

# OPMT 5701

## Matrix Algebra Questions Answers

#1 Find  $C = AB$ , if  $A = \begin{bmatrix} 12 & 14 \\ 20 & 5 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 9 \\ 0 & 2 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (12 \times 3) + (14 \times 0) & (12 \times 9) + (14 \times 2) \\ (20 \times 3) + (5 \times 0) & (20 \times 9) + (5 \times 2) \end{bmatrix} = \begin{bmatrix} 36 & 136 \\ 60 & 190 \end{bmatrix}$$

#2 Find  $C = AB$ , if  $A = \begin{bmatrix} 4 & 7 \\ 9 & 1 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 8 & 5 \\ 2 & 6 & 7 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (4 \times 3) + (7 \times 2) & (4 \times 8) + (7 \times 6) & (4 \times 5) + (7 \times 7) \\ (9 \times 3) + (1 \times 2) & (9 \times 8) + (1 \times 6) & (9 \times 5) + (1 \times 7) \end{bmatrix}$$

$$AB = \begin{bmatrix} 26 & 74 & 69 \\ 29 & 78 & 52 \end{bmatrix}$$

#3 Find  $C = AB$ , if  $A = \begin{bmatrix} 7 & 11 \\ 2 & 9 \\ 10 & 6 \end{bmatrix}$   $B = \begin{bmatrix} 12 & 4 & 5 \\ 3 & 6 & 1 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (7 \times 12) + (11 \times 3) & (7 \times 4) + (11 \times 6) & (7 \times 5) + (11 \times 1) \\ (2 \times 12) + (9 \times 3) & (2 \times 4) + (9 \times 6) & (2 \times 5) + (9 \times 1) \\ (10 \times 12) + (6 \times 3) & (10 \times 4) + (6 \times 6) & (10 \times 5) + (6 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 117 & 94 & 46 \\ 51 & 62 & 19 \\ 138 & 76 & 56 \end{bmatrix} = C$$

#4 Find (i)  $AB = C$ , and (ii)  $BA = D$ , if  $A = \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 6 & -2 \end{bmatrix}$

Answer:

$$(i) AB = \begin{bmatrix} -2 \times 3 & -2 \times 6 & -2 \times -2 \\ 4 \times 3 & 4 \times 6 & 4 \times -2 \\ 7 \times 3 & 7 \times 6 & 7 \times -2 \end{bmatrix} = \begin{bmatrix} -6 & -12 & 4 \\ 12 & 24 & -8 \\ 21 & 42 & -14 \end{bmatrix}$$

$$(ii) BA = [(3 \times -2) + (6 \times 4) + (-2 \times 7)] = [4]$$

#5 Find the minors and cofactors of the third row, given

$$A = \begin{bmatrix} 9 & 11 & 4 \\ 3 & 2 & 7 \\ 6 & 10 & 4 \end{bmatrix}$$

Answer:

Step 1: Delete row 3 and column 1

$$|M|_{31} = \begin{vmatrix} 11 & 4 \\ 2 & 7 \end{vmatrix} = 69$$

Step 2: Delete row 3 and column 2

$$|M|_{32} = \begin{vmatrix} 9 & 4 \\ 3 & 7 \end{vmatrix} = 51$$

Step 3: Delete row 3 and column 3

$$|M|_{33} = \begin{vmatrix} 9 & 11 \\ 3 & 2 \end{vmatrix} = -15$$

Step 4: Since a cofactor is simply the minor with a particular sign, according to  $|C_{ij}| = (-1)^{i+j} |M_{ij}|$  we find:

$$|C_{31}| = (-1)^4 |M_{31}| = 69$$

$$|C_{32}| = (-1)^5 |M_{32}| = -51$$

$$|C_{33}| = (-1)^6 |M_{33}| = -15$$

#6 Use Laplace Expansion to find the determinant of  $A = \begin{bmatrix} 15 & 7 & 9 \\ 2 & 5 & 6 \\ 9 & 0 & 12 \end{bmatrix}$

(HINT: use column or row with the most zero's)

Answer:

Expand second column

$$|A| = a_{12} |C_{12}| + a_{22} |C_{22}| + a_{32} |C_{32}|$$

$$|A| = (7)(-1) \begin{vmatrix} 2 & 6 \\ 9 & 12 \end{vmatrix} + (5) \begin{vmatrix} 15 & 9 \\ 9 & 12 \end{vmatrix} + 0$$

$$|A| = (7)(-30) + (5)(99)$$

$$|A| = 705$$

#7 Find the inverse for

$$A = \begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix}$$

Where  $A^{-1} = \frac{1}{|A|} \cdot Adj A$

Answer:

Step 1: find the determinant  $|A|$

$$|A| = 4 \begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} - (-2) \begin{vmatrix} 1 & -5 \\ -1 & 4 \end{vmatrix} + 3 \begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix} = 4[(3)(4) - (1)(-1)] - (-2)[(1)(4) - (-5)(-1)] + 3[(1)(1) - (-5)(3)]$$

$$= 52 - 2 + 48$$

$$= 98 \neq 0$$

A is nonsingular

Step 2: Find the cofactor matrix

$$C = \begin{bmatrix} \begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & -5 \\ -1 & 4 \end{vmatrix} & \begin{vmatrix} 4 & -5 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & -5 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 13 & 11 & -7 \\ 1 & 31 & 7 \\ 16 & 6 & 14 \end{bmatrix}$$

Step 3: Transpose the cofactor matrix to get the adjoint

$$C' = Adj A = \begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix}$$

Step 4: (i) Multiply the adjoint by  $\frac{1}{|A|}$

$$A^{-1} = \frac{1}{|A|} \cdot Adj A = \frac{1}{98} \begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix} = \begin{bmatrix} \frac{13}{98} & \frac{1}{98} & \frac{16}{98} \\ \frac{11}{98} & \frac{31}{98} & \frac{6}{98} \\ -\frac{1}{14} & \frac{1}{14} & \frac{1}{7} \end{bmatrix}$$