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Accretion of a ghost condensate by black holes

Andrei V. Frolov
KIPAC/SITP, Stanford University, Stanford, California, 94305-4060, USA
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The intent of this paper is to point out that the accretion of a ghost condensate by black holes could be extremely efficient. We analyze steady-state spherically symmetric flows of the ghost fluid in the gravitational field of a Schwarzschild black hole and calculate the accretion rate. Unlike minimally coupled scalar field or quintessence, the accretion rate is set not by the cosmological energy density of the field, but by the energy scale of the ghost condensate theory. If hydrodynamical flow is established, it could be as high as a tenth of a solar mass per second for 10 MeV scale ghost condensate accreting onto a stellar-sized black hole, which puts serious constraints on the parameters of the ghost condensate model.

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I. INTRODUCTION

Prompted by increasingly precise experimental measurements of cosmological parameters, and, in particular, detection of acceleration of the universe due to an unknown source which looks like a cosmological constant, in the recent years there has been a wide discussion in the literature about modifications of Einstein gravity on cosmological scales as a possible alternative to dark matter and/or energy. However, finding a self-consistent and well-motivated theory which agrees with all the observations is proving to be quite a challenge.

Recently, Arkani-Hamed *et. al.* proposed a model [1], dubbed *a ghost condensation*, which they argued to be consistent with all experimental observations and provide an interesting modification of gravity in the infrared, with potential applications to inflation [2], dark matter and cosmological constant problems. It involves an introduction of a scalar field which develops a nonzero expectation value of its (timelike) gradient in vacuum, due to the nontrivial kinetic term in the action. Such modifications of the scalar field kinetic term were considered earlier on phenomenological grounds in the model known as k-inflation [3,4].

The ghost condensate model has been studied on a perturbative level in effective field theory [1], which already leads to interesting consequences such as star trails [5,6]. We look at the ghost condensate from a slightly different perspective; namely, we would like to investigate its behavior in the strong gravitational field, for instance, near a Schwarzschild black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_n^2, \tag{1}$$

where $f(r) = 1 - r_g/r$, and $r_g = 2Gm$ is a gravitational radius of Schwarzschild black hole of mass m. The problem is similar to interaction of a cosmological scalar field with a black hole [7], so one would expect ghost condensate to be accreted by a black hole.

Accretion of fluid onto a black hole has long been an important problem in astrophysics [8]. Spherically symmetric steady-state fluid accretion onto a Schwarzschild black hole was derived in Ref. [9]. Minimally coupled scalar field [10] and quintessence [7,11] accrete onto black holes as well, although the accretion rate is limited by the cosmological density of the field [7]. Accretion of exotic matter fields can lead to unusual results. For instance, accretion of a phantom energy (which violates energy dominance conditions) *decreases* the black hole size [12].

In this paper, we calculate the steady-state accretion rate of the ghost condensate by a black hole, and point out that it could be extremely efficient. This puts serious constraints on the parameters of the ghost condensate model.

II. GHOST CONDENSATE AS A FLUID

The ghost condensate model adds a nonminimally coupled scalar field to the Einstein theory of gravity. However, instead of the usual kinetic term

$$X = -(\nabla\phi)^2, \tag{2}$$

the action is assumed to involve a more complicated function of the field gradient squared

$$S = \int\left\{\frac{R}{16\pi G} + M^4 P(X)\right\}\sqrt{-g}d^4x, \tag{3}$$

as well as higher-derivative terms. We will ignore higher-derivative terms in what follows. They complicate calculations significantly, and we are concerned with large scale flows, while one would expect higher-derivative terms to be important on short scales.

