Workshop 5: Demography

In the last workshop, we analyzed a simple dispersal model by iteratively applying a transition matrix. Demographic models are a class of linear models where the transition matrix takes a special form (the "Leslie matrix"). Here, we'll analyze demographic models using the techniques that we developed in class.

Squirrels*

*all data here is entirely made up

A scientist is studying a colony of squirrels. She finds that they produce, on average, 0 surviving daughters per female in the first year of life, 6 in the second year of life, 10 in the third year of life, and 2 in each year of life beyond this. She also finds that they have only a 20% chance of surviving to the second year, 25% chance of surviving from the second to the third year, and 10% chance of surviving from the third year to the fourth year. Individuals age four and over have a 5% probability of surviving to each subsequent age (e.g., remaining in the oldest age class). Construct the 4×4 Leslie matrix, **L**, for this population [answer is at this bottom of the document, but try to fill the below matrix in on your own first].

```
L <- matrix(c(x,x,x,x,
              x,x,x,x,
              x,x,x,x,
              x,x,x,x), ncol=4, byrow=TRUE)
```

1. Use the `eigen` command to find the eigenvalues and right eigenvectors of **L**. If you assign your `eigen` command to an object, say `r.vals.vecs`, you can use

   ```
   r.vals.vecs$val
   r.vals.vecs$vectors
   ```

   to extract eigenvalues (R puts them into a vector) and eigenvectors (R puts them into a matrix with each eigenvector as a column). From these, extract the leading eigenvalue and the leading eigenvector. Normalize the leading eigenvector, so that its elements sum to 1 (e.g., so that it is a "distribution"). Let's denote this right eigenvector $\vec{v}_{\text{right}}$.

2. Based on the above, what is the long-term growth rate? What proportion of the population will be in each age class once it has reached its stable age distribution?

3. Starting with an initial population, $\vec{N}_0$, that contains 1 female in each of the four age classes, use a `for` loop to calculate $\vec{N}_{100}$. Confirm that this distribution of individuals matches the stable age distribution.

4. Repeat the above with a few different initial age distributions and numbers of iterations to convince yourself that, for the demographic parameters considered here, the population quickly converges to its stable age distribution, regardless of the initial distribution.

5. Here's some R code that writes a function to raise a matrix to the n^{th} power. Note that you should be able to read through this function and understand how it works.

```
matrix.pow <- function(A,n) {
  if(n>1) {
    B <- A
    for(i in 2:n) A <- A %*% B
  }
  return(A)
}
```

   Repeat question 3. using this matrix power function in place of a `for` loop.

6. The leading "left eigenvector" of $\mathbf{L}$ (let's call this $\vec{v}_{\text{left}}$), if scaled properly, contains reproductive values for individuals in each age class. We can calculate the left eigenvectors by calculating the *right* eigenvectors of $\mathbf{L}^{\text{T}}$ (the transpose of $\mathbf{L}$). However, to ensure that our leading left eigenvector contains reproductive values, we must scale it (multiply it by a scalar), so that it satisfies the dot product $\vec{v}_{\text{left}} \cdot \vec{v}_{\text{right}} = 1$, where $\vec{v}_{\text{right}}$ is the leading right eigenvector we calculated in question 1 (the stable age distribution). Calculate this left eigenvector and confirm that it is, indeed, a left eigenvector by checking that:

$$\vec{v}_{\text{left}}\mathbf{L} = \lambda\vec{v}_{\text{left}}$$

   where λ is the leading eigenvalue that you found in question 1. Note that the vector must be in row format for the multiplication with $\mathbf{L}$, as written, to make sense. Additionally, verify that the dot product $\vec{v}_{\text{left}} \cdot \vec{v}_{\text{right}}$ does, indeed, equal 1.

7. Based on your left eigenvector, which age class has the highest reproductive value?

8. Compute the **total population size** after 10 generations if you start with only a single individual in a single age class (do this separately for each age class). In other words, calculate the total population size after 10 generations when starting with each of the following:

$$\vec{N}_0 = [1, 0, 0, 0]$$
$$\vec{N}_0 = [0, 1, 0, 0]$$
$$\vec{N}_0 = [0, 0, 1, 0]$$
$$\vec{N}_0 = [0, 0, 0, 1]$$

   Do these agree with your inference from question 7?

$$\mathbf{L} = \begin{bmatrix} 0 & 6 & 10 & 2 \\ 0.2 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.1 & 0.05 \end{bmatrix}$$