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Prospects and problems of tachyon matter cosmology

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Abstract

We consider the evolution of FRW cosmological models and linear perturbations of tachyon matter rolling towards a minimum of its potential. The tachyon coupled to gravity is described by an effective 4d field theory of string theory tachyon. In the model where a tachyon potential $V(T)$ has a quadratic minimum at finite value of the tachyon field T_0 and $V(T_0) = 0$, the tachyon condensate oscillates around its minimum with a decreasing amplitude. It is shown that its effective equation of state is $p = -\varepsilon/3$. However, linear inhomogeneous tachyon fluctuations coupled to the oscillating background condensate are exponentially unstable due to the effect of parametric resonance. In another interesting model, where tachyon potential exponentially approaches zero at infinity of T, rolling tachyon condensate in an expanding Universe behaves as pressureless fluid. Its linear fluctuations coupled with small metric perturbations evolve similar to these in a pressureless fluid. However, this linear stage changes to a strongly non-linear one very early, so that the usual quasi-linear stage observed at sufficiently large scales in the present Universe may not be realized in the absence of the usual particle-like cold dark matter.

1. Introduction

There are many faces of superstring/brane cosmology which come from different corners of M/String theories. In particular, people search for potential candidates to explain early Universe inflation, present day dark energy and dark matter in the Universe. One of the string theory constructions, tachyon on D-branes, has been recently proposed for cosmological applications by Sen [1]. A relatively simple formulation of the unstable D-brane tachyon dynamics in terms of effective field theory stimulates one to investigate its role in cosmology [2].

The rolling tachyon in the string theory may be described in terms of an effective field theory for the tachyon condensate T which in the flat space–time has a Lagrangian density

$$\mathcal{L} = -V(T)\sqrt{1 + \partial_\mu T \partial^\mu T}. \tag{1}$$

The tachyon potential $V(T)$ has a positive maximum at $T = 0$ and a minimum at T_0 with $V(T_0) = 0$. We consider two models: with a finite value of T_0 and with a minimum at infinity, as illustrated in Fig. 1. In both cases one encounters interesting possibilities for cosmological applications.

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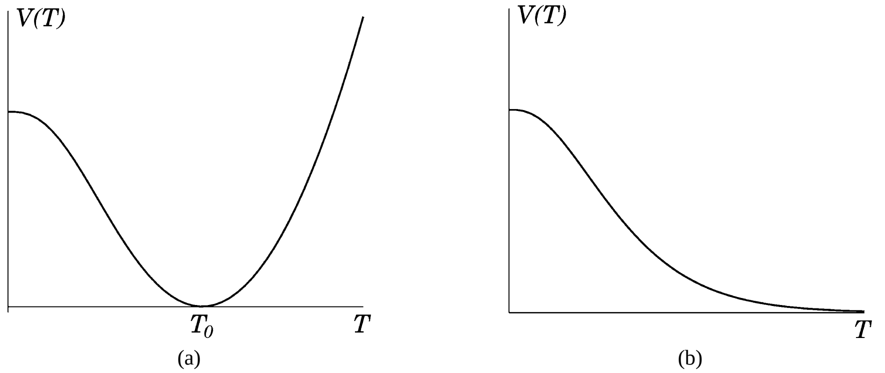


Fig. 1. Tachyon matter potentials with a minimum at a finite (a) and the infinite (b) value of the field. The potentials near the minimum are taken to be: (a) $V(T) = \frac{1}{2}m^2(T - T_0)^2$, (b) $V(T) = V_0 e^{-T/T_0}$.

In the case of finite T_0, we consider quadratic expansion around the minimum of the potential $V(T) \approx \frac{1}{2}m^2(T - T_0)^2$. As we will show, in this case the tachyon matter has negative pressure and may be considered a candidate for quintessence.