As the ghost field ϕ is not directly coupled to other fields, its dimensionality and normalization are arbitrary. If the field ϕ is chosen to have dimension of length, the field gradient X and the function P are dimensionless, and

*Electronic address: afrolov@stanford.edu

the only dimensional quantity in the ghost sector is M, which sets the overall energy scale of the ghost condensate. The specific form of the function $P(X)$ is not rigidly fixed, although the defining feature of the ghost condensate model is that P has a minimum at nonvanishing (timelike) value of the field gradient. Because of that, the ghost field rolls even in its vacuum state. The simplest choice for P with this property is

$$P(X) = \frac{1}{2}(X - A)^2, \tag{4}$$

illustrated in Fig. 1. One could also add a cosmological constant term Λ, but since it is not accreted by a black hole, we will not discuss it further.

Variation of the action (3) with respect to the ghost field ϕ yields equation of motion

$$\nabla_\mu[P'(X)\nabla^\mu\phi] \equiv \frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}P'(X)\partial^\mu\phi] = 0. \tag{5}$$

The equation of motion is implied by conservation of the stress-energy tensor, which for the ghost condensate is

$$T_{\mu\nu} = 2M^4P'(X)\phi_{;\mu}\phi_{;\nu} + M^4P(X)g_{\mu\nu}. \tag{6}$$

Configurations with $P'(X) \equiv 0$ solve the equation of motion identically for any spacetime metric. However, such configurations are indistinguishable gravitationally from a purely Einstein theory, as the stress-energy tensor becomes trivial as well.

The stress-energy tensor of the ghost condensate (6) can be transformed into that of a perfect fluid

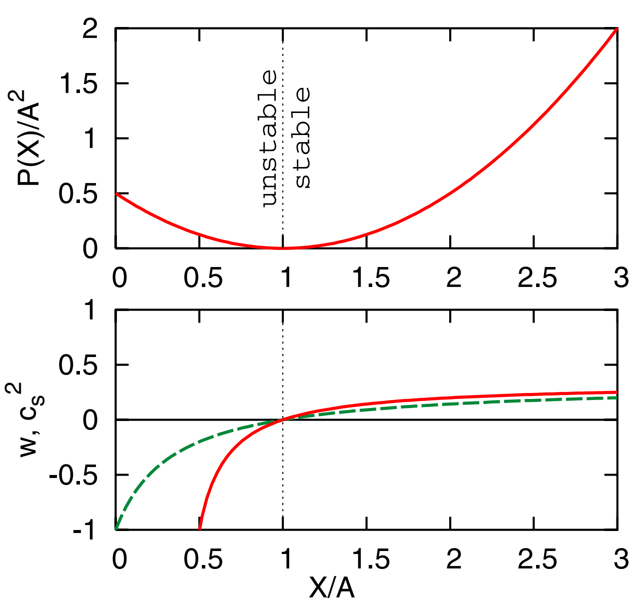

FIG. 1 (color online). Ghost condensate kinetic term (top) and equivalent fluid description (bottom). Equation of state w and sound speed c_s^2 are shown by dashed curves and solid curves, respectively.

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{7}$$

by a formal identification

$$\rho = M^4(2XP' - P), \quad p = M^4P, \quad u_\mu = \frac{\phi_{;\mu}}{\sqrt{X}}. \tag{8}$$

The fluid analogy is very useful in understanding the physics behind the solutions of the ghost equation of motion (5), although it is not an exact correspondence. Unlike ordinary fluids, ghost condensate is *irrotational*, that is, the vorticity tensor of the flow u^μ vanishes identically

$$\omega_{\alpha\beta} = \frac{1}{2}(u_{\alpha;\mu}q^\mu_\beta - u_{\beta;\mu}q^\mu_\alpha) \equiv 0, \tag{9}$$

where $q_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$. This is a direct consequence of the vector flow u^μ being derived from a scalar.

