Errata for original printing of

Control Theory for Physicists, Cambridge Univ. Press, 2021
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List of corrections known as of December 21, 2025

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Thanks to Joseph Lucero and Carvin Müns for sending in reports of typos and other mistakes.

Book typos

The label (PE) indicates an error introduced during production.
Long right arrow $\longrightarrow$ means "should be".

1. p. 143, Eq. 4.41: the column vector $\begin{pmatrix} k_1 \\ !!k_2 \end{pmatrix}$ should be a row vector $\begin{pmatrix} k_1 & k_2 \end{pmatrix}$. (PE)

2. p. 229, line above Eq. 6.24: $\longrightarrow$ implies that $\cancel{\ln}$ $\det(N\mathcal{H}) = N^K \det \mathcal{H}$.

3. p. 233–234, Eqs. 6.32 and 6.34: $\dot{x}_1 = -\lambda_1 x_1 + b_1 u$.

4. p. 235, Eq. 6.37: There should not be the dt term in the right-most expression.

5. p. 257, margin sketch at right: The label $\mathbf{I}(t)$ should be $\lambda(t)$. (PE)

6. p. 277, Eq. 7.45: $\delta^2 J' = \delta^2 J + \int_0^\tau \mathrm{d}t \left[\delta\boldsymbol{\lambda}^\mathsf{T} \left(\boldsymbol{A}\,\delta\boldsymbol{x} + \boldsymbol{B}\,\delta\boldsymbol{u} - \delta\dot{\boldsymbol{x}} \right) \right]$.

7. p. 279, diagrams on upper right: on left axis, $p \longrightarrow \pi$. (PE)

8. p. 287, Fig. 7.8a: again, $p \longrightarrow \pi$. (PE)

9. p. 355, Problem 8.1b: It should read,

   Show that the recurrence relations for the time-dependent Kalman filter become

   $$\boldsymbol{P}^y_{k+1} = \boldsymbol{C}\boldsymbol{P}_k\boldsymbol{C}^\mathsf{T} + \boldsymbol{Q}_\xi\,, \qquad \boldsymbol{P}^{xy}_k = \boldsymbol{P}_k\boldsymbol{C}^\mathsf{T}\,, \qquad \boldsymbol{L}^*_{k+1} = \boldsymbol{A}\boldsymbol{P}^{xy}_k\left(\boldsymbol{P}^y_k\right)^{-1},$$
   $$\boldsymbol{P}^*_{k+1} = \boldsymbol{A}\boldsymbol{P}^*_k\boldsymbol{A}^\mathsf{T} + \boldsymbol{Q}_\nu - \boldsymbol{L}^*_{k+1}\boldsymbol{P}^y_k\,\boldsymbol{L}^{*\mathsf{T}}_{k+1}\,.$$

   In the above equations, because the covariances $\boldsymbol{P}^{xy}$ and $\boldsymbol{P}^y$ depend on observations y_k for the prediction observer, it makes more sense to define them with an index k. By contrast, the current observer uses y_{k+1} and thus defines the covariances with $k+1$, too. Cf. below the corresponding correction to the solution.

10. p. 357, Problem 8.7c: The problem should specify that the calculations should use the *prediction observer* for the Kalman estimator. In particular, the observer gain values $L^{\mathsf{T}} \approx (0.09, 0.03)$ given depend on this assumption.

11. p. 370, Figure 9.3b. On left axis, $p \longrightarrow \pi$. (PE)

12. p. 378, margin sketch at left: Left axis should read "Bad systems α". (PE)

13. p. 441, Fig. 10.6e: $y_k \approx r_{k-1}$.

14. p. 442: Notice that in (c), the input (light gray line) gently...

15. p. 488, Problem 11.22. (See corrections to solution, below, too.)

    Part (a): Show that Eq. (11.53) can be rewritten as $\dot{\phi}_k = \omega_k + \varepsilon K \sin(\Theta - \phi_k)$.

    Part (f): ... synchronization threshold $\varepsilon_c = (\sqrt{8/\pi})\,\sigma$, where σ^2 is the variance of the oscillator-frequency distribution.

<u>Solutions typos</u>:

1. **Prob. 2.1.** There are issues in all parts (a,b,c):

   (a) i. In the definition of the Lagrangian, the expression for potential energy relative to the bottom equilibrium position at $\theta = 0$ should read $mg\ell(1 - \cos\theta)$ and not $1 - mg\ell\cos\theta$. The "1" was for scaled units, but here they have not yet been scaled. Since constants in the Lagrangian do not appear in the equations of motion, there is no consequence for the equations given.

   ii. In the derivation of the equations of motion for the cart-pendulum system, the differentiation operator should be in Roman font, in two places:

   - The x equation is then $\mathrm{d}_t \partial_{\dot{x}} L - \partial_x L = u$
   - The θ equation is $\mathrm{d}_t \partial_{\dot{\theta}} L - \partial_\theta L = d$

   The mistake was particularly unfortunate since the equation for $\theta(t)$ also includes an external torque disturbance, $d(t)$.

   iii. The variable y_ℓ should be defined as $y_\ell = -\ell\cos\theta$. The change in sign makes no difference since we use only $\dot{y}_\ell^2$ in the derivation.

   (b) The same mistake as in (a) occurs in the Lagrangian: Because the center of mass is at the middle of the stick, the potential energy term should read $\frac{1}{2}mg\ell(1 - \cos\theta)$ and not $1 - \frac{1}{2}mg\ell\cos\theta$.