In the case when $T_0 \to \infty$, we use exponential asymptotic of the potential $V(T) = V_0 e^{-T/T_0}$ derived from the string theory calculations [3–5] (the exact form of the potential from Ref. [3,4] is $V = (1 + \frac{T}{T_0})e^{-T/T_0}$; our qualitative results for late time asymptotics of $T(t)$ do not depend on the pre-exponential factor). Dimensional parameters of the potential are related to the fundamental length scale, $T_0 \sim l_s$, and V_0 is the brane tension. As it was demonstrated by Sen [5], the tachyon matter is pressureless for the potential with the ground state at infinity. In this case tachyon matter may be considered a cold dark matter candidate [1].

It is noteworthy that models of type (1) have already been studied in cosmology on phenomenological grounds. For certain choices of potentials V and non-minimal kinetic terms one can get kinematically driven inflation, "k-inflation" [6]. In particular, a toy model with the potential $V(T) \sim 1/T^2$ with a ground state at infinity may give rise to the power law inflation of the Universe [6–8]. However, it remains to be seen how this potential can be motivated by the string theory of tachyon. The model with $V \equiv \text{const}$ is reduced to the so-called "Chaplygin gas" where the matter equation of state is $p = -\text{const}/\varepsilon$. Such matter was suggested as a candidate for the present dark energy [9,10].

In this Letter, we investigate cosmology with tachyon matter with the string theory motivated potentials of Fig. 1. In Section 2, we write down equations for the tachyon matter coupled to gravity. We focus on self-consistent formulation of the isotropic Friedmann–Robertson–Walker (FRW) cosmology supported by the tachyon matter. It is described by coupled equations for the time-dependent background tachyon field $T(t)$ and the scale factor of the Universe $a(t)$.

One of the lessons of the scalar field theory in cosmology is the possibility of the fast growth of inhomogeneous scalar field fluctuations, as it was found in different situations. Instability of scalar field fluctuations are typical for preheating after inflation due to the parametric resonance [11], or tachyonic preheating after hybrid inflation [12] (which so far has only remote relation with the string theory tachyon). Fluctuations may be unstable in axion cosmology due to parametric resonance [13]. Therefore, we address the problem of stability of linear fluctuations of the rolling tachyon matter.

Consistent investigation of tachyon cosmology, in principle, should be started with the tachyon rolling from the top of its potential which has negative curvature. In this case the setting of the problem is similar to what we met in tachyonic preheating after hybrid inflation [12]. In these cases we expect fast decay of the scalar field into long-wavelength inhomogeneities. Here we assume that the somehow homogeneous tachyon rolls towards the minimum of its potential as the Universe expands, and consider tachyon fluctuations at the latest stages of its evolution.

In Section 3, we develop a formalism for treating small fluctuations of the tachyon field $\delta T(t,\vec{x})$. It is possible to extend the theory of tachyon matter fluctuations by including scalar metric fluctuations. This allows us to address the issue of gravitational instability in tachyon cosmology. Applying this analysis for specific tachyon potentials, we will see that instability of tachyon fluctuations is essential for the whole story of tachyon cosmology.

In Section 4, we consider background cosmological solutions for the tachyon potential with the ground state at finite value T_0. We find that tachyon field is oscillating around the minimum of its potential, while its equation of state (averaged over oscillations) is $p=-\frac{1}{3}\varepsilon$. Then in Section 5, we check the stability of tachyon fluctuations around this background solutions, and find that they are exponentially unstable due to the parametric resonance.

In Section 6, we repeat the analysis for the model where tachyon potential is exponential $V(T)\propto e^{-T/T_0}$ and its ground state is at $T\to\infty$. In this case, background cosmological solution corresponds to the pressureless tachyon with energy density $\varepsilon\propto 1/a^3$. In Section 7, we consider small inhomogeneous tachyon and metric fluctuations, and find gravitational instability of fluctuations around the background solution. Specifically, we find that the linear approximation for fluctuations becomes insufficient very early for the pressureless rolling tachyon. We argue that the rolling tachyon dark matter scenario may have difficulties in explaining gravitational clustering and large scale velocity flows in the Universe.

