Feedback traps and the generalized Landauer limit for erasing information

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Abstract: We review studies based on feedback traps of the fundamental thermodynamic limits for information erasure (generalized Landauer limit). We report the first direct measurement of nonequilibrium system entropy, showing it matches Shannon's expression.

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1. Feedback traps and their applications to the stochastic thermodynamics of small systems

In 2005, Cohen and Moerner developed the Anti-Brownian Electrokinetic (ABEL) trap, a new tool for trapping and manipulating small objects in solution [1]. The idea, to trap small objects by counteracting Brownian motion using a feedback loop, is illustrated in Fig. 1. One measures the position of an object in the trap, for example, by taking a camera image and using image processing to find the object's coordinates. Then one computes and applies the desired force F to the charged particles via a voltage V. To emphasize the generality of the technique, we adopt the more generic name of *feedback trap*. In the trap used for the bulk of the experiments reported here, the loop cycle time was 5 ms, and the bead was 1.5 μm in diameter. The silica bead had a density large enough to confine the particle near the bottom of the sample cell while still allowing horizontal diffusion.

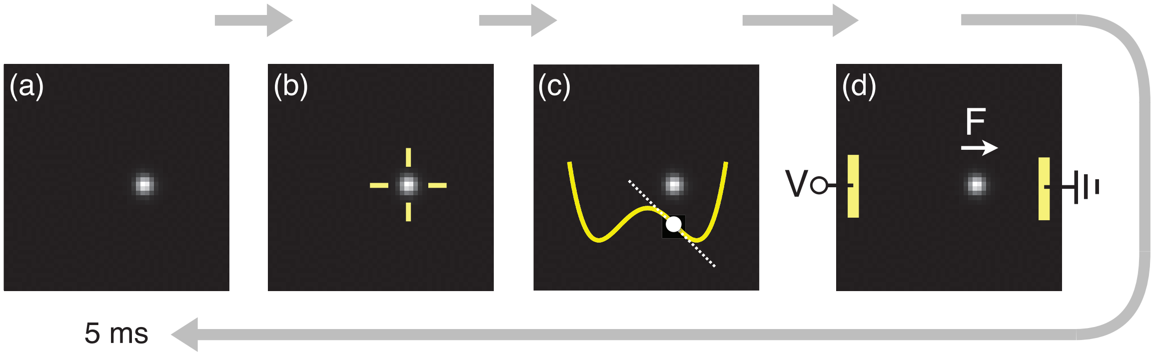

Fig. 1. Feedback trap cycle. (a) Acquire image. (b) Find particle coordinates. (c) Calculate force from virtual potential (here, a double well). (d) Apply corresponding force. Adapted from Ref. [2].

Although feedback traps have largely been used to study single molecules (e.g., [3,4]), Cohen also realized that ABEL traps could create "virtual" potentials where one imposes, via the feedback loop, a rule for applying forces corresponding to a desired potential $U(x)$ [5]. In subsequent work, we improved the stability of the trap and the accuracy of its calibration, to the point where we could study the dynamics and also the thermodynamics of small colloidal particles in solution [2,6,7]. The key technical challenge was to develop a calibration procedure that allows for accurate work measurements on a single particle, for experimental runs that last hours, or even days, a task that was achieved using recursive maximum likelihood estimation to measure continuously the physical properties of the particle [8].

We then used the trap to explore connections between thermodynamics and information theory. Previously, optical tweezers had been used to test Landauer's principle [9], that erasing a one-bit memory should require, on average, a work of at least $k_B T$ ln 2 [10,11]. Using our feedback trap, we performed the highest-precision measurement of

Landauer's principle to date, showing that erasing one bit of information requires, on average, at least $k_B T \ln 2$ of work [2].

In our study, as illustrated at left in Fig. 2, we created a virtual double-well potential whose two wells model a one-bit memory. The protocol then started from a system that could be in either well with equal probability. Then, by manipulating the barrier height and tilt, we ended up in a state where the potential returned to its initial shape but where the probability to being in a reference well was one. The average work to erase for slow protocols was $0.72 \pm 0.08\ k_B T$, consistent with the expected $\ln 2 \approx 0.69$ factor.

