BISC-838, Population Dynamics

January 25, 2021

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Office Location: Irrelevant

- Syllabus

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- Course format

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- Introductions

This is a class about **models**.

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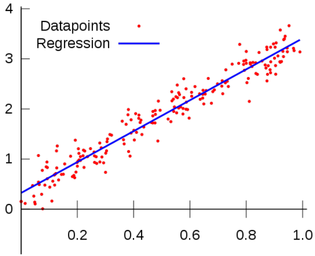

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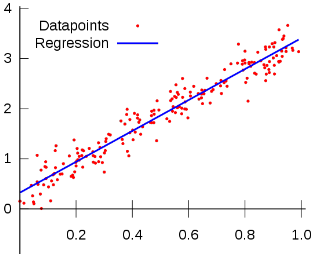

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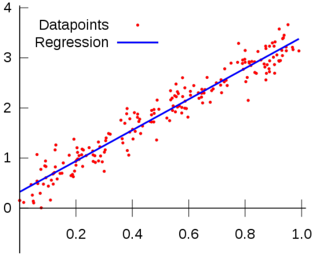

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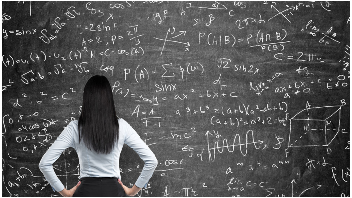

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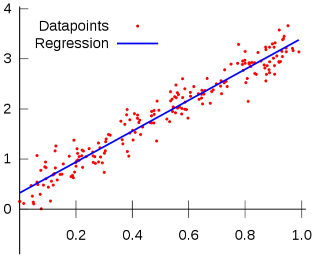

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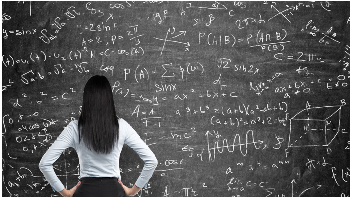

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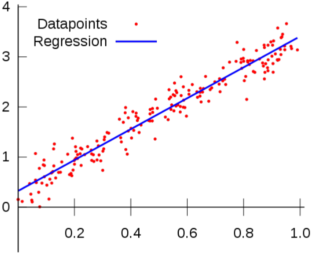

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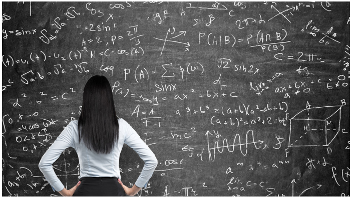

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These can intersect (e.g., you could fit statistical models to generate parameter estimates that then go into a mathematical model). We will focus on the latter here.

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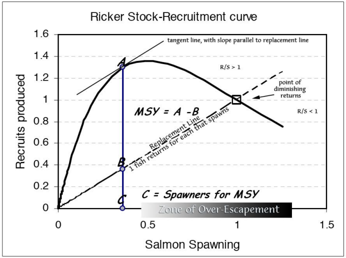

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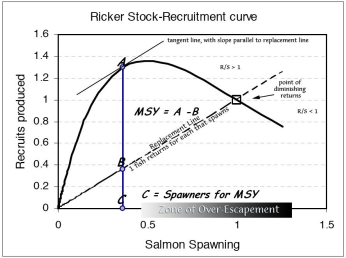

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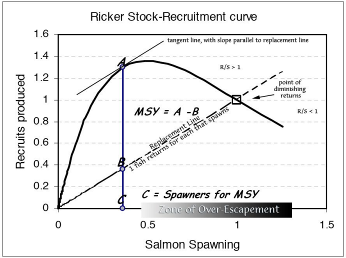

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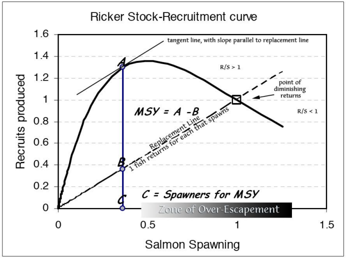

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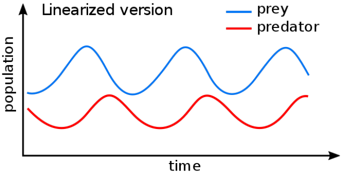
Predator-prey models (stable limit-cycles)