Important parameters of the fluid are its equation of state and sound speed

$$w \equiv \frac{p}{\rho}, \qquad c_s^2 \equiv \frac{dp}{d\rho} = \frac{p'}{\rho'}. \tag{10}$$

For the ghost condensate with kinetic term (4), they are

$$w = \frac{X - A}{3X + A}, \qquad c_s^2 = \frac{X - A}{3X - A}. \tag{11}$$

The equation of state and the sound speed change from dustlike in the minimum $X = A$ to radiationlike for large displacements $X \gg A$, as shown in Fig. 1. Configurations with $X < A$ are unstable, as the sound speed squared becomes negative.

Cosmological expansion of the universe dilutes the density of the homogeneous ghost condensate and drives its gradient toward the minimum of the kinetic term P [1]. Dust-like equation of state of the ghost condensate near its minimum lends itself to interpretation of ghost condensate as a dark matter in late-time cosmology [1]. If this is the case, the energy density of the ghost condensate ρ is the energy density of the dark matter, and displacement of the ghost condensate from the minimum is small, $P' = \rho/(2M^4A)$, but nonzero. "Modification of gravity" depends on the excitations in the ghost condensate, which displace the ghost condensate from its minimum and carry energy density [1]. Indeed, as the ghost condensate with $P' \equiv 0$ is *identical* to the cosmological constant, such a restriction would greatly diminish the attractiveness of the ghost condensate model. In view of the above, we assume the ghost condensate distribution which is homogeneous far from the black hole, but could be displaced from the minimum (by a small amount).

III. STEADY-STATE ACCRETION

Steady-state accretion means that the flow of the field (i.e. its gradient) does not change with time, that is $\mathcal{L}\mathcal{L}_{\partial t}(\nabla_\mu\phi) = \partial_\mu\partial_t\phi = 0$, which in turn implies that

$\partial_t \phi$ is constant (and can be set to one by a choice of the field normalization). Therefore, a general steady-state spherically symmetric field configuration is of the form

$$\phi = t + \psi(r), \tag{12}$$

and, in Schwarzschild spacetime (1), has a gradient

$$X = \frac{1 - (\partial_r^* \psi)^2}{f(r)}, \tag{13}$$

where we introduced a "tortoise" derivative $\partial_r^* \equiv f(r)\partial_r$.

For steady-state accretion of the spherically symmetric ghost condensate profile (12) onto a Schwarzschild black hole (1), the equation of motion (5) becomes

$$\partial_r^*(r^2 P' \partial_r^* \psi) = 0, \tag{14}$$

which can be immediately integrated to yield the flow conservation equation

$$P' \partial_r^* \psi = \alpha \frac{r_g^2}{r^2}. \tag{15}$$

The meaning of the dimensionless constant of integration α becomes clear if one looks at the accretion rate

$$\dot{m} = 4\pi r^2 T_t^r = 2\alpha \cdot 4\pi r_g^2 M^4, \tag{16}$$

which does not depend on r and describes a steady-state transfer of mass from infinity into a black hole. The numerical value of the coefficient α is picked by the solution of the flow Eq. (15) that is regular at the horizon and becomes homogeneous far from the black hole.

The flow Eq. (15) is actually algebraic in $\partial_r^* \psi$, and could be analyzed for an arbitrary function P. We will restrict our discussion to the ghost condensate with kinetic term (4) and further assume $A \leq 1$, as the choice $A > 1$ places the solution (12) on the unstable branch of the kinetic term far from a black hole and is not physically relevant. Introducing the shorthand notation $v \equiv \partial_r^* \psi$ and $x \equiv f(r)$, the flow Eq. (15) can be written as

$$\left(\frac{1 - v^2}{x} - A\right)\frac{v}{(1 - x)^2} = \alpha. \tag{17}$$

Solutions $v(x)$ for various values of α are shown in Fig. 2. Although cubic Eq. (17) can be directly solved in radicals, the flow is more readily analyzed using standard phase space diagram techniques.

