Numerical Analyses: Graphs

January 29, 2021

Being able to "see" the results of a model is particularly important.

Often, the best first step in analysing a model is to graph the equations.

Today we will look at graphs that illustrate the behavior exhibited by the equations that we have developed.

	Discrete Time	Continuous Time
Exponential Growth	$N[t+1] = (1+r)N[t]$	$\frac{dN[t]}{dt} = rN[t]$
Logistic Growth	$N[t+1] = \left(1 + r\left(1 - \frac{N[t]}{K}\right)\right)N[t]$	$\frac{dN[t]}{dt} = r\left(1 - \frac{N[t]}{K}\right)N[t]$

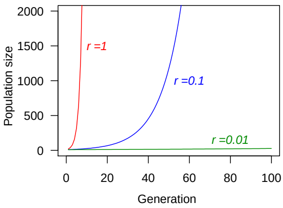
In this model, there is one parameter (r), where $(1 + r)$ is the average number of offspring per parent (we called this λ when we looked at this model).

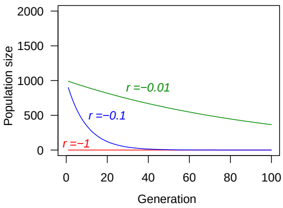
We will write a **recursive** function in R to generate values at different time points.

```
N <- function(r, N0, t) {
  if(t==0) {
    return(N0)
  } else {
    return((1+r) * N(r, N0, t-1))
  }
}
```

```
N(r=0.1,N0=10,t=10)
25.93742
```

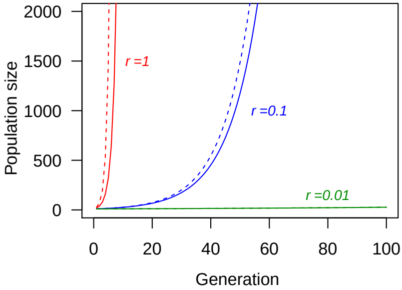
```
round(sapply(1:10, N, r=0.1, N0=10))
11 12 13 15 16 18 19 21 24 26
```

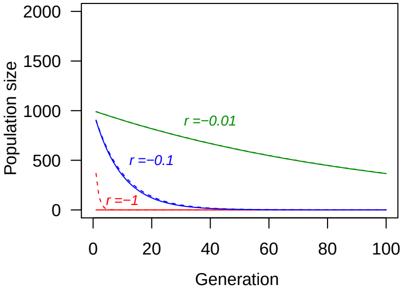




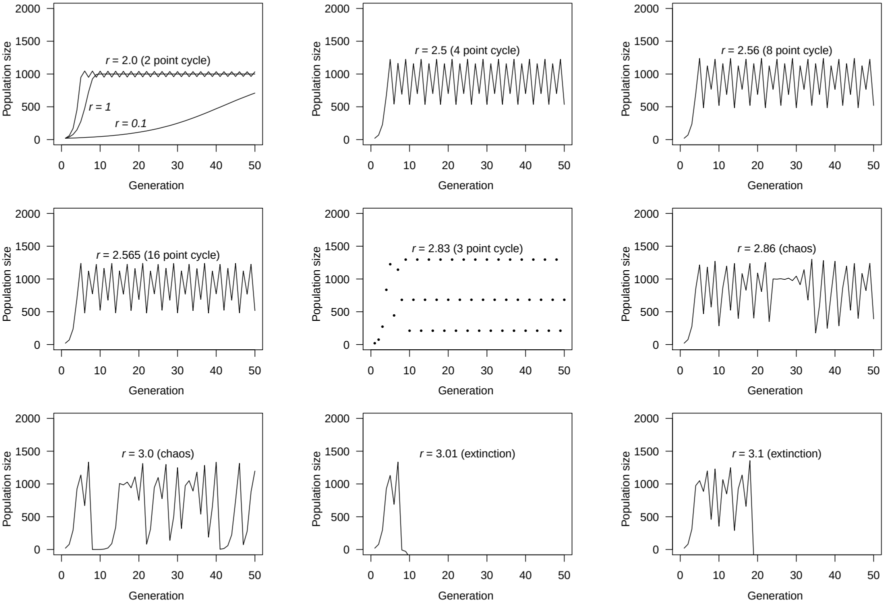
We can directly compare the discrete model (where growth is compounded per generation) and continuous model (where growth is compounded continuously):

Discrete:	`n[t]=(1+r)^t*n0`	(solid line)
Continuous:	`n[t]=exp(r*t)*n0`	(dashed line)

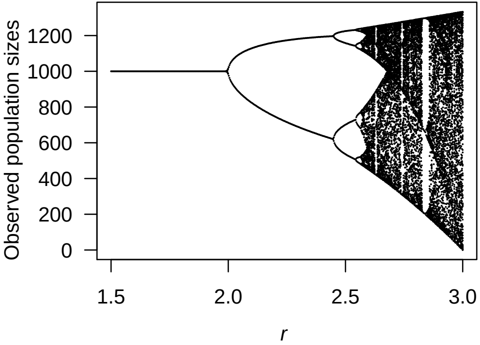




There are two parameters (K, r). The behavior doesn't change much with different values of K (we'll use $K = 1000$), but is **extremely** sensitive to the value of r.



A bifurcation plot (with $K = 1000$)



The dynamics of the logistic equation are bizarre, to say the least.

The periodic cycles and the chaotic fluctuations may not, however, be particularly relevant.

Hassell (1976) studied 28 insect populations. 26 of them exhibited values of r leading to a stable equilibrium point, one led to a periodic cycle (the Colorado potato beetle), and only one led to chaotic behavior (blowflies under laboratory conditions).

The logistic model is extremely sensitive to the manner in which it is modeled.

The continuous and discrete time formulations agree when r is small, but lead to completely different predictions when r is large.

Unlike for discrete time models, we cannot just "iterate" a differential equation to plot it. Instead, we must solve the differential equation for a general solution (which is difficult - here is some code that does it in Maple).

```
ode:=diff(n(t),t)=r*n(t)*(1-n(t)/K):
ics:=n(0)=n0:
dsolve({ode,ics},n(t));
```