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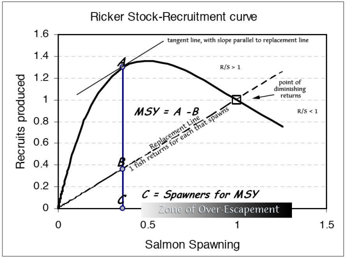

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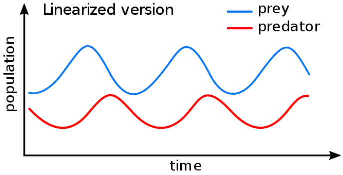

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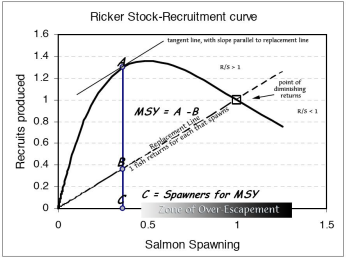

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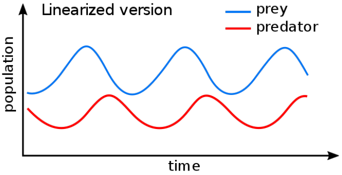

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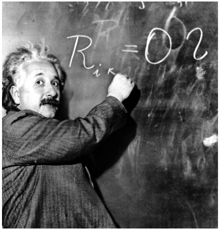
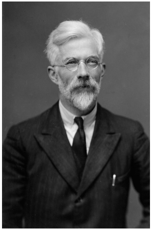
However, they lack the realism to be accurately model specific systems.

Introduction





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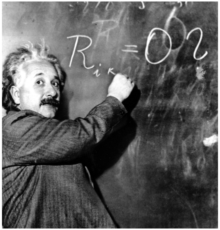

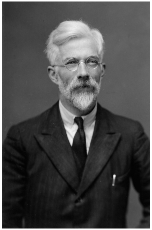

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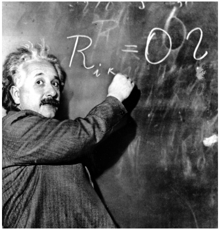
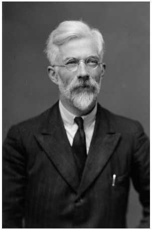

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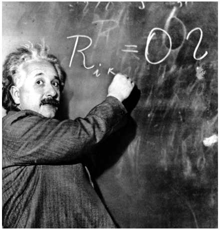

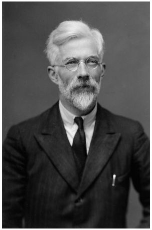

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Our goal in this class is to learn to use computer simulations to model basical population dynamics. Ultimately, in three weeks, you should be able to analyze your own models!

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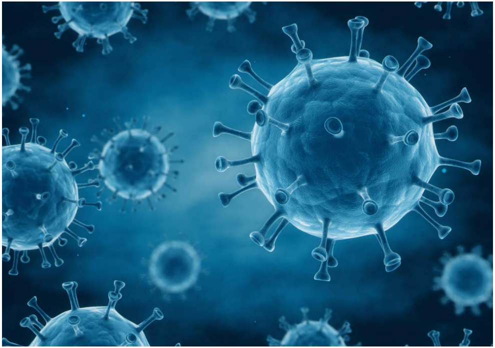
A good example is a paper by Phillips:

Phillips, A. N., 1996. Reduction of HIV Concentration During Acute Infection: Independence from a Specific Immune Response. Science 271:497-499.



Excerpts from the paper:

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- These equations can be explained as follows.

Activated, uninfected CD4 lymphocytes arise at a constant rate $\Gamma\tau$, where Γ is the rate at which new, uninfected CD4 lymphocytes arise and τ is the proportion that are activated and are removed by HIV-independent death at rate μ or by infection at rate βV.