   (c) In the second line of equations in the paragraph "Inverting the matrix...", the term $-\frac{3}{4}\sin\theta\cos\theta$ should be $-\frac{3}{4}\dot{\theta}^2\sin\theta\cos\theta$. Similarly, in the first-order equations, the corresponding term in the right-hand side of the $\dot{x}_4$ equation should read $-\frac{3}{4}x_2^2\sin x_3\cos x_3$.

2. **Prob. 3.15** Add a comment after the display equation $\tau = \cdots = \sqrt{\frac{\ell}{3g}}$: In terms of the pendulum period $T = 2\pi\sqrt{\ell/g}$, the maximum delay $\tau/T = 1/(2\pi\sqrt{3}) \approx 0.092$, meaning the maximum delay is a bit less than 10% of the period. This severe limit

arises from the tension between the need for a minimum gain to stabilize the vertical equilibrium and a maximum gain to keep the feedback stable.

3. **Prob. 4.2a.** Below first display equation: If we did not care about $x_2(t)$, ...
   At the end of the same paragraph, the last sentence was cut off and should read as follows: That is much less demanding than asking it follow a prescribed path in a prescribed way.

4. **Prob. 7.5c.** In the solution to part (c), there is a sentence giving boundary conditions for λ at time $t = \tau$. These conditions actually apply for a different version of the problem, where a "soft" condition for the pendulum is imposed instead of a "hard" one. The former was used in an earlier version of the problem and corresponds to a "terminal cost" $\phi = \frac{1}{2}[\theta(t) - \pi]^2 + \frac{1}{2}\dot{\theta}^2$. (Note that the "elastic" constants are taken $= 1$.) In the actual problem studied in the book, the pendulum is constrained at the end. Hence, there are four conditions on θ, even though it is a second-order equation of motion and no conditions on the second-order adjoint equation. Of course, the two sets are coupled, so that the four conditions on a fourth-order system are appropriate.

5. **Prob. 7.13.** The last equation should read

$$\boldsymbol{Q} + \boldsymbol{SA} + \boldsymbol{A}^\mathsf{T}\boldsymbol{S} - \boldsymbol{SBR}^{-1}\boldsymbol{B}^\mathsf{T}\boldsymbol{S} = \boldsymbol{0}\,,$$

6. **Prob. 7.15a.** In spelling out the solution $u(t) = -\mathrm{sat}[\lambda(t)]$, the limit on the middle condition should be $-1 < \lambda < 1$ and not $0 < \lambda < 1$. That is,

$$u(t) = -\mathrm{sat}[\lambda(t)] = \begin{cases} -1, & \lambda \geq 1 \\ -\lambda, & -1 < \lambda < 1 \\ +1, & \lambda \leq -1\,. \end{cases}$$

7. **Prob. 8.1b.** The very last line of the derivation should read

$$\begin{aligned} \boldsymbol{P}^*_{k+1} &= \boldsymbol{A}\boldsymbol{P}^*_k\boldsymbol{A}^\mathsf{T} + \boldsymbol{Q}_\nu - \boldsymbol{A}\boldsymbol{P}^{xy}_k\,(\boldsymbol{P}^y_k)^{-1}\,(\boldsymbol{P}^{xy}_k)^\mathsf{T}\,\boldsymbol{A}^\mathsf{T} \\ &= \underbrace{\boldsymbol{A}\boldsymbol{P}^*_k\boldsymbol{A}^\mathsf{T}}_{\text{dynamics}} + \underbrace{\boldsymbol{Q}_\nu}_{\text{disturbances}} - \underbrace{\boldsymbol{L}^*_{k+1}\,\boldsymbol{P}^y_k\,\boldsymbol{L}^{*\mathsf{T}}_{k+1}}_{\text{observations}}\,, \end{aligned}$$

   The steps above are correct. The qualitative point is that the variance of the state estimator at time $k+1$ depends on observations up through time k in the prediction observer, whereas it also incorporates information at time $k+1$ for the current observer.

8. **Prob. 8.7f.** The standard deviation of $\hat{x}$, after transients have died away, is closer to 0.04, not 0.08.

9. **Prob. 11.22.** (See also corrections to problem statement in book.)

   Part (a): First, we note that we can separate the equation defining the order parameter,

$$\mathcal{K} = K\,\mathrm{e}^{\mathrm{i}\Theta} = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\phi_j}\;.$$

into real and imaginary equations:

$$K \cos\Theta = \frac{1}{N}\sum_{j=1}^{N} \cos \phi_j \,, \qquad K \sin\Theta = \frac{1}{N}\sum_{j=1}^{N} \sin \phi_j \,.$$

Part (e): For the first equation, Taylor expanding g to second order about ω_0 gives

$$g(\omega_0 + \varepsilon K \ \sin\psi) \approx g(\omega_0) + \frac{1}{2} g''(\omega_0)(\varepsilon K \ \sin\psi)^2 \,.$$

Then substitute:

$$\begin{aligned} 1 &= \varepsilon \int_{-\pi/2}^{\pi/2} \mathrm{d}\psi \cos^2\psi \ \left[g(\omega_0) + \frac{1}{2} g''(\omega_0)(\varepsilon K \ \sin\psi)^2 \right] \,, \\ &\approx \varepsilon \frac{\pi}{2} g(\omega_0) + \varepsilon^3 K^2 g''(\omega_0) \frac{\pi}{16} \,. \end{aligned}$$

Appendix typos (for version posted on CUP website):

1. The margin figures are on the opposite side as called out in the text. That is, every statement of the form "see left" should be "see right," and vice versa. (PE)

2. p. 57, 2nd line from top: P(theta | Y,X) $\longrightarrow$ P(θ | Y,X).