2. Cosmology with rolling tachyon matter

A rolling tachyon is associated with unstable D-branes, and self-consistent inclusion of gravity may require higher-dimensional Einstein equations with branes. Still, in the low energy limit, one expects that the brane gravity is reduced to the four-dimensional Einstein theory [14].

In this section, we consider tachyon matter coupled with Einstein gravity in four dimensions. Tachyon matter is described by the phenomenological Lagrangian density (1), where derivatives are covariantly generalized with respect to the metric $g_{\mu\nu}$, $\partial_\mu\to\nabla_\mu$. We use the metric with signature $(-,+,+,+)$. The model is given by the action

$$S=\int d^4x\,\sqrt{-g}\times\left(\frac{R}{16\pi G}-V(T)\sqrt{1+\nabla_\mu T\nabla^\mu T}\right). \tag{2}$$

The Einstein equations which follow from (2) are

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G\left(\frac{V}{\sqrt{1+\nabla_\alpha T\nabla^\alpha T}}\nabla_\mu T\nabla_\nu T-g_{\mu\nu}V\sqrt{1+\nabla_\alpha T\nabla^\alpha T}\right), \tag{3}$$

and the field equation for the tachyon is

$$\nabla_\mu\nabla^\mu T-\frac{\nabla_\mu\nabla_\nu T}{1+\nabla_\alpha T\nabla^\alpha T}\nabla^\mu T\nabla^\nu T-\frac{V_{,T}}{V}=0. \tag{4}$$

Let us apply these equations to a spatially flat ($K=0$) FRW cosmological model

$$ds^2=-dt^2+a^2(t)\,d\vec{x}^{\,2}. \tag{5}$$

For this geometry, the energy–momentum tensor of tachyon matter in the right-hand side of Eq. (3) is reduced to a diagonal form $T^\mu_\nu=\mathrm{diag}(-\varepsilon,p,p,p)$ where the energy density ε is positive

$$\varepsilon=\frac{V(T)}{\sqrt{1-\dot{T}^2}} \tag{6}$$

and the pressure p is negative or zero

$$p=-V(T)\sqrt{1-\dot{T}^2}. \tag{7}$$

Equation for the evolution of the scale factor follows from (3)

$$\frac{\dot{a}^2}{a^2}=\frac{8\pi G}{3}\frac{V(T)}{\sqrt{1-\dot{T}^2}}. \tag{8}$$

Equation for the time-dependent rolling tachyon in an expanding Universe follows from (4)

$$\frac{\ddot{T}}{1-\dot{T}^2}+3\frac{\dot{a}}{a}\dot{T}+\frac{V_{,T}}{V}=0. \tag{9}$$

Note that the tachyon potential enters the field equation in a combination $(\ln V)_{,T}$.

In the following sections we consider background solutions of Eqs. (8) and (9) for two models of the tachyon potentials $V(T)$ from Fig. 1.

3. Fluctuations in rolling tachyon

The issue of stability of a FRW background with respect to small spatially inhomogeneous fluctuations is often essential in cosmology. In this section, we provide a formalism for treating linear inhomogeneous scalar fluctuations in tachyon cosmology. Let us consider small inhomogeneous perturbation of the tachyon field $\delta T(t,\vec{x})$ around time-dependent background solution $T(t)$ of Eq. (9):

$$T\big(t,\vec{x}\big) = T(t) + \delta T\big(t,\vec{x}\big). \tag{10}$$

As we will see, for one of our examples of tachyon potentials $V(T)$, instability of tachyon fluctuations grows and becomes non-linear very quickly. Therefore, first we write down the equation for fluctuations $\delta T(t,\vec{x})$ ignoring expansion of the Universe and ignoring coupling of tachyon fluctuations to metric fluctuations.

