Dynamic Risk Sharing with Escrow*

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Abstract

I analyze the role of escrowed withholding in dynamic risk sharing with information and commitment frictions between a risk-averse agent with stochastic income and an insurance platform. A fixed limited portion of the agent's income can be withheld in escrow. I consider three endogenously incomplete markets settings with different obstacles to risk sharing: limited commitment, private information due to hidden income, and both limited commitment and private information. I show that the withholding technology alleviates the commitment problem and improves risk sharing, including possibly to full insurance; however, it is ineffective in the private information settings. In the model with both commitment and information frictions, I show that private information is the binding constraint.

Keywords: risk sharing, limited commitment, private information, mechanism design, withholding, escrow

JEL codes: D82, D86, G52

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1 Introduction

The ability of economic agents to share risk is frequently hindered by contractual or market frictions stemming from limited commitment or private information (e.g., Townsend 1994 and 2016; Ligon, Thomas and Worrall 2002). In the absence of perfect enforcement, agents may renege on an insurance arrangement when their income is high. In the presence of private information, agents may under-report their income to avoid making contributions or pay premia. Both frictions make financial markets and contracts endogenously incomplete and can result in imperfect risk sharing.

In this paper, I analyze the efficacy of a specific contract instrument – escrowed withholding of income – for addressing commitment, information, or both obstacles to risk sharing. I consider an infinite-horizon setting in which a risk-neutral insurance platform contracts with risk-averse agents who face idiosyncratic income fluctuations. I solve for and characterize the optimal dynamic risk-sharing contract based on the history of observed or reported income. The contract is designed to respect voluntary participation and contributions (no reneging) at all times in the limited commitment settings, and to ensure truthful reporting of the income state in the private information settings.

The key feature of the model is allowing the technological possibility that, as part of the risk-sharing arrangement, the agent and insurance platform can agree ex-ante, before the realization of the income shock, to set and implement a fixed withholding amount held in escrow. To ensure feasibility in every income state, the withholding amount is limited to be less or equal to the agent's base (lowest possible) income level. The escrow mechanism is particularly relevant in the context of digital financial platforms or decentralized finance (DeFi) applications where smart contracts can be used to automate withholding of funds (programmable escrow) and enforce payment obligations while being unable to observe agents' actual income realizations.

I analyze the escrow mechanism in three dynamic contract settings: (i) limited commitment with ex-post observable income; (ii) private information (hidden income) with commitment; and (iii) a setting featuring both limited commitment and hidden income.

The analysis yields contrasting results, depending on the underlying friction. In the setting with limited commitment alone (e.g., Thomas and Worrall, 1988; Kocherlakota, 1996; Ligon, Thomas and Worrall, 2002), I show that escrowed withholding can increase efficiency and the degree of consumption smoothing. Specifically, the contract with escrowed withholding lowers the agent's payoff from reneging in the high-income state. If the maximum feasible escrow is sufficiently large, the first-best allocation (full insurance) can be implemented, eliminating the limited commitment friction entirely. More generally, for any feasible positive escrow level, the mechanism strictly improves welfare by relaxing the limited commitment constraint. This implies that, holding the insurance platform's payoff constant, ex-ante the agent prefers the escrow contract to the contract without.

In contrast to the results for limited commitment, I show that the escrow mechanism is ineffective in a setting with private information (hidden income, e.g., as in Townsend, 1982 or Thomas and Worrall, 1990). In this setting the binding constraint is the agent's incentive to misreport high income as low. Since, by definition the withholding amount is fixed before the realization of the income state and is non-contingent, it does not alter the agent's marginal incentives regarding truth telling. Consequently, I show that the constrained-optimal contract in the hidden income model with escrow is identical to that in the standard model without escrow.

Finally, I analyze a setting in which both hidden income and limited commitment frictions are present. I first prove that the truth-telling constraint is stricter than the limited commitment constraint. That is, a contract that induces a high-income agent to report truthfully rather than claiming low income automatically satisfies the limited commitment constraints. Since the private information friction is the binding constraint and because, as argued before, escrowed withholding does not alleviate the private information friction, the escrow technology provides no welfare gains in the setting with both information and commitment frictions.

The paper relates to the theoretical literature on recursive contracts and optimal risk sharing in dynamic settings with either limited commitment or private information (e.g., see Townsend, 1982; Thomas and Worrall, 1988 and 1990; Kocherlakota, 1996; Cole and Kocherlakota, 2001; and the review in Golosov et al., 2016). Zilberman et al. (2019) also consider a setting with both commitment and information frictions and show that the combined truth-telling and commitment constraints eliminate the long-run 'immiseration' result inherent in models with only private information (e.g., Thomas and Worrall, 1990; Atkeson and Lucas, 1992) whereby agent consumption and present-value utility drift down, and instead a non-degenerate stationary utility distribution obtains.[1] In contrast, I analyze the role of escrow in this setting.

By considering escrowed withholding as part of the contract design, this paper also relates

[1]See also Dovis (2019) who studies the optimal arrangement between a sovereign borrower and foreign lenders under both lack of commitment and private information in a production economy with imports and exports.

to Rampini and Viswanathan (2010) who study the role of collateral in financial risk management and to Karaivanov et al. (2023) on using escrow in digital safety nets. The paper's results suggest that while (digital) escrow technologies can mitigate enforcement problems or strategic default, they may be ineffective in addressing frictions stemming from private information.

The escrow mechanism analyzed here has a possible application in the design of decentralized finance (DeFi). Smart contracts—self-executing code running on digital platforms—enable credible commitment to future transfers by programmatically locking funds ex-ante (e.g., see Szabo, 1996; Cong and He, 2019; Karaivanov et al. 2023), as assumed and modelled here. By automating the computation and execution of state- and history-contingent transfers, these technologies can further reduce transaction and verification costs (Catalini and Gans, 2020). However, while smart contracts can enforce payments programmatically, thus addressing commitment frictions, they remain dependent on external mechanisms ("oracles") to verify off-platform income realizations or other relevant data. The paper's results contribute to formalizing the limits of such digital technologies by comparing and contrasting commitment/enforcement with information frictions.

The remainder of the paper is organized as follows. Section 2 outlines the dynamic risk-sharing model. Section 3 presents the main theoretical results for the three settings with different contractual frictions and computed numerical examples and comparisons of consumption smoothing. Section 4 concludes.

2 Model

I analyze the optimal dynamic risk-sharing arrangement between an insurance platform ("the insurer") and a large number (continuum) of ex-ante identical risk-averse agents. The agents have preferences represented by the strictly increasing and strictly concave utility of consumption function $u(c)$ and discount factor $\beta \in (0, 1)$.

Every period an agent draws income

$$y = \underline{y} + \varepsilon$$

where $\underline{y} > 0$ is known fixed "base income" and ε is a discrete stochastic income shock with possible values $\{\varepsilon_1, ..., \varepsilon_S\}$ ordered in increasing magnitude, $0 = \varepsilon_1 < ..., < \varepsilon_S$, taken with respective probabilities $Prob(\varepsilon = \varepsilon_i) = \pi_i, \ \forall i = 1, ..., S$. The income realizations are assumed i.i.d. over time and across agents.

The agents contract with a competitive risk-neutral insurer over an infinite time horizon. The constrained-optimal risk-sharing contract maximizes the agent's discounted utility subject to commitment or incentive-compatibility constraints, as specified below.

The insurer is conceptualized as a (digital/programmable) platform that is technologically capable to withhold in escrow up to amount $\bar{f} \geq 0$ from the agent's base income flow. Note that the platform does not necessarily observe the agent's ex-post realized income – this depends on the assumed contractual frictions (see below).

Assumption 1
The maximum feasible escrow amount $\bar{f}$ satisfies

$$\bar{f} \leq \underline{y}$$

Assumption 1 ensures that in every possible income state $i = 1, ...S$ there are sufficient funds to implement ex-ante agreed-upon escrowed withholding. A possible interpretation is that, with the agent's consent and authorization (see Corollary 1), an amount f satisfying $f \leq \bar{f} \leq \underline{y}$ out of the agent's base income $\underline{y}$ can be routed through the platform and (programmatically) withheld and locked in escrow unconditionally and non-contingently.[2] For example, the base income could be a pension or other fixed transfer payment. Any additional income, $\varepsilon_i + \underline{y} - f$, remains in the agent's discretionary control. The parameter $\bar{f}$ can be interpreted as measuring the strength or scope of the escrow technology, with the special case $\bar{f} = 0$ corresponding to the standard model without withholding.

To facilitate parallels with the literature, denote the agent's total income in state $i = 1, .., S$ by

$$y_i = \underline{y} + \varepsilon_i$$

and expected income by

$$\bar{y} = E(y) = \underline{y} + \sum_{i=1}^{S} \pi_i \varepsilon_i$$

The platform provides insurance to the risk-averse agent via contingent transfers τ_i, which are either contributions/premia paid by the agent to the platform or payouts/indemnities paid to the agent by the platform and in general depend on the history of the agent's observed or reported income. The platform can also borrow or save externally at gross interest rate $R = \beta^{-1}$, i.e., unlike in Kocherlakota (1996), there is no resource constraint linking total payouts and premia across agents in a given period. Thus, we can think of the insurer as contracting separately with each agent while keeping track of the agent's income

[2] Another possible interpretation is fixed-amount garnishment.

history/reports, e.g., see Karaivanov and Townsend (2014).

