OPMT 5701
Matrix Algebra Questions Answers

#1 Find$C = AB$, if A=$\begin{bmatrix} 12 & 14 \\ 20 & 5 \end{bmatrix}$ B=$\begin{bmatrix} 3 & 9 \\ 0 & 2 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (12 \times 3) + (14 \times 0) & (12 \times 9) + (14 \times 2) \\ (20 \times 3) + (5 \times 0) & (20 \times 9) + (5 \times 2) \end{bmatrix} = \begin{bmatrix} 36 & 136 \\ 60 & 190 \end{bmatrix}$$

#2 Find $C = AB$, if A=$\begin{bmatrix} 4 & 7 \\ 9 & 1 \end{bmatrix}$ B=$\begin{bmatrix} 3 & 8 & 5 \\ 2 & 6 & 7 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (4 \times 3) + (7 \times 2) & (4 \times 8) + (7 \times 6) & (4 \times 5) + (7 \times 7) \\ (9 \times 3) + (1 \times 2) & (9 \times 8) + (1 \times 6) & (9 \times 5) + (1 \times 7) \end{bmatrix}$$

$$AB = \begin{bmatrix} 26 & 74 & 69 \\ 29 & 78 & 52 \end{bmatrix}$$

#3 Find C=AB, if $A = \begin{bmatrix} 7 & 11 \\ 2 & 9 \\ 10 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 12 & 4 & 5 \\ 3 & 6 & 1 \end{bmatrix}$

Answer:

$$AB = \begin{bmatrix} (7 \times 12) + (11 \times 3) & (7 \times 4) + (11 \times 6) & (7 \times 5) + (11 \times 1) \\ (2 \times 12) + (9 \times 3) & (2 \times 4) + (9 \times 6) & (2 \times 5) + (9 \times 1) \\ (10 \times 12) + (6 \times 3) & (10 \times 4) + (6 \times 6) & (10 \times 5) + (6 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 117 & 94 & 46 \\ 51 & 62 & 19 \\ 138 & 76 & 56 \end{bmatrix} = C$$

#4 Find (i) $AB = C$, and (ii) $BA = D$, if $A = \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 & -2 \end{bmatrix}$

Answer:

(i) $$AB = \begin{bmatrix} -2 \times 3 & -2 \times 6 & -2 \times -2 \\ 4 \times 3 & 4 \times 6 & 4 \times -2 \\ 7 \times 3 & 7 \times 6 & 7 \times -2 \end{bmatrix} = \begin{bmatrix} -6 & -12 & 4 \\ 12 & 24 & -8 \\ 21 & 42 & -14 \end{bmatrix}$$

(ii) $BA = [(3 \times -2) + (6 \times 4) + (-2 \times 7)] = [4]$

#5 Find the minors and cofactors of the third row, given

$$A = \begin{bmatrix} 9 & 11 & 4 \\ 3 & 2 & 7 \\ 6 & 10 & 4 \end{bmatrix}$$

Answer:

Step 1: Delete row 3 and column 1

$$|M|_{31} = \begin{vmatrix} 11 & 4 \\ 2 & 7 \end{vmatrix} = 69$$

Step 2: Delete row 3 and column 2
$|M|_{32} = \begin{vmatrix} 9 & 4 \\ 3 & 7 \end{vmatrix} = 51$
Step 3: Delete row 3 and column 3
$|M|_{33} = \begin{vmatrix} 9 & 11 \\ 3 & 2 \end{vmatrix} = -15$
Step 4: Since a cofactor is simply the minor with a particular sign, according to $|C_{ij}| = (-1)^{i+j}\,|M_{ij}|$ we find:
$|C_{31}| = (-1)^4\,|M_{31}| = 69$
$|C_{32}| = (-1)^5\,|M_{32}| = -51$
$|C_{33}| = (-1)^6\,|M_{33}| = -15$

#6 Use Laplace Expansion to find the determinant of $A = \begin{bmatrix} 15 & 7 & 9 \\ 2 & 5 & 6 \\ 9 & 0 & 12 \end{bmatrix}$
(HINT: use column or row with the most zero's)
Answer:
Expand second column
$|A| = a_{12}\,|C_{12}| + a_{22}\,|C_{22}| + a_{32}\,|C_{32}|$
$|A| = (7)(-1)\begin{vmatrix} 2 & 6 \\ 9 & 12 \end{vmatrix} + (5)\begin{vmatrix} 15 & 9 \\ 9 & 12 \end{vmatrix} + 0$
$|A| = (7)(-30) + (5)(99)$
$|A| = 705$

#7 Find the inverse for
$A = \begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix}$

Where $A^{-1} = \frac{1}{|A|} \cdot Adj\,A$

Answer:

Step 1: find the determinant $|A|$
$|A| = 4\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -5 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix} = 4\,[(3)\,(4) - (1)(-1)] - (-2)\,[(1)(4) - (-5)(-1)] + 3\,[(1)(1) - (-5)(3)]$
$= 52 - 2 + 48$
$= 98 \neq 0$
A is nonsingular

Step 2: Find the cofactor matrix
$$C = \begin{bmatrix} \begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & -5 \\ -1 & 4 \end{vmatrix} & \begin{vmatrix} 4 & -5 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & -5 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 13 & 11 & -7 \\ 1 & 31 & 7 \\ 16 & 6 & 14 \end{bmatrix}$$
Step 3: Transpose the cofactor matrix to get the adjoint
$C\prime = Adj\,A = \begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix}$
Step 4: (i) Multiply the adjoint by $\frac{1}{|A|}$
$$A^{-1} = \frac{1}{|A|} \cdot Adj\,A = \frac{1}{98}\begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix} = \begin{bmatrix} \frac{13}{98} & \frac{1}{98} & \frac{16}{98} \\ \frac{11}{98} & \frac{31}{98} & \frac{6}{98} \\ -\frac{1}{14} & \frac{1}{14} & \frac{1}{7} \end{bmatrix}$$