Linearizing the field equation (4) (without the Hubble friction term) with respect to small fluctuations δT and performing Fourier decomposition $\delta T(t,\vec{x}) = \int d^3k\, T_k(t) e^{i\vec{k}\vec{x}}$ of the linear fluctuations, we obtain evolution equation for the time-dependent Fourier amplitudes $T_k(t)$

$$\frac{\ddot{T}_k}{1-\dot{T}^2} + \frac{2\dot{T}\ddot{T}}{(1-\dot{T}^2)^2}\dot{T}_k + \big[k^2 + (\log V)_{,TT}\big]T_k = 0. \tag{11}$$

Next we consider tachyon fluctuations coupled with metric perturbations in an expanding Universe. Small scalar metric perturbations around a FRW background can be written in the longitudinal gauge as

$$ds^2 = -(1+2\Phi)\,dt^2 + (1-2\Psi)a^2(t)\,d\vec{x}^2. \tag{12}$$

Now we have to linearize the Einstein equations (3) and the field equation (4) with respect to small fluctuations δT, Φ and Ψ. Then it follows that $\Phi = \Psi$ for tachyon cosmology (as well as in many other cases, in particular, for minimally coupled scalar field cosmology).

Fortunately, the useful formalism for cosmological scalar fluctuations for the class of models which includes the theory (2) was developed in Ref. [6] (in connection with "k-inflation"). This is exactly what we need to pursue the investigation of small cosmological fluctuations with tachyon matter. Using results of [6], from (3) and (4) we obtain two coupled equations for the time-dependent Fourier amplitudes $T_k(t)$ and $\Phi_k(t)$,

$$\left(\frac{T_k}{\dot{T}}\right)^{\cdot} = \left(1 - \frac{1}{4\pi G}\frac{k^2}{a^2}\frac{(1-\dot{T}^2)^{3/2}}{V\dot{T}^2}\right)\Phi_k, \tag{13}$$

and

$$\frac{(a\Phi_k)^{\cdot}}{a} = 4\pi G \frac{V\dot{T}^2}{(1-\dot{T}^2)^{1/2}}\frac{T_k}{\dot{T}}. \tag{14}$$

Introducing the Mukhanov variable v_k, which is related to the potential Φ_k as

$$\frac{v_k}{z} = \frac{5\varepsilon+3p}{3(\varepsilon+p)}\Phi_k + \frac{2}{3}\frac{\varepsilon}{\varepsilon+p}\frac{\dot{\Phi}_k}{H}, \tag{15}$$

where energy density ε and pressure p are given by Eqs. (6) and (7), $H = \dot{a}/a$, and

$$z = \frac{\sqrt{3}\,a\dot{T}}{(1-\dot{T}^2)^{1/2}}, \tag{16}$$

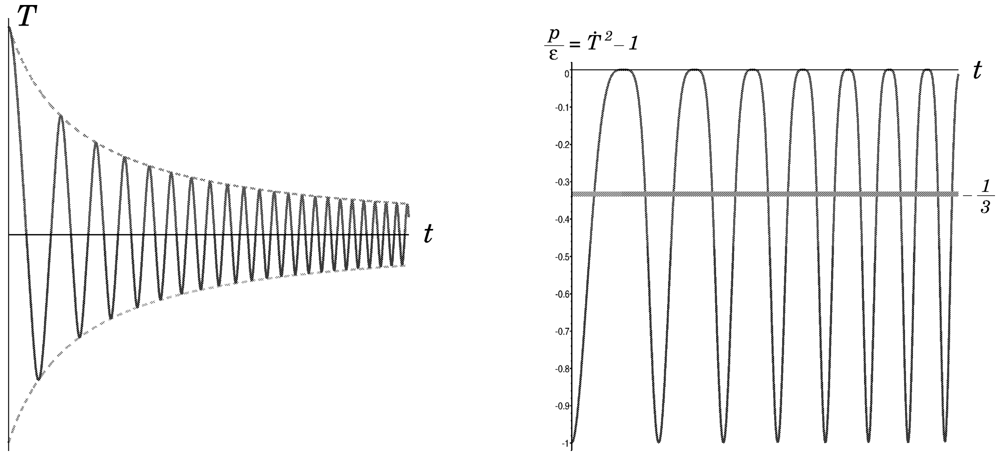
Eqs. (13) and (14) can be reduced to a single second order equation for v_k

$$v_k'' + \left(\big(1-\dot{T}^2\big)k^2 - \frac{z''}{z}\right)v_k = 0, \tag{17}$$

where prime ($'$) stands for derivative with respect to the conformal time $d\eta = dt/a(t)$. We will use this equation for analysis of coupled tachyon and metric fluctuations in an expanding Universe in Section 7.

