between parties may be freely arranged at any time. This problem has been examined by many authors since Ronald Coase's important article,[12] although Coase was not entirely or even primarily interested in the case of costless transactions. On the contrary, he was mainly interested in the costless-transaction case as a convenient expository device, a polar case. The Coase Theorem, a phrase coined and used by later writers, states roughly that regardless of the initial assignment of rights, if rights can be traded costlessly, the resulting allocation of rights will be efficient. Furthermore, if the income effects resulting from the transfer of rights can be neglected, the allocation of rights after bargaining will be independent of the initial assignment of those rights.

The questions that we shall ask here are, what transactions will take place between the parties, and what will the final amounts of X and Y be after all transactions have taken place? To set the stage let us define the initial property rights as follows: Xavier, the injurer, has complete control over the amount of X chosen as well as the liability for any bills that the liability rules assess him for. Similarly, Yvonne the victim has complete control over Y and responsibility for any bills that are assessed to her.

In the first situation we shall analyze, where the liability rule in force is no liability, all bills emitted by the liability generator must be paid by Yvonne. We have shown that the equilibrium amounts of X and Y in the absence of transactions will be $X = 0$ and $Y = Y^f(0)$. The expected cost to her is $C_y(0, Y^f(0))$, which is the same as the social cost since Xavier pays nothing.

Free bargaining assures that Yvonne will continue to pay Xavier to increase X until the marginal value of X to Xavier and to Yvonne are equal. The marginal value of X to Xavier is simply W_x; the marginal value to Yvonne is $A\ P_x(X, Y)$. This condition is exactly that in equation (3), and the cost-minimizing choice of Y by Yvonne assures us of (4). These two together are the conditions of the social optimum. The cost to Yvonne of attaining (X_Ω, Y_Ω) is

$$W_y Y_\Omega + A(1 - P(X_\Omega, Y_\Omega)) + \text{Payment to X}. \tag{A1}$$

The payment is undetermined except that it is bounded from below by $W_x X_\Omega$, the minimum amount that Xavier would accept in order to provide X_Ω, and from above by $C_S(0, Y^f(0)) - C_S(X_\Omega, Y_\Omega)$, the maximum that Yvonne would be willing to pay. If X is provided competitively, then the competitive price of X would be W_x and the payment to the injurer would be exactly $W_x X_\Omega$. In any event, the socially optimal amount of X and Y will be chosen and the net social cost will be $C_S(X_\Omega, Y_\Omega)$.

[12] R. H. Coase, *supra* note 1.

The result for the second liability rule, the rule of strict liability, is exactly the same, as is the argument leading to the result, except that the roles of the injurer and victim are reversed. The amount of X and Y chosen this time by Xavier will be X_Ω and Y_Ω, and the net social cost will be $C_S(X_\Omega, Y_\Omega)$. The only difference between the results of the first and second liability rules is a transfer of income. In the first case, when there is no liability, the victim, Yvonne, pays the whole social cost plus a net payment to Xavier. In the case of strict liability, Yvonne not only pays nothing but receives some net payment from Xavier which depends on relative bargaining strength, while Xavier must pay the whole social cost.

I turn now to the case of costless transactions when more complicated liability rules, such as the negligence rule, are in force. Consider the negligence rule. What transactions will take place when they are free?

In the absence of transactions X_Ω and Y_Ω will be chosen, as we have shown above. Thus the victim, Yvonne, can be assured that the injurer will set X at X_Ω without her having to pay a bribe. The benefit of having the injurer increase X above X_Ω would be less than the minimum cost of the increase, W_x. Similarly, Xavier would have no reason to bribe Yvonne to increase Y, because he is not liable. Therefore, under the negligence rule no transactions will take place and the level of avoidance measures chosen by the injurer and the victim will be X_Ω and Y_Ω. The cost to the injurer will be W_xX_Ω, and the expected cost to the victim will be $W_yY_\Omega + A\,(1 - P(X_\Omega, Y_\Omega))$. The results for the rest of the liability rules can easily be shown to be similar to those of the negligence rule.