The insurer is assumed to fully commit to the contract. On the agent's side, I consider the following commitment and/or information frictions:

- **limited commitment** (e.g., Thomas and Worrall, 1988): after income y_i is realized, the agent may consider reneging on a due insurance premium and quit the contract. The contract must provide an incentive, i.e., sufficiently high present-value utility for the agent to participate and not quit in any period, as well as not renege on a due transfer (e.g., when the agent has high income). There is no private information problem.

  The outside option for the agent upon reneging has present value V^{out}. As in most related work, I assume that the outside option equals the autarky value obtained by the agent consuming his or her own income in each period,

  $$V^{out} = \frac{E(u(y))}{1-\beta} \tag{OO}$$

  where $E(u(y))$ is the expected utility from consuming one's income every period.

- **private information** (hidden income, e.g., Townsend, 1982; Thomas and Worrall, 1990): the agent's income is unobservable to the insurer and is self-reported by the agent. The contract provides incentives for truth telling by conditioning transfers on the history of income reports. There is no commitment problem.

- **both limited commitment and private information**: both the limited commitment and hidden income frictions are simultaneously present. The optimal insurance contract must provide incentives for staying in, not reneging, and reporting truthfully in all time periods and for all income histories.

The timing is as follows:

(i) the agent and platform agree to enter a risk-sharing contract (which may include escrowed withholding of fixed amount f) specifying transfers τ_i and promised utility w_i in each income state $i = 1, ...S$.

(ii) agent income y_i is realized and, if applicable, amount f from the base income $\underline{y}$ is withheld in escrow.

(iii) in the limited commitment settings the agent can decide to renege on the contract (not make a due payment) and take the outside option (autarky forever). If the agent leaves, the contract relationship ends at this point.

(iv) if the agent remains in the contract, the transfer τ_i is implemented and promised utility w_i is updated, based on the agent's observed or reported income history depending on the setting.

For most of the analysis, to obtain analytical results I assume that there are two possible income realizations ($S = 2$) – low income, $y_L = \underline{y}$ and high income, $y_H = \underline{y} + \varepsilon$ where $\varepsilon > 0$, occurring with respective probabilities $\pi_L \in (0, 1)$ and $\pi_H = 1 - \pi_L$.

3 Results

3.1 Limited Commitment

I first analyze a setting in which the only contractual friction is limited commitment by the agent. I allow for both ex-ante and ex-post commitment problems. First, as in Phelan (1995) the agent could quit if her present-value utility (captured by the promised utility variable in the contract) falls below the agent's outside option value V^{out}. Second, the agent could consider reneging on a due insurance premium after the income is realized if not paying and going to autarky dominates staying in the contract. The optimal contract addresses both these possibilities by ensuring that, at any time and after any income history, the agent does not want to leave the contract or renege on a premium. The insurer observes the agent income after it is realized.

Formally, following the literature (Spear and Srivastava, 1987; Thomas and Worrall, 1988; Karaivanov and Townsend, 2014), I write the contracting problem as a recursive dynamic program with state variable promised utility, w, i.e., the agent's present-value discounted utility from the current period onward. The timing within each model time period t is: (i) the agent begins with current promised utility state $w_t = w$; (ii) income $y_t = y_i$ is realized, implying transfer τ_i (positive or negative); (iii) the agent either (a) pays a premium or receives an indemnity, depending on the sign of τ_i, and enters next period with promised utility $w_{t+1} = w_i$, or (b) reneges on the arrangement by consuming $c_t = y_i$ and thereafter continues in autarky with present-value V^{out}.

Problem LC

$$\Pi(w) = \max_{\{\tau_i, w_i\}} \quad \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t.} \quad \pi_H(u(y_H + \tau_H) + \beta w_H) + \pi_L(u(y_L + \tau_L) + \beta w_L) = w \tag{PK}$$

$$u(y_i + \tau_i) + \beta w_i \geq u(y_i) + \beta V^{out}, \quad \forall i \tag{LC}$$

$$w_i \geq V^{out}, \ \forall i \tag{continued participation}$$

for all $w \geq V^{out}$ (ex-ante participation), with w_0 given. In Problem LC, V^{out} is the agent's outside option, the autarky value from (OO), and the insurer can borrow and save at gross interest $R = \beta^{-1}$. The choice variables are the insurance transfers τ_i, which are positive when the agent receives an insurance payout/indemnity and negative when the agent pays a net premium/contribution, and the future promised utility w_i in each income state i. The promised utility state variable w captures the history of income realizations and determines the present value of future transfers to or from the agent. In a digital platform promised utility can be represented by a non-monetary token balance updated via the platform's code.

Constraint (PK) is the promise-keeping constraint ensuring that the insurer delivers the contracted present-value utility w to the agent. The limited commitment constraints (LC) ensure that the agent has no incentive to renege on a due transfer and walk away after any income realization. The left hand side of (LC) is the value of staying in the contract for income state $i = L, H$, while the right hand side is the value of going to the outside option of autarky forever with income y_i in hand. The last constraint, on future promises w_i, ensures the continued participation by the agent who would be better off to switch to autarky if the in-contract utility value w_i falls below V^{out} in some history.

I set the initial promised utility w_0 so that the insurance platform breaks even on the contract ex-ante and in expectation, that is, w_0 solves

$$\Pi(w_0) = 0.$$

However, in any period, $t = 1, 2, ..., +\infty$, the insurer is allowed to run a surplus or deficit with an agent.

Denote by superscript "lc" the contract elements (policy functions), (τ_i^{lc}, w_i^{lc}) for $i = \{L, H\}$ in Problem LC – each of them being a function of the current state w. Let also c_i^{lc} be the in-contract consumption in state i,

$$c_i^{lc} = y_i + \tau_i^{lc}, \ \ \forall i$$

Problem LC is well-known from the literature, e.g., see Thomas and Worrall, (1988) or

Ljungqvist and Sargent (2018) for detailed analysis. The main results and contract properties are summarized in Lemma 1. Note that the lower bound V^{out} on promised utility does not affect the solution properties since promised utility either stays the same or increases in the optimal contract. I then focus on allowing escrowed withholding in Problem LC and analyze its effect on the constrained-optimal risk-sharing contract.

Lemma 1. *The optimal contract solving Problem LC has the following properties:*
(a) $\forall w \in [V^{out}, u(y_H) + \beta V^{out})$ *partial insurance obtains – constraint (LC) binds for* $i = H$ *and the optimal contract is:*

$$w_H^{lc} = u(y_H) + \beta V^{out},\ u(c_H^{lc}) = (1-\beta) w_H^{lc},\ w_L^{lc} = w, \text{ and } c_L^{lc}(w) \text{ solving}$$

$$\pi_L(u(c_L^{lc}) + \beta w) + \pi_H(u(c_H^{lc}) + \beta w_H^{lc}) = w$$

(b) $\forall w \geq u(y_H) + \beta V^{out}$ *full insurance obtains – the optimal contract is:*

$$w_i^{lc} = w \text{ and } u(c_i^{lc}) = (1-\beta) w,\ \forall i.$$

(c) *At zero ex-ante expected profits, that is, initial promised utility* w_0 *set so that* $\Pi(w_0) = 0$, *if*

$$\frac{u(\bar{y})}{1-\beta} \geq u(y_H) + \beta V^{out}, \tag{L1}$$

then the insurance platform optimally provides full insurance, $c_i^{lc} = c^{fi} = \bar{y}$, $\forall i$ *by setting* $w_0 = w^{fi} = \frac{u(\bar{y})}{1-\beta}$. *Conversely, if*

$$\frac{u(\bar{y})}{1-\beta} < u(y_H) + \beta V^{out}, \tag{L1a}$$

then full insurance is infeasible at zero profits and the platform provides partial insurance.

Proof: see Appendix.

When the model parameters satisfy inequality (L1) in Lemma 1(c), the platform can deliver full insurance at zero expected profits by setting $w_0 = \frac{u(\bar{y})}{1-\beta}$, i.e., it delivers equal consumption across all income states while respecting the limited-commitment constraints. This corresponds to the first-best outcome. This case arises if the discount factor β is relatively close to 1 (the agent is patient) and/or the high income level y_H is not too large relative to the mean income $\bar{y}$. A higher degree of risk aversion also makes (L1) more likely to be satisfied.

Below I focus on the partial-insurance case characterized in part (a) of Lemma 1, where the limited commitment problem has a bite and constraint (LC) binds for the high-income state $i = H$. In this case the agent has to be offered more than the mean income $\bar{y}$ whenever y_H is realized to be induced to stay in the contract, resulting in partial insurance. For $i = L$ the incentive to renege (the r.h.s. of (LC)) is low and the agent optimally receives an insurance payout ($\tau_L > 0$) – see the Lemma 1 proof. Thus, constraint (LC) does not bind for $i = L$ and I remove it from the analysis of Problem LCE below.

Escrow

Suppose now that escrowed withholding is technologically possible in the limited-commitment setting. Specifically, suppose that the agent and platform agree ex-ante that an amount $f \leq \bar{f}$ out of the agent's base income $\underline{y}$ can be withheld in escrow, while all additional income remains in the agent's discretion. Corollary 1 below shows that, holding the platform's expected payoff constant, it is always beneficial for the agent in terms of ex-ante expected utility to accept such arrangement when technologically feasible.