Both at the horizon ($x = 0$) and infinity ($x = 1$), all flow trajectories converge to one of three roots

$$x = 0\colon\ v_0 = 0, \pm 1 \qquad x = 1\colon\ v_1 = 0, \pm\sqrt{1 - A}. \tag{18}$$

All flow trajectories must start and end at these roots, and they do not intersect except at the critical points. The critical points are defined as the points where the full differential of (17),

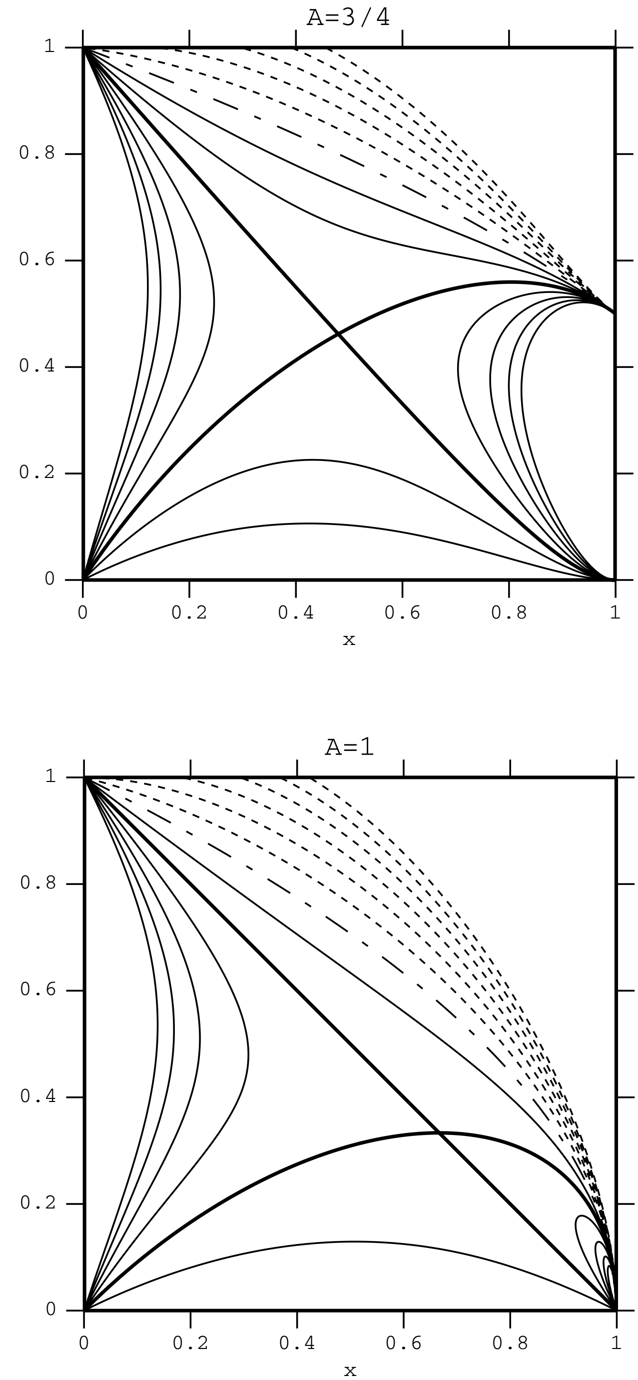

FIG. 2. Flow diagrams $v(x)$ of the ghost condensate accretion onto a black hole for $A = 3/4$ (top) and $A = 1$ (bottom). Inflow, no-flow, and (unstable) outflow branches are shown by solid lines, dot-dash lines , and dotted lines correspondingly. Flow trajectories passing through the critical point are emphasized by thicker lines. Negative v region corresponds to reversed flow direction, and is not shown.

$$-\frac{3v^2 - 1 + Ax}{x(1 - x)^2}\,dv - \frac{(1 - v^2)(1 - 3x) + 2Ax^2}{x^2(1 - x)^3}\,v\,dx = 0, \tag{19}$$

becomes degenerate, i.e., when coefficients in front of dv and dx both vanish. In the positive v region, there is (at most) one critical point

$$v_*^2 = \frac{A + \sqrt{A^2 - 36A + 36}}{18}, \qquad x_* = \frac{1 - 3v_*^2}{A}. \quad (20)$$