4. Negative-pressure tachyon matter

In this section, we consider the model with a potential $V(T)$ with its ground state at a finite value T_0, as it is sketched in the left panel of Fig. 1. Let us assume that tachyon is rolling towards the minimum of the potential T_0. We will approximate the shape of the tachyon potential around the minimum by a quadratic form $V(T) \approx \frac{1}{2}m^2(T-T_0)^2$. Despite quadratic form of the potential, tachyon motion around T_0 is not harmonic, since $\ln V$ but not V is involved in the



(a) (b)

Fig. 2. (a) Background tachyon oscillations in the model with $V(T) = \frac{1}{2}m^2(T - T_0)^2$. (b) Background oscillations of the tachyon equation of state. The horizontal line $p/\varepsilon = -1/3$ is the time-averaged equation of state.

tachyon equation of motion. For the same reason parameter m drops out of the field equation (9). It is convenient to use tachyon field in units of T_0 and time t also in units of T_0. Parameter m, however, is involved in the energy density of tachyon $\varepsilon \propto m^2/t^2$. The choice of $m \sim l_s^{-1} \sim M_p$ may bring the value of ε to the required density of dark energy.

Numerical solution of Eqs. (9) and (8) reveals that the tachyon begins to oscillate around the minimum of the potential very soon, within a time interval of several T_0, as shown in the left panel of Fig. 2. The amplitude of the oscillations is decreasing with time due to the Hubble friction term in Eq. (9). The envelope curve (dashed line) in the left panel of Fig. 2 shows the amplitude decreasing as $1/t$. As we will see below, this time-dependence of the amplitude exactly corresponds to the (time-averaged) equation of state p/ε which will be found for the tachyon matter in this model. Also, note that the period of oscillations is decreasing with time. Tachyon oscillations in this model are not only non-harmonic, but also non-periodic.

The instant value of the ratio of energy density (6) and pressure (7), $p/\varepsilon = \dot{T}^2 - 1$, is oscillating with time, as shown in the right panel of Fig. 2. Although the amplitude of oscillations T is decreasing with time, the amplitude of $\dot{T}$ is not changing with time as it is clear from the Fig. 2.

The period of oscillations is very small ($\sim T_0$), so that only the average equation of state is important for cosmological evolution. To find it, we average $\dot{T}^2 - 1$ over several consecutive oscillations. The average value of p/ε, shown as the horizontal line at the right panel of Fig. 2, is independent of time and equal to

$$\left\langle \frac{p}{\varepsilon} \right\rangle = -\frac{1}{3}. \tag{18}$$

For this type of an equation of state, an average value of the energy density is diluted as $\langle\varepsilon\rangle \propto a^{-2}$ with the expansion of the Universe. The amplitude of the tachyon oscillations is then decreasing as $1/t$, which is compatible with numerical results. From (8) we find that the averaged scale factor is $a(t) \propto t$. Note that equation of state similar to (18) occurs for a network of cosmic strings.

The equation of state (18) for the quadratic tachyon potential can be easily derived analytically. Indeed, assuming that tachyon is oscillating much faster than the Universe expands, we can treat energy density ε as adiabatic invariant, and write $\dot{T}^2 = 1 - V^2(T)/\varepsilon^2$, where ε is constant over several consecutive oscillations. Then the average value of $\dot{T}^2$ for quadratic potential is

$$\begin{aligned}\langle \dot{T}^2 \rangle &= \frac{\int \dot{T}^2 dt}{\int dt} \\ &= \frac{\oint (1 - V^2(T)/\varepsilon^2)^{1/2} dT}{\oint (1 - V^2(T)/\varepsilon^2)^{-1/2} dT} = \frac{2}{3}.\end{aligned} \tag{19}$$

So, $\langle p/\varepsilon\rangle = \langle \dot{T}^2\rangle - 1 = -1/3$. If shape of the potential around the minimum is not quadratic, but a power-law $V \propto (T - T_0)^n$, the average equation of state is $\langle p/\varepsilon\rangle = -\frac{1}{n+1}$.