The special case $\bar{f} = 0$ corresponds to the standard limited commitment setting in Problem LC. Aside from the technological ability to withhold part f of the agent's base income in escrow, the commitment friction remains present, i.e., the agent can still decide to quit if his/her promised utility is too low or after a high income realization.

The dynamic risk-sharing problem with escrow is stated in Problem LCE, with the additional choice variable f. The contract elements (escrow f, transfers τ_i, and next-period promised utility w_i for each income state i) are optimally chosen to ensure that the agent participates voluntarily and does not renege on a premium after any income history.

Problem LCE

$$\Pi(w) = \max_{\{\tau_i, w_i\},\, f \in [0,\bar{f}]} \quad f + \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t.} \quad \pi_H(u(y_H - f + \tau_H) + \beta w_H) + \pi_L(u(y_L - f + \tau_L) + \beta w_L) = w \qquad \text{(PK)}$$

$$u(y_H - f + \tau_H) + \beta w_H \geq u(y_H - f) + \beta V^{out} \qquad \text{(LCH)}$$

$$w_i \geq V^{out},\ \forall i$$

for $\forall w \geq V^{out}$ and where $f \in [0, \bar{f}]$ is the withholding/escrow amount with upper bound $\bar{f}$.

Theorem 1 characterizes the constrained-optimal contract with limited commitment and escrow. I focus on case (a) from Lemma 1 in which full insurance is not achievable. Clearly, if full insurance is already attainable in Problem LC no additional efficiency gains are possible.

Theorem 1. *Suppose full insurance is not feasible in Problem LC, i.e., inequality (L1) in Lemma 1 holds. Then the optimal insurance contract with escrow solving Problem LCE has the following properties:*

(a) full insurance: the escrow f relaxes the limited commitment constraint (LCH) and can implement full insurance, $c_i^{lce} = \bar{y}$ at zero expected profits if $\bar{f}$ is sufficiently large so that:

$$\frac{u(\bar{y})}{1-\beta} \geq u(y_H - \bar{f}) + \beta V^{out} \tag{E1}$$

(b) improved partial insurance: otherwise, if inequality (E1) does not hold, escrowed withholding relaxes the limited commitment constraint (LCH) and improves the degree of partial insurance relative to Problem LC.

Proof: see Appendix.

Theorem 1 shows that the escrow mechanism allows improved risk sharing and consumption smoothing relative to the standard model in Problem LC and Lemma 1. Specifically, Theorem 1(a) shows that if the escrow upper bound $\bar{f}$ is sufficiently large so that inequality (E1) holds, the insurance platform can provide full insurance while only partial insurance is possible in Problem LC.

In Theorem 1(b), if the maximum feasible escrow amount is relatively smaller so that full insurance cannot be attained, the ability to withhold income in escrow, no matter how small, allows the optimal contract to improve on the degree of partial insurance and consumption smoothing by relaxing the limited commitment constraint through lowering the agent's value of reneging and reducing the gap between c_i^{lce} and first-best consumption $c^{fi} = \bar{y}$ (see the Theorem 1 proof for details).

Note that the optimal insurance contract solving Problem LCE uses the escrow f to collect all or part of the due premia for an agent realizing the high income state, which is offset by adjusting the ex-post transfer amount τ_H^{lce}. In the low income state the withheld income f is effectively refunded in full as part of the insurance payout τ_L^{lce} received by the agent.

Corollary 1. *Suppose full insurance is not feasible in Problem LC, i.e., inequality (L1) in Lemma 1 holds. Holding the insurer's present-value expected profits fixed, the agent's ex-ante expected present-value utility attained in Problem LCE is larger than in Problem LC.*

Proof: see Appendix.

Corollary 1 to Theorem 1 shows that the initial promised utility level that guarantees zero present-value profits for the insurer, $\Pi(w_0) = 0$ is larger in Problem LCE compared to in Problem LC,

$$w_0^{lce} > w_0^{lc}$$

This means that ex-ante the agent would always voluntarily agree to the arrangement locking base income funds $f \leq \underline{y}$ in escrow when such technology is available. Note that the limited commitment problem remains relevant with respect to the agent's additional income beyond the portion of base income $\underline{y}$ that can be withheld in each income state.

3.2 Hidden Income

Consider next a setting in which risk sharing is constrained by the presence of private information. Specifically, assume that the agent's income is unobserved by the platform and the agent must be induced to report it truthfully. I refer to this setting as "hidden income". There is no commitment problem and an insurance contract is assumed fully enforceable. I relax this assumption and consider both private information and limited commitment simultaneously in Section 3.3.

The timing is as follows: (i) the agent enters the period with current promised utility w; (ii) the agent's income is realized (unobserved to the platform); (iii) the agent makes an income state report, i to the platform which triggers transfer τ_i and next-period promised utility w_i.

Without escrow, this is a well-known problem, e.g., see Townsend (1982), Thomas and Worrall (1990), or Ljungqvist and Sargent (2018). They prove that only the downward truth-telling constraints bind at optimum, i.e., the contract ensures that an agent with high income y_H (who would have to pay a premium) reports the true state $i = H$ instead of reporting $i = L$ (lying) and receiving transfer τ_L and promised utility w_L. The contracting problem is:

Problem HI

$$\Pi(w) = \max_{\{\tau_i, w_i\}} \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t. } \pi_H(u(y_H + \tau_H) + \beta w_H) + \pi_L(u(y_L + \tau_L) + \beta w_L) = w \quad \text{(PK)}$$

$$u(y_H + \tau_H) + \beta w_H \geq u(y_H + \tau_L) + \beta w_L \quad \text{(TT)}$$

Call the contract elements (policy functions), $\{\tau_i^{hi}, w_i^{hi}\}$ – each of them a function of the state w. The agent reports income state $i = H$ or $i = L$ and receives contract (τ_i^{hi}, w_i^{hi}) where τ_i^{hi} is the transfer, either a contribution (premium) or an indemnity (payout). The agent's consumption c_i^{hi} equals the agent's *actual* income y plus the transfer,

$$c_i^{hi} = y + \tau_i^{hi}, \quad \forall i$$

Lemma 2. *The optimal insurance contract solving Problem HI satisfies, $\forall w$:*

(a) $\tau_L^{hi} > \tau_H^{hi}$ *and* $w_H^{hi} > w_L^{hi}$ *– the transfer and promised utility are monotonic in the income state*

(b) $c_H^{hi} > c_L^{hi}$ *– partial insurance is provided.*

Proof: see Thomas and Worrall (1990) or Ljungqvist and Sargent (2018).

In the hidden income setting the constrained-optimal insurance contract always features partial insurance because of the information friction, expressed in the truth-telling constraint (TT). Intuitively, the first-best outcome of full insurance (equal consumption across states $c^{fi} = \bar{y}$ and equal promised utility w^{fi}) is not incentive-compatible with truth telling, as the agent would always report low income and collect the payout $\bar{y} - y_L > 0$ which means that the insurer cannot break even.

Escrow

As in Section 3.1, I next introduce escrowed withholding $f \in [0, \bar{f}]$ in Problem HI. The contracting problem is stated in Problem HIE, with the additional choice variable f. The contract elements (escrow f, transfers τ_i, and next-period promised utility w_i) are optimally chosen to ensure that the agent reports the income state truthfully.

Problem HIE

$$\Pi(w) = \max_{\{\tau_i, w_i\},\, f \in [0,\bar{f}]} \quad f + \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t.} \quad \pi_H(u(y_H - f + \tau_H) + \beta w_H) + \pi_L(u(y_L - f + \tau_L) + \beta w_L) = w \qquad \text{(PK)}$$

$$u(y_H - f + \tau_H) + \beta w_H \geq u(y_H - f + \tau_L) + \beta w_L \qquad \text{(TT)}$$

Theorem 2 characterizes the optimal insurance contract with private information (hidden income) and escrow.

Theorem 2. *The optimal insurance contract with escrow solving Problem HIE implies the same consumption and promised utility profile as in Problem HI.*

Proof: (see Appendix)

Theorem 2 shows that, unlike in the limited commitment setting in Section 3.1, if the obstacle to risk-sharing is private information due to hidden income, escrowed withholding cannot improve the maximum attainable degree of partial insurance and welfare. That is, the solutions (constrained-optimal consumption and promised utility as functions of w) of Problem HI and Problem HIE coincide.

The intuition for the Theorem 2 result is that there is no feasible way to relax the truth-telling constraint (TT) by use of the escrow f since, by construction, f is a fixed ex-ante set portion of the agent's base income independent of the subsequent income state. Hence the escrow cannot provide additional incentives relative to what already can be done via the contingent transfers τ_i and promised utility w_i.

3.3 Both Limited Commitment and Hidden Income

I now analyze a setting with both limited commitment and private information due to hidden income, as in Zilberman et al. (2019). I first characterize the contracting problem without escrowed witholding and then introduce the possibility of escrow $f \in [0, \bar{f}]$.

