Lecture 32 - Galaxy formation and evolution

What's Important:

- galaxy formation
- galactic evolution

Text: Carroll and Ostlie, Chap. 24, Sec. 25.3

Galaxy formation

There are several different aspects to galaxy formation:
development of density fluctuations, as early as T = 3000 K
aggregation of a gas cloud to form a galaxy
aggregation of small galaxies to form a larger one
collision of galaxies, that may result in the "knock-out" of stars

The development of density fluctuations is not yet covered in this course (see end of lecture). We start with a calculation of the free-fall time for a galaxy, going back to a calculation performed previously:

$$t_{ff} = (3\pi / 32G\rho_o)^{1/2}, \qquad (32.1)$$

where ρ_o is the initial density of the medium.

As an example, we showed before that t_{ff} for a giant molecular cloud with $n = 10^{14}$ m^{-3} is a short 5100 years. A more typical, and lower, value of ρ_o gives 10^5 years.

Let's do a calculation of the Milky Way, assuming

$M = 6 \times 10^{11}$ solar masses (including dark matter)
$= 6 \times 10^{11} \cdot 2.0 \times 10^{30}$
$= 1.2 \times 10^{42}$ kg.

R = 100 kpc (outer limit of halo as detected)
$= 100 \cdot 10^3 \cdot 3.09 \times 10^{16}$
$= 3.09 \times 10^{21}$ m.

Hence:

$\rho_o = M / (4\pi R^3 / 3)$
$= 1.2 \times 10^{42} / (4\pi [3.09 \times 10^{21}]^3 / 3)$
$= 9.7 \times 10^{-24}$ kg/m^3.

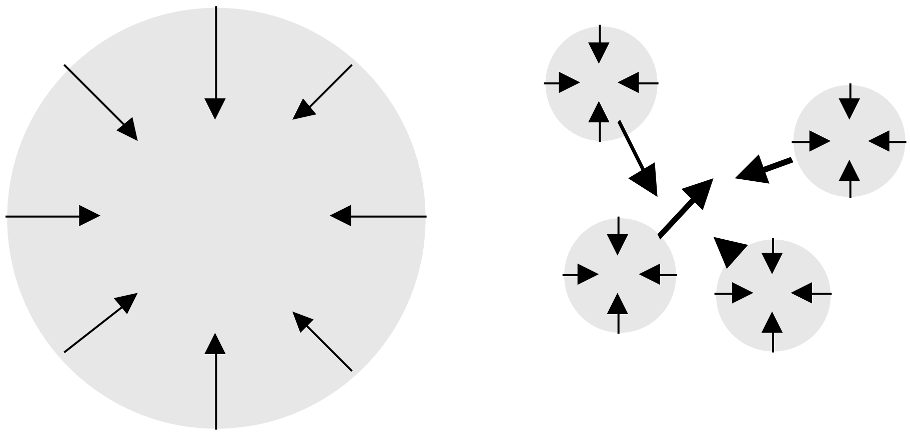
From this, the free fall time is

$t_{ff} = (3\pi / [32 \cdot 6.63 \times 10^{-11} \cdot 9.7 \times 10^{-24}])^{1/2}$
$= 2.14 \times 10^{16}$ seconds
$= 6.8 \times 10^8$ years = 0.7 billion years.

This seems to be a reasonable enough estimate, although it's not the whole story. First, the time frame is that of free-fall, assuming that the energy released in the process can be rapidly radiates away. If the "cooling time" is lower, then the process will be slowed down.

The free-fall model was proposed in 1962 by Eggen, Lynden-Bell and Sandage, and is an extreme "top-down" model, if applied without modification. Evidence that the single event picture is incomplete for the Milky Way includes:

· unlike the stars in the rotating disk, the net rotational velocity of the halo stars is close to zero; equal populations of stars rotate in both directions. This suggests a turbulent initial state.
· there is a spread in age of the stars, which is larger than the collapse time; the globular clusters seem to span 14-17 billion years ago, similar to the thick disk.



These observations may be evidence for an additional "bottom up" contribution to galaxy formation: perhaps there were small proto-galaxies of mass 10^6 to 10^8 solar masses, which then aggregated with a larger object. This would spread out the aggregation time. Further, smaller density fluctuations are more probable than larger ones; hence, small protogalaxies are likely to have been present.

Disk thickness (C&O, Sec. 24.2)

We have described previously how the radius of a star is related to its temperature and core pressure. In the context of a disk-shaped galaxy, analogous quantity is the thickness of the galactic disk. The calculation has several steps:

· find the gravitational potential energy as a function of z, the vertical displacement from the galactic plane
· equate the potential energy with the thermal energy scale, 3/2 $k_B T$
· solve for h as a function of T.

Gauss' law for gravity

Both Coulomb;s law and Newton's Law of Gravity have the same functional form:

$$F_{couloomb} = (1/\,4\pi\varepsilon_o)\, Q_1 Q_2 / r^2 \qquad F_{gravity} = G\, M_1 M_2 / r^2$$

In first year, we obtain Gauss' Law for electric charges as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{4\pi Q_{inside}}{4\pi\varepsilon_0}$$

where **E** is the electric field (**F**/*e*) and **A** is an area element. To find the gravitational equivalent of this expression, we make the replacements:

$(1/4\pi\varepsilon_o) \rightarrow G$
$Q \rightarrow M$ or m
$q\mathbf{E} \rightarrow m\mathbf{g}$.