Although tachyon matter in the model has negative pressure, apparently it is short of explaining the present acceleration of the Universe. Combination of cosmological observations of CMB fluctuations, large scale structure clustering and high red shift supernovae constrains the equation of state to be lower than $\langle p/\varepsilon\rangle < -0.6$ [16] (or even $\langle p/\varepsilon\rangle < -0.76$ at 95% c.l. according to [17]).

As we will see in the next section, background tachyon dynamics in this model is unstable with respect to small spatially inhomogeneous fluctuations, and homogeneous tachyon oscillations will decay. It will be interesting to find what will be the final configuration of tachyon matter in this model and what may be its potential application to cosmology.

5. Fluctuations in tachyon matter with negative pressure

In a realistic cosmological scenario, it is expected that the tachyon field has small, quantum or classical, inhomogeneous fluctuations. In this section, we check the stability of tachyon fluctuations around the background solution discussed in the previous section. For the moment, let us ignore the expansion of the Universe. Then we only have to solve equation (11) to find behaviour of fluctuations. Although formally some of the coefficients in Eq. (11) are singular when the background field $T(t)$ crosses zero in the case of quadratic potential, it is possible to switch to regular variables and overcome this technical inconvenience. Numerical solution of the fluctuation equation (for example, for $k = 10$ in units of T_0) is shown in Fig. 3.

General theory of linear equations with periodic coefficients predicts the presence of stability and instability bands of momenta k. For unstable modes, the amplitude is increasing exponentially as $T_k(t) \sim e^{\mu_k t}$. For the value of k in Fig. 3 the amplitude of fluctuations is increasing with time exponentially fast, by an order of magnitude in one background oscillation, say $T_k(t)$ increases by a factor of 10^{10} in ten oscillations! The physical reason is amplification due to the parametric resonance. This can be clearly

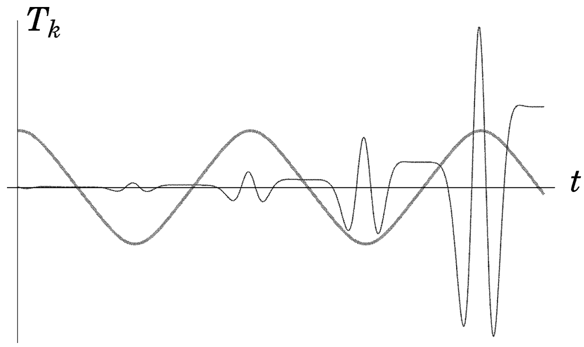


Fig. 3. Instability of fluctuations $T_k(t)$ in the model with the quadratic potential (scales are linear).

seen if one rewrites equation (11) in the form of the oscillator-like equation, where the effective frequency is oscillating with time. This effect can be described by the theory of broad parametric resonance [15].

Since period of oscillation ($\sim T_0$) is tiny compared to the cosmological time, and fluctuations become significant within several background oscillations, one can ignore expansion of the Universe in this analysis. Thus we conclude that, in the model with the quadratic potential, small tachyon fluctuations are exponentially unstable and a background tachyon condensate decays into a strongly inhomogeneous field configuration.

Decay of the background tachyon condensate into inhomogeneous fluctuations does not necessarily mean that the Universe becomes inhomogeneous. The tachyon fluid remains homogeneous as a whole, but not as a coherent condensate. Therefore, it remains to be seen, based on the fully non-linear analysis, what will be equation of state of the non-condensate tachyon fluid.