The contracting problem is, $\forall w \geq V^{out}$:

Problem HILC

$$\Pi(w) = \max_{\{\tau_i, w_i\}} \quad \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t.} \quad \pi_H(u(y_H + \tau_H) + \beta w_H) + \pi_L(u(y_L + \tau_L) + \beta w_L) = w \tag{PK}$$

$$u(y_H + \tau_H) + \beta w_H \geq u(y_H + \tau_L) + \beta w_L \tag{TT}$$

$$u(y_i + \tau_i) + \beta w_i \geq u(y_i) + \beta V^{out}, \ \forall i \tag{LC}$$

$$w_i \geq V^{out}, \ \forall i$$

Constraint (PK) is the promise-keeping constraint, (TT) is the downward truth-telling constraint as in Problem HI, and (LC) are the limited-commitment constraints ensuring that the agent does not renege on a transfer as in Problem LC. As in Section 3.1, the constraints $w_i \geq V^{out}$ ensure the agent's continued participation in the contract.

Theorem 3. *For any $w \geq V^{out}$ the optimal contract $\{\tau_i^{hilc}, w_i^{hilc}\}$, $i = L, H$ solving Problem HILC satisfies:*

(a) if $w = V^{out}$ then $\tau_L^{hilc} = 0$ and $w_L^{hilc} = V^{out}$.

(b) $\tau_L^{hilc} \geq 0$ *– the agent receives a transfer in the low-income state.*

(c) the truth-telling constraint (TT) is stricter than, i.e., implies the limited-commitment constraints (LC).

Proof: see Appendix.

The intuition for the non-negativity of the low-income state transfer τ_L in Theorem 3(b) follows from the objective to provide consumption smoothing to the risk-averse agent. In the low-income state the agent's marginal utility (MU) is high. Hence, to deliver fixed promised utility w at the lowest cost, it is optimal for the insurer to provide additional consumption in precisely that (high MU / low income) state rather than in the high-income state where raising the agent's utility is expensive. A hypothetical contract with $\tau_L < 0$ would mean that an agent is 'taxed' while s/he is consumption poor, which is the opposite of insurance, and would necessitate an inefficient increase in future promised utility w_L (i.e., in future periods including the possibility of high-income states) to satisfy the limited commitment and promise keeping constraints.

Theorem 3(c) is a key result implying that the limited-commitment constraints (LC) are automatically satisfied when truth-telling (TT) and the lower bound on future promises $w_i \geq V^{out}$ are imposed. That is, the limited-commitment problem (the possibility of the agent reneging on a contribution) expressed by constraints LC) does not constrain optimal risk sharing over and above the truth-telling constraint (TT).

Note, however, that the ex-ante continued participation constraints $w_i \geq V^{out}$ in Problem HILC in general do affect the contract compared to imposing constraint (TT) alone as in Problem HI. Indeed, previous research, e.g., Thomas and Worrall (1990), has shown that in Problem HI without imposing a lower bound on promises w_i it is optimal to set $w_L^{hi}(w) < w$ and $w_H^{hi}(w) > w$ – i.e., promised utilities spread out and, in addition, drift downward, with immiseration in the long run. Consequently, Theorem 3 implies that Problem HILC is equivalent to solving a modified Problem HI imposing the additional constraints $w_i \geq V^{out}$, i.e., restricting the range of promised utilities.

I next introduce escrowed withholding into the risk-sharing problem with both limited commitment and private information. The contracting problem is:

Problem HILCE

$$\Pi(w) = \max_{\{\tau_i, w_i\},\, f \in [0, \bar{f}]} \quad f + \pi_H(-\tau_H + \beta\Pi(w_H)) + \pi_L(-\tau_L + \beta\Pi(w_L))$$

$$\text{s.t.} \quad \pi_H(u(y_H - f + \tau_H) + \beta w_H) + \pi_L(u(y_L - f + \tau_L) + \beta w_L) = w \tag{PK}$$

$$u(y_H - f + \tau_H) + \beta w_H \geq u(y_H - f + \tau_L) + \beta w_L \tag{TT}$$

$$u(y_i - f + \tau_i) + \beta w_i \geq u(y_i - f) + \beta V^{out}, \ \forall i \tag{LC}$$

$$w_i \geq V^{out}, \ \forall i$$

Theorem 4 shows that, as in Theorem 2 for the setting with private information alone, allowing escrow in Problem HILC does not improve on the constrained-optimal degree of insurance. The escrow mechanism does relax the ex-post limited-commitment constraints (LC) as in Section 3.1., however, as shown in Theorem 3(c), these constraints are already satisfied at optimum when truth telling (TT) is imposed. Hence, relaxing the limited-commitment constraints yields no efficiency gains in the HILC setting since as shown in Theorem 2 the escrow f cannot relax the truth-telling constraint (TT).

Theorem 4. *The optimal insurance contract with escrow solving Problem HILCE implies the same consumption and promised utility profile as in Problem HILC.*

Proof: see Appendix.

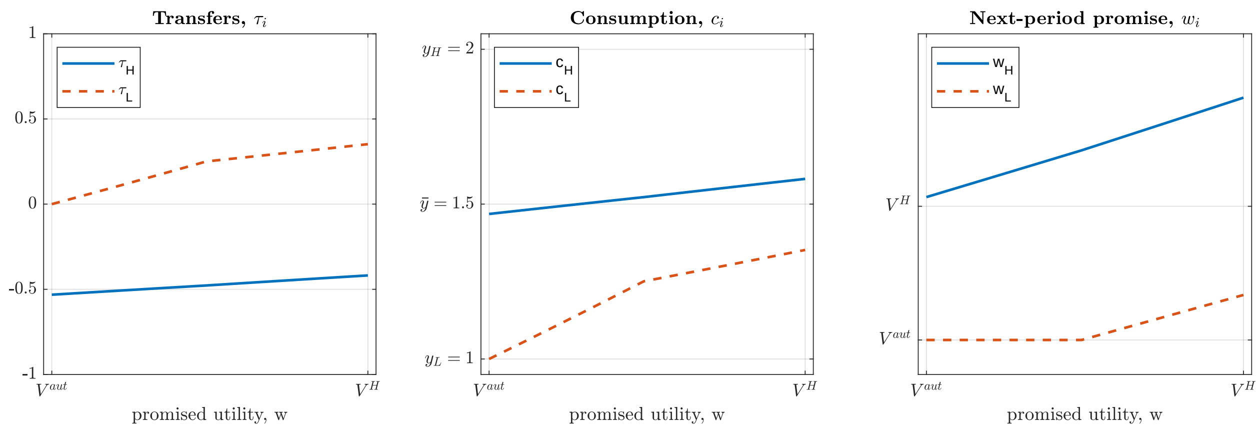
To provide further intuition about the properties of the risk-sharing contract with both limited commitment and hidden income, define the Lagrange multipliers in Problem HILC as follows: μ on (PK), γ on (TT), λ_i on (LC), and η_i on $w_i \geq V^{out}$, for $i = L, H$. Given Theorem 3(c), constraints (LC) are slack ($\lambda_i = 0$) and the problem solution satisfies the following first-order conditions:

$$\text{low income:} \quad \frac{1}{u'(c_L)} = \mu - \frac{\gamma}{\pi_L}\frac{u'(y_H + \tau_L)}{u'(c_L)} \quad \text{and} \quad -\Pi'(w_L) = \mu - \frac{\gamma}{\pi_L} + \frac{\eta_L}{\beta\pi_L} \tag{1}$$

$$\text{high income:} \quad \frac{1}{u'(c_H)} = -\Pi'(w_H) = \mu + \frac{\gamma}{\pi_H} \tag{2}$$

I proceed to illustrate the constrained-optimal dynamic insurance contract solving Problem HILC (and, equivalently, HILCE) via numerical simulation. As in Townsend (1982), assume quadratic utility $u(c) = c - bc^2$, where the parameter value $b = 0.075$ is chosen so that the function is strictly concave over the simulation consumption range. The other simulation parameters are: discount factor, $\beta = 0.95$ and income levels $y_H = 2$ and $y_L = 1$ with probabilities $\pi_L = \pi_H = 0.5$.

Figure 1: Problem HILC/HILCE Solution Properties



Notes: The figure displays simulation results for preferences $u(c) = c - bc^2$ with $b = 0.075$ and discount factor $\beta = 0.95$. The income levels are $y_L = 1$ and $y_H = 2$, with probabilities $\pi_L = \pi_H = 0.5$. The solution to Problem HILC/HILCE is illustrated: the constrained-optimal state-contingent transfers (left panel), consumption (middle panel), and future promises (right panel), each plotted as function of the current promised value w over the range $V^{out} = 26.25$ to $V^H \equiv u(y_H) + \beta V^{out} = 26.64$.

The solution to problem HILC/HILCE is displayed on Figure 1 for the state space $[V^{out}, V^H]$ where $V^H \equiv u(y_H) + \beta V^{out}$ is the value w_H^{lc} from Problem LC (see Lemma 1). The optimal contract has the following properties, $\forall w \in [V^{out}, V^H]$:

(a) the low-income transfer, τ_L^{hilc} is non-negative and increases in w from 0 at $w = V^{out}$ to about 0.35 at $w = V^H$ (see Fig. 1). The high-income transfer, τ_H^{hilc} is negative (the agent pays a premium) and strictly increasing in w.

(b) optimal consumption satisfies $c_H^{hilc}(w) > c_L^{hilc}(w)$ – there is partial insurance. This result follows directly from the first-order conditions (1) and (2) given $\gamma > 0$ (constraint TT binds), $u' > 0$, and $\pi_i > 0$.

