ASSIGNMENT #1

PHYS 490 - Relativity and Gravitation

Due Wednesday, October 10

PROBLEMS:

1. Carroll 1.4
2. **Useful formulas:**

   a) Show that $\Gamma^{\beta}_{\alpha\beta} = \partial_\alpha \ln\sqrt{g}$, where $g = \det(g_{\mu\nu})$.

   b) If V^μ is a vector, show that

$$V^{\mu}{}_{;\mu} = \frac{1}{\sqrt{g}}\,\partial_\mu\left(\sqrt{g}\,V^\mu\right)$$

   c) If $F^{\mu\nu}$ is an antisymmetric tensor, show that

$$F_{\mu\nu;\lambda} + F_{\lambda\mu;\nu} + F_{\nu\lambda;\mu} = F_{\mu\nu,\lambda} + F_{\lambda\mu,\nu} + F_{\nu\lambda,\mu}$$

$$F^{\mu\nu}{}_{;\nu} = \frac{1}{\sqrt{g}}\,\partial_\nu\left(\sqrt{g}\,F^{\mu\nu}\right)$$

$$F^{\mu\nu}{}_{;\mu\nu} = 0$$

   (These provide a way to write Maxwell's equations without explicit use of Christoffel symbols.)

   d) If $T^{\mu\nu}$ is a symmetric tensor, show that

$$T_{\mu}{}^{\nu}{}_{;\nu} = \frac{1}{\sqrt{g}}\,\partial_\nu\left(\sqrt{g}\,T_{\mu}{}^{\nu}\right) - \frac{1}{2}\,T^{\alpha\beta}\,\partial_\mu g_{\alpha\beta}$$

3. Carroll 3.3
4. Carroll 3.5
5. Carroll 3.8 *(Hint: Calculate components of $R^{\alpha\beta}{}_{\mu\nu}$, it will make contractions easier. I will show you later how to do this kind of calculation using computer algebra software, but you have to do this once in your life by hand.)*