6. Pressureless tachyon matter

In this section, we consider tachyon cosmology for a tachyon potential having its ground state at infinity and decaying sufficiently fast:

$$T^2 V(T) \to 0, \quad T \to \infty, \tag{20}$$

as sketched in the right panel of Fig. 1 for a particular example of exponential potential. Background cosmological solutions of Eqs. (8) and (9) for this case very quickly (several T_0) enter the regime where tachyon is rolling very fast and $\dot{T}$ approaches unity. Let us write

$T(t) = t + \theta(t)$, $|\theta| \ll t$ (note that $\dot\theta < 0$). To the first order in θ, Eq. (9) reduces to

$$\frac{\ddot\theta}{\dot\theta} = 6\frac{\dot a}{a} + 2\frac{V'}{V}\bigg|_{T=t} \tag{21}$$

which can be easily integrated. We obtain:

$$\dot\theta = -\frac{1}{2}\left(\frac{a}{a_0}\right)^6 \frac{V^2}{V_0^2}\bigg|_{T=t},$$
$$1 - \dot T^2 = -2\dot\theta, \qquad a_0,\ V_0 = \text{const}, \tag{22}$$

$$\varepsilon = V_0\left(\frac{a_0}{a}\right)^3, \qquad p = -\left(\frac{a}{a_0}\right)^3 \frac{V^2}{V_0}\bigg|_{T=t},$$
$$|p| \ll \varepsilon. \tag{23}$$

Thus, for a wide class of potentials (20) the tachyon at late times behaves as a dust-like matter, as was first discovered by Sen [1,5] in flat space–time. We see that inclusion of gravity leads to the scaling of the tachyon energy density with the scale factor: $\varepsilon \propto a^{-3}$, which is just the right one for a cold dark matter. This makes tachyon matter with such potentials a cosmological dark matter (not dark energy!) candidate. Note that formulas (22) and (23) apply, in particular, both for the radiation dominated stage where $a(t) \propto \sqrt{t}$ where tachyon contribution to gravity is subdominant, and for $a(t) \propto t^{2/3}$ where tachyon gravitationally dominates.

In case of the exponentially decaying potential $V = V_0 e^{-T/T_0}$, Eq. (22) leads to

$$T(t) = t + \frac{T_0}{4}\left(\frac{a}{a_0}\right)^6 e^{-2t/T_0}, \tag{24}$$

and then pressure vanishes with time exponentially fast. Without expansion of the Universe, the solution (24) corresponds to that of Sen [18].

7. Cosmological fluctuations for pressureless tachyon

The crucial property of cosmological dark matter without pressure is the growth of cosmological fluctuations which form a developed large scale structure. The large scale structure of the Universe is ranging from non-linear clustered halos of galaxies and clusters of galaxies, quasi-linear structures at scales of superclusters and voids, and linear fluctuations at very large scales. It is essential that at quasi-linear and non-linear stages dark matter is displaced from the homogeneous distribution due to flows generated by fluctuations of gravitational potential, and gravitationally bound halos have high velocity dispersions.

In this section, we investigate clustering properties of the pressureless tachyon matter. We assume rolling tachyon matter domination, so that the law of the Universe expansion is $a(t) \propto t^{2/3}$.

We begin with the linear analysis of cosmological fluctuations, using formalism of Section 3. For a moment, consider the case without expansion of the Universe and without coupling to gravitational perturbations. Then it follows from Eq. (11) for the exponential potential that fluctuations are not growing, $T_k = \text{const}$.