(c) consumption in the low income state, $c_L^{hilc}(w)$ equals $y_L = 1$ at $w = V^{out}$ and strictly increases in w for $w > V^{out}$, with a kink at the value $\hat{w}$ where the bound on promised utility $w_L \geq V^{out}$ stops binding (see the right panel). At that value the multiplier η_L in FOC (1) becomes 0.

(d) consumption in the high-income state, $c_H^{hilc}(w)$ is strictly increasing in w starting from a value lower than $\bar{y} = E(y)$ at $w = V^{out}$.

(e) next-period promised utility, $w_i^{hilc}(w)$ satisfies:

– $w_L^{hilc}(w) = V^{out}$ for low values of w (the lower bound on w_L binds and $\eta_L > 0$ in FOC (1)), up to a threshold $\hat{w} > V^{out}$. For $w > \hat{w}$, $w_L^{hilc}(w) > V^{out}$ and $w_L^{hilc}(w)$ is strictly increasing in w.

– $w_H^{hilc}(w)$ strictly increasing in w for all $w \in [V^{out}, V^H]$.

3.4 Comparing consumption smoothing

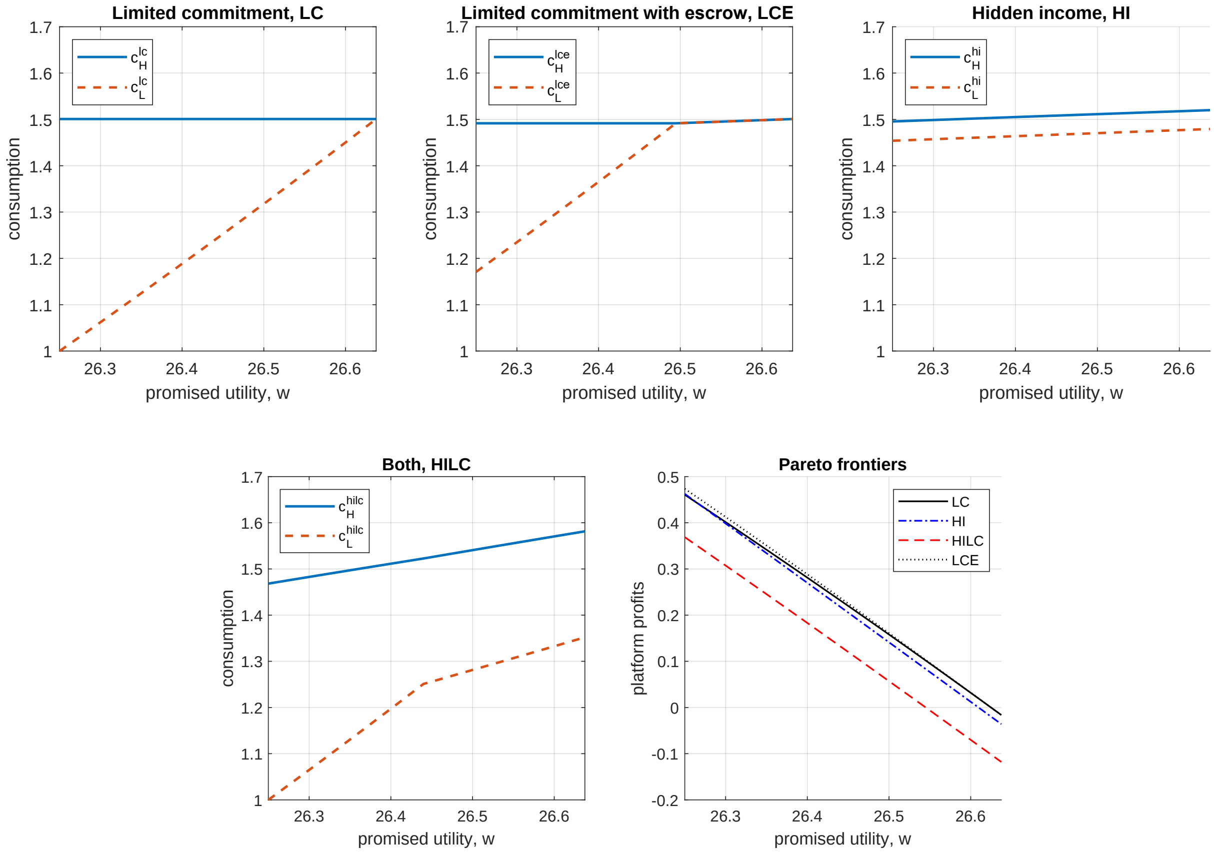
I numerically compute the solutions to Problems LC, LCE, HI(E) and HILC(E) and use the simulated policy functions to illustrate the level of consumption smoothing attained in the different contractual settings. Figure 2 displays the constrained-optimal consumption values in each income state, c_H and c_L, as functions of current promised utility w on the interval V^{out} to $V^H = u(y_H) + \beta V^{out}$.

In Problem LC (the top-left panel of Figure 2), as shown in Lemma 1, if the agent obtains high income, then her consumption c_H^{lc} (the solid line) is strictly below y_H (the agent pays an insurance premium) and is independent of the promised utility value w. In contrast, low-income consumption c_L^{lc} (the dashed line) is strictly increasing in the promise w. Intuitively, the insurer collects funds in the low marginal utility (high-income) state H and uses them to insure the agent against the low-income state L, see also Figure 3 below.

The top-centre panel of Figure 2 (Problem LCE) illustrates the effect of introducing escrow f in the limited commitment setting. Observe that low-income consumption level, c_L^{lce} is always larger than its counterpart c_L^{lc} in Problem LC while the high-state consumption remains the same, i.e., better consumption smoothing is provided. In addition, there is a sub-range of promised utility values, from $u(y_H - \bar{f}) + \beta V^{out}$ to $V^H = u(y_H) + \beta V^{out}$, in which full insurance, $c_H^{lce} = c_L^{lce}$, is attained – see Theorem 1.

The hidden income setting (HI), displayed in the top-right panel of Figure 2, always features a positive consumption differential between the two income states, $c_H^{hi} - c_L^{hi} > 0$. The differential is required to provide incentives for truth telling – see Lemma 2. Unlike in the limited-commitment setting, the platform can provide good consumption insurance (relatively high c_L^{hi} close to $\bar{y} = 1.5$) even for low promised utility values w, since without a commitment problem future promised utility w_L^{hi} can be reduced to below V^{out}. This incentive device is not available in the LC and HILC settings as the agent would quit the contract in such case. However, unlike in the LC problem where full insurance is attained after the agent realizes the highest income for the first time (Thomas and Worrall, 1988), in the HI setting consumption always remains state-contingent.

Figure 2: Risk sharing with different frictions – comparison

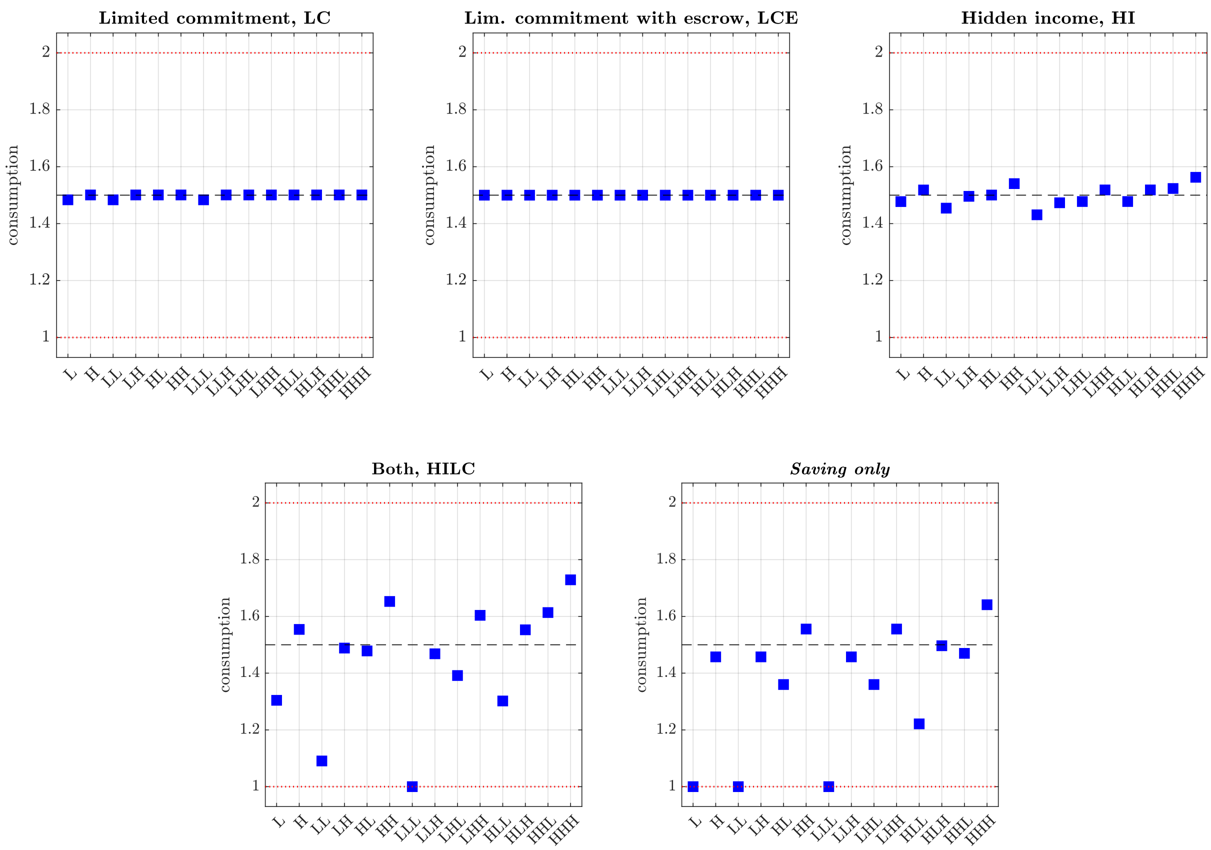


Notes: The figure displays simulation results for preferences $u(c) = c - bc^2$ with $b = 0.075$ and discount factor $\beta = 0.95$. The output levels are $y_H = 2$ and $y_L = 1$ with probabilities $\pi_L = \pi_H = 0.5$, and $\bar{f} = 0.2$. The figure panels display optimal consumption as function of the promised utility state w w over the range $V^{out} = 26.25$ to $V^H = u(y_H) + \beta V^{out} = 26.64$ respectively in problems LC, HI and HILC. Each simulation is initialized at the initial promised utility w_0 that attains zero ex-ante expected profits for the insurer, $\Pi(w_0) = 0$.