The replacement is

$$\oint q\mathbf{E}\cdot d\mathbf{A} = \frac{4\pi q Q_{inside}}{4\pi\varepsilon_0}$$

$$\int m\mathbf{g}\bullet d\mathbf{A} = -4\pi G m M_{\rm inside}$$

or

$$\int \mathbf{g}\bullet d\mathbf{A} = -4\pi G M_{\rm inside}. \qquad (32.2)$$

The minus sign is needed because the gravitational attraction between masses is attractive, whereas the Coulomb force between like-sign charges is repulsive.

We apply this equation to a cylinder with end cap area A and total height $2h$ straddling the galactic disk. Then, ignoring the direction of $g(h)$

$$2Ag(h) = 4\pi G M_{\rm inside}. \qquad (32.3)$$

To find the mass enclosed by the mathematical cylinder, we need to know the density distribution. We said previously that the galactic density drops exponentially with z which we now parametrize as

$$\rho(z) = \rho_o \exp(-z/h),$$

where the height h is the 1/e location of the density. The enclosed mass on both sides of the midplane is then

$$M_{inside} = 2\int_0^h \rho_0 \exp(-z/h)A\,dz = 2\rho_0 Ah\int_0^1 \exp(-x)\,dx$$

The integral is just $(1 - e^{-1}) = 0.632..$, so

$$M_{\rm inside} = 1.264..\rho_o Ah. \qquad (32.4)$$

Placing Eq. (32.4) into (32.3) gives:

$$2Ag(h) = 4\pi G \bullet 1.264..\rho_o Ah$$

or

$$g(h) = 2.53\,\pi G\rho_o h. \qquad (32.5)$$

Not unexpectedly, g grows linearly with h (contrast with spherical distribution). To find the potential energy as a function of h, we just write (32.5) as a function of z and integrate from $z = 0$ to h:

$$U(h) = \int_0^h mg(z)\,dz = \int_0^h 2.53\pi m G\rho_0 z\,dz = 1.26\pi m G\rho_0 h^2 \qquad (32.6)$$

This is the potential energy of a test mass m. Equating (32.6) to the average thermal energy of the particle $3/2\ k_B T$ yields

$$3/2\ k_B T = 1.26\pi G m\rho_o h^2,$$

or

$$h = \left(\frac{3k_BT}{2.53\pi Gm\rho_0}\right)^{1/2} . \qquad (32.7)$$

As expected, the thickness of the disk increases with temperature.

Example

We said before that gravitational collapse can lead to temperatures in the 10^6 K range. What is the thickness of a gaseous disk with $T = 10^6$ K and ρ_o = 0.15 solar masses per cubic parsec? Use:

m = hydrogen atom = 1.67×10^{-27} kg.

$\rho_o = 0.15 \bullet 2.0 \times 10^{30} / (3.09 \times 10^{16})^3 = 1.02 \times 10^{-20}$ kg/m^3.

Then

$h = (3 \bullet 1.38 \times 10^{-23} \bullet 10^6 / [2.53\pi \bullet 6.67 \times 10^{-11} \bullet 1.67 \times 10^{-27} \bullet 1.02 \times 10^{-20}])^{1/2}$

$= 6.8 \times 10^{19}$ m

= 2190 pc

= 2.2 kpc.

This is comparable to the scale height of the disk as measured today.

Local group (C&O, Sec. 25.3)

It should come as no surprise that the aggregation heirarchy doesn't stop at galaxies. Although the number of members in a collection of galaxies is not as large as the 10^8 to 10^{11} stars in a galaxy, nevertheless, the spatial proximity of the members, and their mutual interaction through gravity is clear. Galactic aggregates are classified as

groups

50 galaxies or less
typically 1 Mpc across
speeds of 150 km/s.

clusters

more than 50 galaxies
much larger - 6 Mpc (check)
higher relative speeds -

The Milky Way is a member of a local group of about 30 galaxies, whose two prominent members are the Milky Way and Andromeda, accounting for 90% of the visible mass of the group. The Milky Way and Andromeda mutually attract:

current distance = 770 kpc = $770 \times 10^3 \times 3.09 \times 10^{16} = 2.4 \times 10^{22}$ m.
current speed = 119 km/s = 1.19×10^5 m/s.

Even if the relative speed of the two galaxies does not increase as they approach, the galaxies will collide in

$2.4 \times 10^{22} / 1.19 \times 10^5 = 2.0 \times 10^{17}$ seconds
= 6.3 billion years

Just another thing to worry about when the Sun goes out.

One can use the speed and distance of the two galaxies to determine the mass of the galaxies, using Kepler's Laws. The resulting calculation gives a mass which is about 20 times the estimated visible mass of the galaxies, another piece of evidence in favour of dark matter.

Density fluctuations in the early universe

Calculations of the growth of density fluctuations and the Jeans mass are given in Carroll and Ostlie, Sec. 28.1.