Now let us consider tachyon fluctuations including expansion of the Universe and coupling to a gravitational potential Φ. Substituting the background solution (24) for the pressureless tachyon matter into Eq. (17), one can see that the coefficient in front of k^2 (which plays the role of the sound speed for the tachyon matter) vanishes exponentially fast. This means that the growth of linear tachyon fluctuations is scale independent, similar to that of the standard cold dark matter scenario. Then the solution of (17) is $v_k = z$, and the left-hand side of Eq. (15) is constant. From this we immediately get the time evolution of fluctuations Φ_k and T_k

$$\Phi_k(t) = \text{const}, \qquad T_k(t) = \Phi_k \cdot t. \tag{25}$$

Linear metric fluctuations are constant, similar to that in the cold dark matter scenario. However, the fluctuations in the tachyon field are growing, in contrast to the simplified analysis above where we neglected coupling to the metric fluctuations and expansion of the Universe. The growth of tachyon fluctuations $T_k \propto t$ cannot be obtained without these ingredients. Thus, tachyon fluctuations are unstable due to the effects of gravitational instability in an expanding Universe.

However, a linear approximation for the rolling tachyon/gravity system works only during a very short time interval (of the order of tens of T_0). Indeed, let us inspect the energy density of tachyon matter in the model with the exponential potential not assuming it to be homogeneous and using the perturbed space–time metric (12):

$$\varepsilon = \frac{V_0 e^{-T/T_0}}{\sqrt{1-(1-2\Phi)\dot{T}^2 + a^{-2}(\nabla_{\vec{x}} T)^2}}. \tag{26}$$

For fluctuations of δT, we have $\delta T \simeq \Phi(\vec{x})t$ where $\Phi(\vec{x})$ describes the initial spatial profile of fluctuations. The full tachyon field including fluctuations is

$$T(t,\vec{x}) = t + (T_0/4)(a/a_0)^6 e^{-2t/T_0} + \Phi(\vec{x})t. \tag{27}$$

The numerator of the expression (26) vanishes as e^{-t/T_0}, while the denominator evolves as $(a/a_0)^6 \times e^{-2t/T_0} + a^{-2}t^2(\nabla_{\vec{x}}\Phi)^2$. The linear approximation works during a very short time interval while $(\nabla_{\vec{x}}\Phi)^2 \lesssim e^{-2t/T_0}$. When this inequality breaks, linear analysis becomes insufficient. For cosmological fluctuations $\Phi \sim 10^{-5}$, the linear theory for δT is valid during a time interval of order of $10T_0$. Recall that for the standard cold dark matter scenario the linear stage lasts during a significant fraction of the present age of the Universe.

8. Summary

We considered cosmological solutions of rolling tachyon condensate T for two models of tachyon potential $V(T)$. There are different levels at which one can theoreticize about $T(t)$ in an expanding Universe. Systematic approach suggests for us to begin with a theory of tachyon field rolling down from the top of its potential. The curvature of the potential at the origin is negative, and we expect tachyonic instability of long wavelength fluctuations, similar to spinodal instability in usual field theory [12]. Thus there is an issue of initial conditions for rolling tachyon cosmology.

Suppose (by choice of initial conditions where T is displaced from the origin) tachyon evolves towards its ground state as a homogeneous condensate. We considered the model with minimum at finite T_0 and with quadratic approximation of $V(T)$ around the minimum. Then the background tachyon oscillates around the minimum with frequency of order of $1/T_0$, and its (averaged over several oscillations) equation of state is $p/\varepsilon = -1/3$. However, we found from perturbation theory that tachyon fluctuations are exponentially unstable due to the tachyon self-interaction with background oscillations. It means that in this model homogeneous tachyon condensate decays. To answer the question what will be the resulting tachyon configuration, one has to go to the next level beyond the perturbation theory and to consider fully non-linear problem of evolution of non-condensate tachyon fluid.

We also consider homogeneous tachyon rolling towards its ground state in the model with $V(T) \propto e^{-T/T_0}$, including expansion of the Universe. In this model, the background tachyon condensate has vanishing pressure and finite energy density diluting as $\varepsilon \propto 1/a^3$. Considering linear perturbations of tachyon field coupled with small metric perturbations, we found gravitational instability of tachyon field, $\delta T \propto t$. However, linear theory very soon (tens of T_0) becomes irrelevant. Clearly, much more work including a full non-linear analysis is needed to make more certain conclusions about viability of tachyon cosmology.

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