The contract with both limited commitment and private information frictions (Problem HILC), displayed in the bottom-left panel of Figure 2, combines characteristics of both the HI and LC optimal contracts. Specifically, the agent is 'taxed' in the high income state to insure against the low income state like in Problem LC, and the consumption functions c_H^{hilc} and c_L^{hilc} are strictly increasing in w like in Problem HI, albeit with a larger gap in-between because the use of future promised utility as incentive device is constrained by the participation lower bound $w_i^{hilc} \geq V^{out}$.

Lastly, the bottom-right panel of Figure 2 displays the Pareto frontiers in each setting, namely plotting the present-value discounted profit for the insurer, $\Pi(w)$ vs. the agent's ex-ante expected present-value utility, w in the respective optimal contracts. For the chosen simulation parameters the LC and HI Pareto frontiers lie close to each other, while the LCE frontier extends rightward of the LC frontier. Notably, the HILC setting with both commitment and information frictions delivers significantly lower expected utility for each fixed level of platform profits.

Figure 3: Risk sharing with different frictions – consumption



Notes: The figure displays simulation results for preferences $u(c) = c - bc^2$ with $b = 0.075$ and discount factor $\beta = 0.95$. The output levels are $y_H = 2$ and $y_L = 1$ with probabilities $\pi_L = \pi_H = 0.5$, and $\bar{f} = 0.2$. Each figure panel displays the optimal consumption level for each possible income history of length 1, 2 or 3 periods indicated on the horizontal axis (for example, history LHL denotes state L in period 1, H in period 2, and L in period 3) for Problems LC, LCE, HI, and HILC. Each simulation is initialized at the promised utility w_0 which attains zero ex-ante expected profits for the insurer, $\Pi(w_0) = 0$. The solutions are also compared (in the right-most panel) to a *Saving Only* setting (see footnote 6). The dashed line in all panels corresponds to the full-insurance (first-best) consumption level $\bar{y} = 1.5$; the dotted red lines correspond to the y_L and y_H income levels.

Figure 3 illustrates further the mechanics of consumption smoothing in the presence of the alternative considered frictions by plotting the optimal consumption values for different income histories. The plotted consumption values are computed for the initial promise w_0 (in general different for each setting) at which the platform breaks even ex-ante, i.e., solving $\Pi(w_0) = 0$. Specifically, Figure 3 plots the optimal consumption for all possible income histories of lengths 1, 2 or 3 time periods. For example, the history of length 1 labelled "L" on the horizontal axis means low income in period 1, history "HH" (of length 2) means high income in periods 1 and 2, and history "LLH" (of length 3) means low income in periods 1 and 2 and high income in period 3, and so on. As a comparison benchmark, note that the first-best outcome (full insurance) corresponds to consuming $c_i^{fi} = \bar{y} = 1.5$ in each history, while autarky (consumption equal to income, $c_i = y_i, \forall i$) would correspond to consuming $y_H = 2$ whenever current income is high (i.e., in histories H, LH, HH, LLH, LHH, HLH, HHH) and consuming $y_L = 1$ otherwise.

Figure 3 shows that the limited commitment settings achieve the highest degree of consumption smoothing, with the escrow technology (panel LCE in the top-centre) narrowing the gap between the high and low consumption values and achieving very close to full insurance.[3] The history-dependent consumption values vary sizably more in the hidden income setting HI (the top-right panel of Fig. 3), where notably histories with consecutive low or consecutive high incomes require a larger spread in consumption to maintain truth-telling incentives.

When both the commitment and information frictions are present (panel "Both, HILC" in the bottom row of Figure 3), the degree of consumption smoothing across income state histories is significantly worsened, underlying the Pareto frontier findings in Fig. 2. The last panel (bottom row, right) also compares the LC, LCE, HI and HILC solutions with the exogenously-incomplete "saving only" setting, in which the agent starts with zero savings and can accumulate, at gross rate of return R, or run down buffer-stock savings.[4] We see that, in terms of state-contingent consumption, the HILC setting resembles the saving-only setting although HILC provides more insurance against income histories L and LL.

[3] The reason why c_L^{lc} and c_H^{lc} are numerically close in the simulation is that the break-even initial promised utility w_0 in the LC model is very close to the upper bound of the state space V^H, see the Pareto frontier in Figure 2.

[4] In the saving only (S) setting, the agent solves the following dynamic problem:

$$V^S(a) = \max_{\{a_i'\}} \sum_{i \in L,H} \pi_i \big(u(y_i + Ra - a_i') + \beta V^S(a_i') \big)$$

where a is current savings, a_i' are next-period savings in income state i, and $a_0 = 0$.

4 Conclusions

I analyze escrowed withholding of income in dynamic risk-sharing with commitment or information frictions, considered both separately and jointly. I model a long-term dynamic risk-sharing contract where an insurance platform can withhold ex-ante a fixed limited portion of the agent's base income in escrow. The main result is that the escrow technology can be used to mitigate or resolve the limited commitment problem but is ineffective in the settings with private information alone and with both private information and limited commitment.

In the limited-commitment setting, the escrow mechanism allows lowering the agent's value of reneging and thus diminishes the agent's incentive to not pay due insurance premia when his/her income is high. This relaxes the commitment constraints resulting in improved consumption smoothing. In contrast, when the obstacle to risk sharing is private information (hidden income), the escrow mechanism does not lead to an efficiency improvement since the agent's incentive to misreport the income state depends on the utility differential between reporting high versus low income. Since the withholding amount is fixed ex-ante before the income state is realized, it does not affect the agent's truth-telling constraint and cannot provide additional incentives beyond what can be achieved via state-contingent transfers and promised utility alone.

In the scenario with both limited commitment and hidden income frictions present at the same time, I prove that the private information friction dominates – that is, satisfying truth-telling implies satisfying the limited commitment constraints. Consequently, escrowed withholding also does not improve efficiency and consumption smoothing in this setting. Numerical simulations illustrate and compare the constrained-optimal risk-sharing contracts in the considered settings characterizing their ability to provide consumption smoothing relative to autarky, full insurance, and buffer stock saving benchmarks.

The analysis in this paper assumes for simplicity that the escrowed withholding technology is costless. Digital platforms and programmatic escrow can indeed provide such service at very low cost. Given the welfare gains present in the limited commitment model a (sufficiently low) cost can be incorporated without changing the main results.

While the paper's results are mostly theoretical, they offer practical implications for the design of (digital) financial and insurance platforms. A key takeaway is that technological solutions for locking funds such as smart contracts or programmatic escrow are best suited for situations where the primary risk is contract repudiation or reneging on due payments. In contrast, in settings where client income verification is costly or infeasible, escrowed withholding would be insufficient and ineffective for improving risk sharing and instead mechanisms designed to elicit truthful revelation of unobserved states are the binding requirement.

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Appendix – Proofs

Lemma 1

(a) and (b) The solution to Problem LC has been characterized in previous work, e.g., see Thomas and Worrall (1988) or Ljungqvist and Sargent (2018) (hereafter LS), with contract properties as stated in parts (a) and (b) of Lemma 1. Note also, that for $\tau_L^{lc} > 0$, which holds for any $w > V^{out}$ (see LS for proof), constraint (LC) is slack in the low-income state $i = L$ since $u(y_L + \tau_L) + \beta w_L > u(y_L) + \beta V^{out}$ given $w_L \geq V^{out}$.

(c) Calling $w^{fi} = \frac{u(\bar{y})}{1-\beta}$ note that inequality (L1), $\frac{u(\bar{y})}{1-\beta} \geq u(y_H) + \beta V^{out}$, is equivalent to

$$u(\bar{y}) + \beta w^{fi} \geq u(y_H) + \beta V^{out}$$

which implies that both (LC) constraints are satisfied and hence the insurance platform can implement full insurance at zero ex-ante expected profits ($\Pi(w_0) = 0$) by setting $w_0 = w^{fi}$ and offering the contract:

$$\tau_i^{lc} = \bar{y} - y_i \;\text{ (i.e., } c_i^{lc} = c^{fi} = \bar{y}\text{) and } w_i^{lc} = w^{fi} = \frac{u(\bar{y})}{1-\beta}, \quad \text{for } \; i = L, H \tag{FI}$$

Note that profits are zero both ex-ante and in each period.