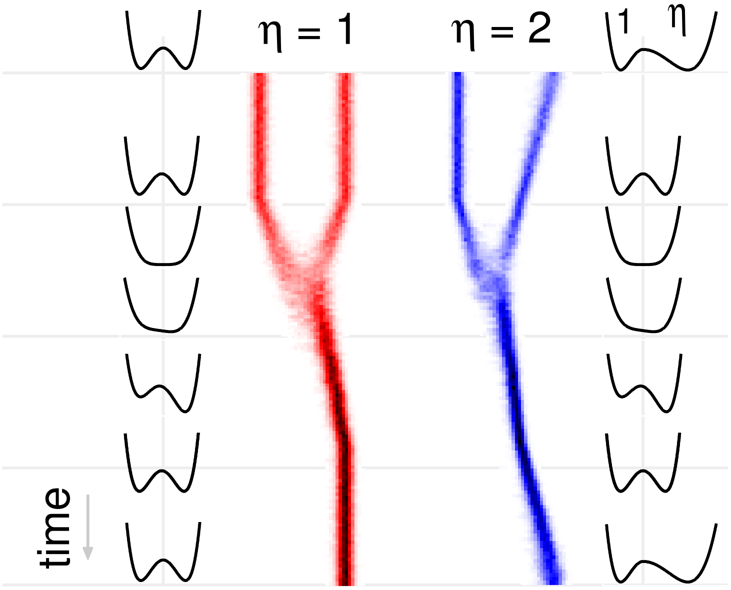

Fig. 2. Erasure protocols and trajectories for symmetric ($\eta = 1$) and asymmetric ($\eta = 2$) feedback traps. Trajectories are compiled into two-dimensional histograms. Adapted from Ref. [12].

We can also explore nonequilibrium generalizations of the Landauer principle [12]. At right in Fig. 2, we show erasure in an asymmetric potential, where the right well is stretched by a factor $\eta = 2$ relative to the left half. Here, we take advantage of the unique flexibility of the feedback trap to define precisely such potential shapes. In such a memory, the system is initially out of equilibrium, as the entropy contributions to the free energy differ by a factor of $\ln \eta$. Erasing slowly then requires, on average, just $0.25 \pm 0.07\ k_B T$, in accordance with predictions by Sagawa and Ueda [13] and notably below the classic ln-2 limit. The reduction can be explained qualitatively by noting that when the system starts with a particle in the right well, the initial and final wells have the same size. But half the time, the system starts with a particle at left that ends in the right well. In these cases, the particle in effect can do work as it expands, which reduces the total work required for the erasure process. By contrast, if the system were erased to the smaller, left well, we would expect on average a *higher* work required, as half the cases require effectively compressing a well.

In another set of experiments, we have also developed protocols that can erase fractional bits of information, demonstrating for the first time that the average work is proportional to the Shannon entropy of a two-state system. The Shannon entropy is

$$H(p) = -p \ln p - (1-p) \ln\ 1-p\,, \tag{1}$$

where p is the probability for a particle to end up in the reference well. When $p = 1$, there is complete one-bit erasure ($H = 0$), and the average work $k_B T\,(\ln 2 - H)$ is $k_B T\, \ln 2$, in accordance with Landauer's original result. The result shows that the Shannon entropy has direct thermodynamic significance: It is the quantity that appears in nonequilibrium generalizations of the first and second laws of thermodynamics for fluctuating, mesoscopic systems.

In exploring nonequilibrium generalizations of the Landauer principle, one surprise was that protocols that are naive variations of the complete-erasure protocol fail to reach the predicted thermodynamic bounds, even when carried out arbitrarily slowly [14]. Indeed, it is all too easy to inadvertently create irreversible steps when manipulating mesoscopic systems [12]. Only a more subtle protocol that ensures global equilibrium when barriers are raised and lowered can reach the expected thermodynamic bounds. Thus, in Fig. 2, the first step in the asymmetric case at right should compress the larger well to make the system symmetric. It is then erased as for the symmetric case; the well is expanded to regain the initial form of the potential. A naive protocol that simply lowers the barrier with unequal probabilities in both wells would require *more work* on average than the protocol shown, no matter how slowly it is carried out.

In closing, we emphasize that these experiments on generalizations of the Landauer limit require great flexibility for choosing and manipulating the form of the applied potential. Such flexibility is difficult, perhaps impossible, to achieve using optical tweezers but is readily accomplished using feedback traps.

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