Regularity at the horizon for ingoing flow demands that $v_0 = 1$, while proper fall-off at infinity requires $v_1 = 0$. For $A < 1$, the only flow trajectory that connects the two is the one that passes through the critical point (20), as it is clear from the top panel of Fig. 2. The flow starts out subsonic at infinity, and turns supersonic at the critical point. Thus, the accretion rate for the steady-state flow is set by the local conditions at the supersonic transition, which happens in the immediate vicinity of the black hole, and depends little on the actual boundary conditions at infinity. The coefficient α is calculated by evaluating Eq. (17) at the critical point (20). The resulting expression is straightforward, but cumbersome for arbitrary A, so we will not write it down here. Instead, we will show the graph of α as a function of A in Fig. 3. The coefficient α decreases monotonically from $3\sqrt{3}/2$ at $A = 0$ to 1 at $A = 1$. Note that it does not vanish even as displacement of the ghost condensate from the minimum approaches zero.

Asymptotics of the ghost condensate profile far from the black hole are easy to analyze directly from the flow Eq. (17). For the slow flow ($v \ll 1$), v^2 term in Eq. (17) can be neglected, and we have

$$v \simeq \alpha \frac{(1-x)^2}{x^{-1} - A}, \qquad \psi \simeq \frac{\alpha}{A} r_g \ln \frac{r}{Ar_g + (1-A)r}. \quad (21)$$

As can be seen from the above expressions, sufficiently far from the black hole the field gradient v falls off at infinity as r^{-2}, while the field profile levels off as r^{-1}. The field profile is influenced by the black hole and deviates from homogeneous significantly inside a "sphere of influence" of radius $r_i = Ar_g/(1-A)$.

The case of $A = 1$ is special, and is shown on the bottom panel of Fig. 2. The three roots at infinity merge into one triple root at $v_1 = 0$, and one can get from

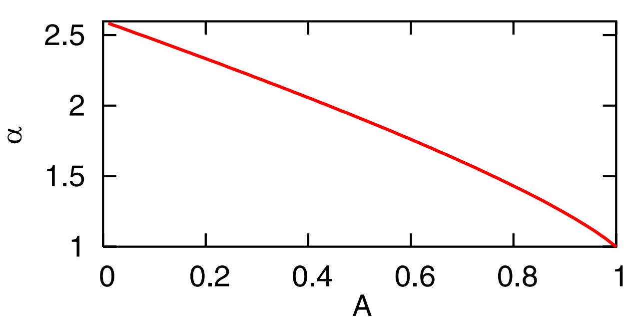

FIG. 3 (color online). Dependence of the accretion rate coefficient α on A.

infinity to horizon without going through a critical point. These solutions correspond to a dustlike flow with $0 \le \alpha < 1$, and are always supersonic. However, their gradient v falls off at infinity only as $r^{-\frac{1}{2}}$, which means that the ghost field does not become homogeneous far from a black hole, but in fact grows as $r^{\frac{1}{2}}$. In particular, the trivial solution ($P' \equiv 0$, $\alpha = 0$) is

$$\phi = t + 2r_g^{\frac{1}{2}}\left[r^{\frac{1}{2}} - r_g^{\frac{1}{2}} \operatorname{arctanh}\sqrt{\frac{r_g}{r}}\right]. \quad (22)$$

The likely reason behind the change in field asymptotic is that spherically symmetric dust accretion is not steady-state. The accretion rate is ever growing, as the dust from larger and larger volume falls inside the black hole.