If the opposite inequality holds, i.e.,

$$\frac{u(\bar{y})}{1-\beta} < u(y_H) + \beta V^{out} \tag{L1a}$$

then, from part (a) we know that $\forall w \in [V^{out}, u(y_H) + \beta V^{out})$ partial insurance obtains with the (LC) constraint binding for income state $i = H$. Call $V^H = u(y_H) + \beta V^{out}$ and note that if (L1a) holds then the initial promised utility level w_0 at which $\Pi(w_0) = 0$ must satisfy $w_0 < V^H$, i.e., w_0 is in the partial insurance range described in part (a). To see this, call c^* the absorbing consumption level (see Lemma 1(b)) obtained at $w = V^H$ and defined by $\frac{u(c^*)}{1-\beta} = V^H$ and note that inequality (L1a) implies $c^* > \bar{y}$ by the strict monotonicity of u. But then expected consumption exceeds expected income in each period and hence $\Pi(V^H) < 0$, from where the monotonicity of Π implies $w_0 < V^H$ indeed.

Theorem 1

Denote the constrained-optimal transfers solving Problem LCE by τ_i^{lce}, $i = L, H$ and the optimal escrow amount by f^{lce}.

(a) **escrow attains full insurance**

If inequality (E1), $\frac{u(\bar{y})}{1-\beta} \geq u(y_H - \bar{f}) + \beta V^{out}$ holds,[5] then first-best full insurance, $c_i = \bar{y}$, $\forall i$ can be attained by setting $f^{lce} \in [0, \bar{f}]$ so that,

$$u(y_H - f^{lce}) + \beta V^{out} = \frac{u(\bar{y})}{1-\beta} \tag{T1}$$

Such f^{lce} always exists given (E1), since the function $G(f) = u(y_H - f) + \beta V^{out}$ is strictly decreasing in f and since, as assumed, inequality (L1a) from Lemma 1, $\frac{u(\bar{y})}{1-\beta} < u(y_H) + \beta V^{out}$ holds, i.e., $G(0) > \frac{u(\bar{y})}{1-\beta}$.

Consider the following transfers τ_i^{lce} and promises w_i^{lce}:

$$\tau_i^{lce} = \bar{y} - y_i + f^{lce} \quad \text{and} \quad w_i^{lce} = \frac{u(\bar{y})}{1-\beta} = w^{fi}, \quad \forall i$$

which implies $c_i^{lce} = y_i - f^{lce} + \tau_i^{lce} = \bar{y}$. The limited commitment constraint (LCH) in Problem LCE evaluated at $c_i^{lce} = \bar{y}$, $w_i^{lce} = w^{fi}$ and f^{lce} defined in (T1) is equivalent to:

$$u(\bar{y}) + \beta w^{fi} \geq u(y_H - f^{lce}) + \beta V^{out},$$

which is satisfied since, using (T1), both its l.h.s. and r.h.s. equal $w^{fi} = \frac{u(\bar{y})}{1-\beta}$. Constraint (PK) is also trivially satisfied at $w = w^{fi}$ and $w_i^{lce} = w^{fi}$ and the platform's profits from the contract are zero, as in Lemma 1(b). Therefore, full insurance at zero expected profits (see L is indeed implementable via the contract described above, starting from $w_0 = w^{fi} = \frac{u(\bar{y})}{1-\beta}$.

Note that there is an indeterminacy in the escrow amount f and the transfers – any feasible values $\{\hat{\tau}_i\}$ and $\hat{f} \in [f^{lce}, \bar{f}]$ which satisfy $\hat{\tau}_i - \hat{f} = \bar{y} - y_i$ achieve the same consumption values $\{c_i^{lce}\}$ and respect constraint (LCH). $\square$

(b) **escrow improves partial insurance**

If the upper bound $\bar{f}$ is relatively low so that inequality (E1) does not hold, the insurer cannot implement the first-best full insurance allocation $c_i = \bar{y}$, since even using the maximum feasible escrow level $f^{lce} = \bar{f}$ would result in the agent reneging in income state $i = H$ (remember, Theorem 1 assumes that inequality (L1) from Lemma 1 holds).

However, note that the escrow f still relaxes constraint (LCH) for any $\bar{f} > 0$. Indeed, constraint (LCH) is

$$u(y_H - f + \tau_H) + \beta w_H \geq u(y_H - f) + \beta V^{out}$$

Setting f at its maximum feasible value $f^{lce} = \bar{f}$ therefore lowers the right hand side (the

[5] A simple sufficient condition for (E1) is $\bar{f} \geq y_H - \bar{y}$.

agent's value from reneging on the contract), while the constraint can be always satisfied by adjusting the transfer τ_H in the left hand side as required.

Therefore, following Lemma 1, we obtain that for any promised utility level $w \in [V^{out}, u(y_H - \bar{f}) + \beta V^{out})$, the constrained-optimal contract solving Problem LCE is:

$$w_H^{lce} = u(y_H - \bar{f}) + \beta V^{out} \text{ and } u(c_H^{lce}) = (1-\beta) w_H^{lce}$$

which implies $c_H^{lce} > c_H^{fi} = \bar{y}$, as in Lemma 1(a) – i.e., partial insurance.

Comparing with Lemma 1, note that $w_H^{lce} < w_H^{lc}$ and therefore, using the relationship between c_H and w_H,

$$c_H^{lce} < c_H^{lc},$$

that is, in Problem LCE consumption in the high income state c_H^{lce} is closer to the first-best level $c_H^{fi} = \bar{y}$ compared to its counterpart c_H^{lc} in Problem LC.

Similarly, c_L^{lce} is determined from the promise-keeping constraint (PK) and, because $c_H^{lce} < c_H^{lc}$ and $w_L^{lce} = w$ (see Lemma 1(a)), c_L^{lce} must satisfy $c_L^{lce} > c_L^{lc}$, that is, c_L^{lce} is also closer to $c_L^{fi} = \bar{y}$. Therefore the degree of partial insurance in Problem LCE is strictly improved by the escrow mechanism, as the gap between consumption in the high and low income states shrinks. See Figure 2 in Section 3.4 for a numerical example illustration.■

Corollary 1

In Theorem 1 we showed that, for any given promised utility level $w \in [V^{out}, u(y_H) + \beta V^{out})$, the consumption spread between the high and low income states is reduced (to zero in Theorem 1(a)) in Problem LCE compared to in Problem LC, i.e.,

$$c_H^{lce} - c_L^{lce} < c_H^{lc} - c_L^{lc}$$

Consider a marginal perturbation that, starting at (c_L^{lc}, c_H^{lc}) reduces the consumption spread by lowering high-state consumption by a small amount $dc_H = -\Delta$ where $\Delta > 0$ and increasing low-state consumption by $dc_L > 0$ such that the promise keeping constraint (PK) remains unchanged:

$$\pi_H u'(c_H^{lc})(-\Delta) + \pi_L u'(c_L^{lc}) dc_L = 0 \implies dc_L = \frac{\pi_H}{\pi_L} \frac{u'(c_H^{lc})}{u'(c_L^{lc})} \Delta$$

The resulting change in the insurer's expected net transfer cost is:

$$dC = \pi_H(-\Delta) + \pi_L(dc_L) = \pi_H \Delta \left(\frac{u'(c_H^{lc})}{u'(c_L^{lc})} - 1 \right)$$

Since, under the Corollary assumptions, the Problem LC solution features partial insurance, $c_H^{lc} > c_L^{lc}$, and since $u(\cdot)$ strictly concave implies decreasing marginal utility, we have $u'(c_H^{lc}) < u'(c_L^{lc})$. Therefore, the term in the brackets is strictly negative, implying $dC < 0$, i.e., by providing better consumption smoothing, the insurer can deliver present-value utility w at a strictly lower expected cost (net transfers), as long as the limited-commitment constraints remain satisfied.

The above results imply that, for any given $w \in [V^{out}, u(y_H) + \beta V^{out})$, the maximized platform profit in Problem LCE is strictly higher than that in Problem LC:

$$\Pi^{lce}(w) > \Pi^{lc}(w) \tag{3}$$

The initial promised utility w_0 and resulting contract are pinned down by the zero-profit condition. Call w_0^{lc} and w_0^{lce} the initial promised utilities solving respectively:

$$\Pi^{lc}(w_0^{lc}) = 0 \quad \text{and} \quad \Pi^{lce}(w_0^{lce}) = 0$$

By the Envelope theorem, since the promise keeping constraint binds at optimum, the platform's profit function $\Pi(w)$ is strictly decreasing in the promised utility w. Therefore, inequality (3) implies:

$$w_0^{lce} > w_0^{lc},$$

showing that agreeing ex-ante to the escrow mechanism strictly increases the agent's expected utility.■

Theorem 2

Suppose there exists a feasible contract $(f^{**}, \tau_i^{**}, w_i^{**})$ that is different from the Problem HI solution $(0, \tau_i^{hi}, w_i^{hi})$ and achieves higher objective value in Problem HIE. Then consider the contract:

$$\tilde{f} = 0, \;\; \tilde{\tau}_i = \tau_i^{**} - f^{**}, \;\; \text{and} \;\; \tilde{w}_i = w_i^{**}$$

Note that the objective value in Problem HIE (and HI) remains the same since

$$f^{**} + \pi_H(-\tau_H^{**}) + \pi_L(-\tau_L^{**}) = 0 + \pi_H(-(\tau_H^{**} - f^{**})) + \pi_L(-(\tau_L^{**} - f^{**}))$$

Constraints (PK) and (TT) are also unchanged since

$$\tau_i^{**} - f^{**} = \tilde{\tau}_i - 0 \;\; \text{for} \;\; i = L, H.$$

Therefore, the contract $(\tilde{f}, \tilde{\tau}_i, \tilde{w}_i) = (0, \tilde{\tau}_i, \tilde{w}_i)$ attains the same objective value as contract $(f^{**}, \tau_i^{**}, w_i^{**})$, i.e., by assumption higher than the value from $(0, \tau_i^{hi}, w_i^{hi})$ and is feasible for

Problem HI since it does not use escrow ($\tilde{f} = 0$) and satisfies all constraints. However, this contradicts the optimality of contract $(0, \tau_i^{hi}, w_i^{hi})$ solving Problem HI and hence such $(f^{**}, \tau_i^{**}, w_i^{**})$ does not exist. This implies that the constrained-optimal consumption profiles in Problems HI and HIE are the same and therefore the escrowed withholding technology does not improve the attainable degree of insurance and welfare in the hidden income setting.■

Theorem 3

(a) **The boundary case** $w = V^{out}$.