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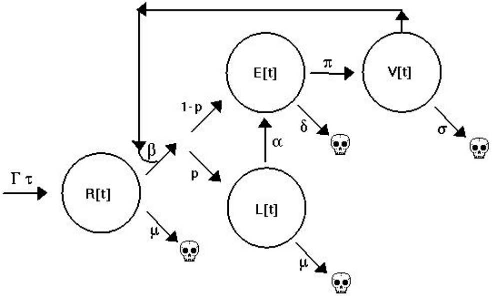
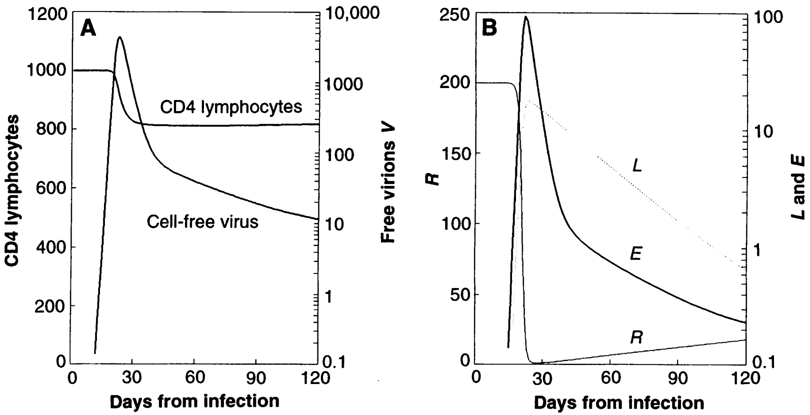
Free virions are produced at rate π by actively infected cells and removed at rate σ.

$$\frac{dV}{dt} = \pi E - \sigma V \tag{3}$$



R[t]: Activated and uninfected CD4 lymphocytes
L[t]: Latently infected CD4 lymphocytes
E[t]: Actively infected CD4 lymphocytes
V[t]: Free virions

Figure: Flow diagram.


A
1200
1000
800
600
400
200
0
10,000
1000
100
10
1
0.1
CD4 lymphocytes
Cell-free virus
Free virions V
0
30
60
90
120
Days from infection
B
250
200
150
100
50
0
R
100
10
1
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L and E
L
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R
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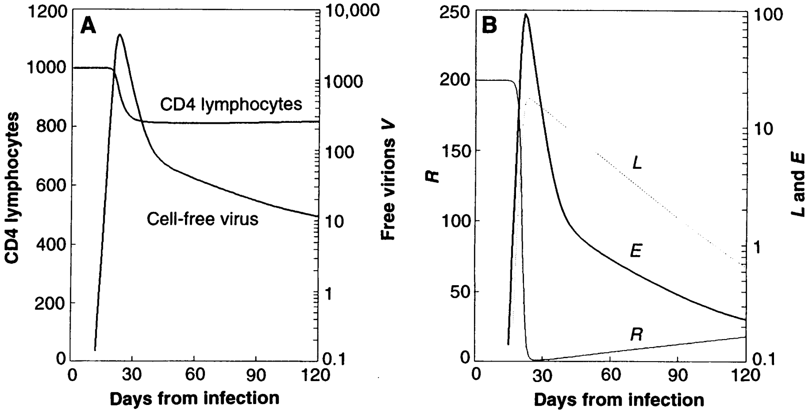

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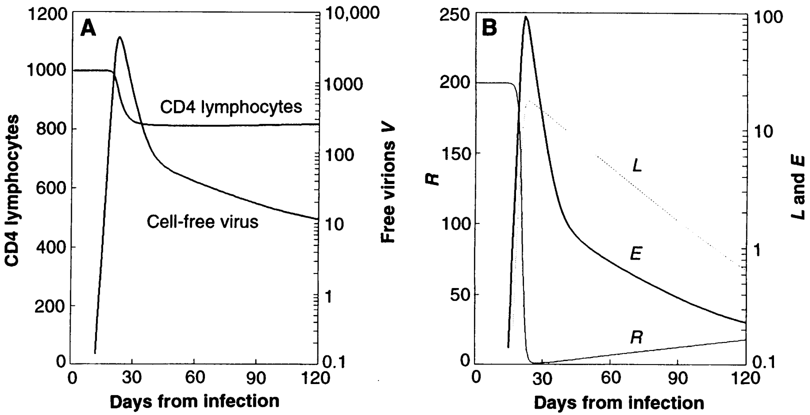

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The model presented here does not include any increases in the rate of removal of HIV after infection and shows such a decrease can still occur.

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This equation will form the basis for the first models we will investigate in this class.