For $A = 1$, the flow trajectory which passes through a critical point ($\alpha = 1$) is simply $v = 1 - x = r_g/r$. The corresponding field profile is

$$\phi = t + r_g \ln\left[\frac{r}{r_g} - 1\right]. \quad (23)$$

Its asymptotic at infinity is also nonhomogeneous, but the growth is only logarithmic. This solution emerges from steady-state flow solutions (21) with $A < 1$ in the limit when the sphere of influence of the black hole grows infinitely large.

IV. DISCUSSION

In the last section, we calculated the steady-state accretion rate of the ghost condensate by a black hole for spherically symmetric flows. The most important result of the calculation is that the dimensionless coefficient α, which determines the accretion rate of the flow, is bounded below by 1 even as the density of the ghost condensate far from a black hole becomes vanishingly small. This means that it is the energy scale M of the ghost condensate theory that sets the accretion rate, and not the cosmological abundance of the ghost condensate field as one might have naively expected.

The physical reason for this is that unlike the usual dark matter, the homogeneous ghost condensate does not have angular momentum and can be prevented from falling onto a gravitating body only by building up pressure support, which requires significant displacement from the minimum and densities of order M^4. This consideration alone might pose significant problems for ghost condensate as a realistic dark matter candidate, as dark matter galaxy halos are thought to be *virialized* and not pressure-supported. This seems impossible to achieve in the framework of the ghost condensate model, unless the condensate becomes strongly nonhomogeneous and breaks up into "particles", in which case it is hard to view it in any sense as a modification of gravity.

Up to a numerical coefficient of order 1, the steady-state accretion rate is equal to the energy density

M^4 falling down through the horizon area $4\pi r_g^2$ at the speed of light. The top value of 10 MeV for the ghost energy scale quoted in [1] corresponds to a rather high density

$$(10\ \mathrm{MeV})^4 = \frac{(10\ \mathrm{MV} \cdot e)^4}{\hbar^3 c^5} = 2.32 \cdot 10^{12} \frac{\mathrm{kg}}{\mathrm{m}^3}. \qquad (24)$$

If the steady-state flow of the kind we considered is established, the accretion rate of a 10 MeV scale ghost condensate by an astrophysical black hole would be enormous

$$\dot{m} = 0.08\alpha \frac{M_\odot}{\mathrm{s}} \left(\frac{r_g}{3\ \mathrm{km}}\right)^2 \left(\frac{M}{10\ \mathrm{MeV}}\right)^4. \qquad (25)$$

To avoid rapid black hole growth and its astrophysical consequences, energy scale M of the ghost condensate should be significantly less than 10 MeV. A stellar-size black hole would double in size over the lifetime of the universe (roughly 14 Gyrs, or $4 \cdot 10^{17}$s) for the ghost energy scale of order 1 keV. This estimate goes down to 10 eV for supermassive ($10^9\ M_\odot$) black holes.

In our calculations, we ignored the backreaction of the accreting ghost condensate matter onto a black hole metric. This is a good approximation for low accretion rates and ghost energy scales. However, if the accretion rate becomes as large as the above estimate, backreaction can no longer be ignored, both because the density of the ghost condensate near the black hole is high and a large supply of ghost matter far from the black hole is needed to sustain the flow. Although proper treatment of backreaction is unlikely to alleviate the problem of excessive accretion rates, as it is caused by the largeness of the effect in the first place, sustainability of the steady-state flow with such enormous accretion rates is questionable. It would seem more likely that the black holes would completely evacuate all ghost matter from their gravitational potential wells, growing in the process.

Spherically symmetric steady-state accretion is an idealized situation, of course. We have not considered time-scale required to establish such flows, the role of initial conditions, motion of the black hole with respect to the condensate, or what happens if the ghost field becomes highly inhomogeneous. All of these are much harder problems, and it might turn out that some factors prevent the accretion from settling into an efficient steady-state regime. Still, having ghost condensate capable of such high accretion rates is alarming, and the issue should be further addressed by the ghost condensate scenario.

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