The result that $\tau_L = 0$ and $w_L = V^{out}$ for $w = V^{out}$ follows as in Zilberman et al. (1999). First, note that the promise-keeping constraint (PK) at $w = V^{out}$ and the (LC) constraints $u(y_i + \tau_i) + \beta w_i \geq u(y_i) + \beta V^{out}$, $\forall i$ imply:

$$\begin{aligned} V^{out} &= \pi_H(u(y_H + \tau_H) + \beta w_H) + \pi_L(u(y_L + \tau_L) + \beta w_L) \\ &\geq \pi_H(u(y_H) + \beta V^{out}) + \pi_L(u(y_L) + \beta V^{out}) = V^{out} \end{aligned}$$

This means that at $w = V^{aut}$ we have,

$$u(y_i + \tau_i) + \beta w_i = u(y_i) + \beta V^{out}, \ \forall i \tag{4}$$

Since u is strictly increasing, equations (4) and the constraints $w_i \geq V^{out}$ then imply

$$\tau_i \leq 0, \quad \forall i \tag{5}$$

Next, use (4) and the truth-telling constraint (TT) to obtain:

$$\begin{aligned} &u(y_H) + \beta V^{out} = u(y_H + \tau_H) + \beta w_H \geq \quad [\textit{using (TT)}] \\ &\geq u(y_H + \tau_L) + \beta w_L = \quad [\textit{using (LC) at i=L}] \\ &= u(y_H + \tau_L) + u(y_L) + \beta V^{out} - u(y_L + \tau_L) \end{aligned}$$

which implies

$$u(y_H) - u(y_L) \geq u(y_H + \tau_L) - u(y_L + \tau_L) \tag{6}$$

By the concavity of u, inequality (6) implies

$$\tau_L \geq 0$$

Combining with the previous results in (5) and (4), we conclude that for $w = V^{out}$

$$\tau_L = 0 \quad \text{and} \quad w_L = V^{out}$$

completing the proof.□

(b) Using a perturbation argument, I next prove that it cannot be optimal to set $\tau_L < 0$ for any $w > V^{out}$, i.e., asking the agent to pay a premium in the low income state. Suppose not, i.e., assume $\tau_L^{hilc} < 0$ for some $w > V^{out}$. I will show that the value of the Problem HILC objective would increase if the insurer raised τ_L slightly and adjusted the continuation utility w_L to keep constant the agent's total value $V_L = u(y_L + \tau_L) + \beta w_L$ from reporting $i = L$.

Note first that if $\tau_L < 0$ then $c_L < y_L$. This implies, using the $i = L$ limited commitment constraint, $w_L > V^{out}$ and so decreasing w_L is possible. Consider a small increase in the low income transfer $d\tau_L = \epsilon > 0$ and reduce the promised utility in the low state by

$$dw_L = -\frac{u'(c_L)\epsilon}{\beta}$$

to keep the agent's total value $V_L = u(y_L + \tau_L) + \beta w_L$ constant, thus ensuring that constraint (PK) and constraint (LC) for $i = L$ remain satisfied. Keep the other contract elements τ_H and w_H unchanged, hence constraint (LC) for $i = H$ is unaffected. Finally, note that constraint (TT) remains satisfied too, since its l.h.s. does not change while it r.h.s. changes by

$$u'(y_H + \tau_L)\epsilon - \beta\frac{u'(c_L)\epsilon}{\beta} = \epsilon(u'(y_H + \tau_L) - u'(y_L + \tau_L)) < 0$$

by the strict concavity of u and $y_H > y_L$. Therefore the perturbed contract $(\tau_L + d\tau_L, w_L - dw_L, \tau_H, w_H)$ is feasible for Problem HILC.

Next, evaluate the effect of the perturbation on the insurer's profits:

$$d\Pi = \pi_L \left(\underbrace{-\epsilon}_{\text{higher payout}} + \underbrace{\beta\Pi'(w_L)dw_L}_{\text{change in future value}} \right)$$

By the Envelope Theorem, $\Pi'(w) = -\mu(w)$ where $\mu(w)$ is the marginal cost of providing agent utility in state w (the multiplier on the PK constraint). We thus obtain:

$$d\Pi = \pi_L \left[-\epsilon - \beta\mu(w_L) \left(-\frac{u'(c_L)\epsilon}{\beta} \right) \right] = \pi_L\epsilon \left[\mu(w_L)u'(c_L) - 1 \right]$$

To prove that the contract perturbation increases profits ($d\Pi > 0$), and hence $\tau_L < 0$ is suboptimal, it therefore remains to show that:

$$\mu(w_L) > \frac{1}{u'(c_L)} \tag{7}$$

Note that at $w = V^{out}$, as shown in part (a) constraint (LC) for $i = L$ is redundant as it is implied by (PK) and (LC) for $i = H$. The FOC with respect to τ_L at $w = V^{out}$ then implies:

$$\mu(V^{out}) = \frac{1}{u'(y_L)} + \frac{\gamma}{\pi_L}\frac{u'(y_H)}{u'(y_L)}$$

or, since $\gamma \geq 0$ and u is strictly increasing,

$$\mu(V^{out}) \geq \frac{1}{u'(y_L)} \tag{8}$$

By standard arguments, e.g., Thomas and Worrall (1990) or Ljungqvist and Sargent (2018), since the utility function u is strictly concave and the insurer is risk-neutral, the profit function $\Pi(w)$ is strictly concave and thus the insurer's marginal cost $-\Pi'(w) = \mu(w)$ is strictly increasing in w. Therefore, using (8), $w_L > V^{out}$ implies:

$$\mu(w_L) > \mu(V^{out}) \geq \frac{1}{u'(y_L)}$$

In addition, $\tau_L < 0$ implies $c_L < y_L$ and, because u is concave, $u'(c_L) > u'(y_L)$, that is,

$$\frac{1}{u'(y_L)} > \frac{1}{u'(c_L)}$$

Combining the above inequalities, we obtain:

$$\mu(w_L) > \frac{1}{u'(y_L)} > \frac{1}{u'(c_L)}$$

which implies that inequality (7) indeed holds, and hence the resulting change in profits $d\Pi$ is strictly positive if the insurer increases τ_L starting from $\tau_L < 0$ while reducing w_L to keep all constraints satisfied.

Intuitively, if $\tau_L < 0$ (i.e., $c_L < y_L$) the agent's marginal utility in the low-income state is high which makes increasing current consumption c_L a more efficient way to deliver utils to the agent than the future promise w_L (because future-period income could also be high and the agent's marginal utility low). We conclude that at the Problem HILC solution $\tau_L < 0$ is impossible and therefore $\tau_L \geq 0$ for all $w \geq V^{out}$, with $\tau_L = 0$ for $w = V^{out}$.$\square$

(c) Note that since the HI contract requires $w_L \geq V^{out}$, then as shown in parts (a) and (b), for $\tau_L \geq 0$, we have

$$u(y_H + \tau_L) + \beta w_L \geq u(y_H) + \beta V^{out} \tag{9}$$

Inequality (9) implies that the truth-telling constraint (TT)

$$u(y_H + \tau_H) + \beta w_H \geq u(y_H + \tau_L) + \beta w_L$$

is stricter than, and therefore implies, the limited commitment constraint (LC) for $i = H$,

$$u(y_H + \tau_H) + \beta w_H \geq u(y_H) + \beta V^{out}$$

The other (LC) constraint for $i = L$,

$$u(y_L + \tau_L) + \beta w_L \geq u(y_L) + \beta V^{out}$$

is also directly implied by $\tau_L \geq 0$ and $w_L \geq V^{out}$.■

Theorem 4

It is easy to show that the Problem HILC solution can be implemented in Problem HILCE in an analogous way as done for Problem HIE in Theorem 2. Therefore, choosing $f > 0$ again cannot improve the degree of insurance relative to Problem HILC since the limited commitment constraints (LC) do not bind at the optimum and the escrow cannot relax the truth-telling constraint (TT) as shown in Theorem 2.■