14 Oscillations

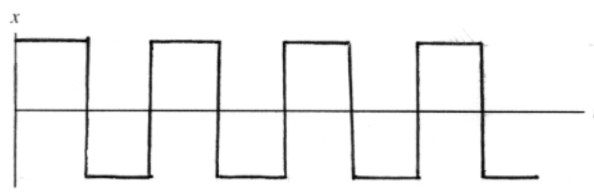
14.1 Simple Harmonic Motion

1. Give three examples of *oscillatory* motion. (Note that circular motion is not the same as oscillatory motion.)

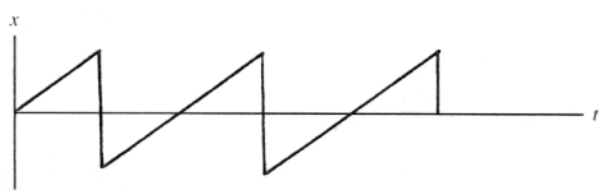
   1. A mass hanging from a spring
   2. A tennis ball being volleyed back and forth
   3. Washboard road bumps
   4. A beating heart
   5. AC electric current and voltage
   6. A pendulum swinging

2. On the axes below, sketch three cycles of the displacement-versus-time graph for:

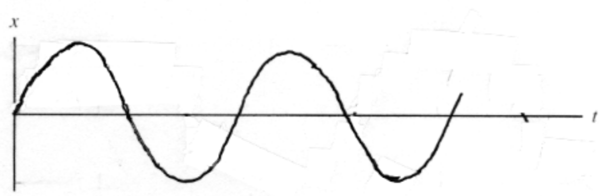
   a. A particle undergoing symmetric periodic motion that is *not* SHM.



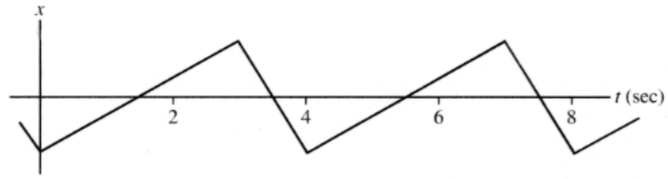
   b. A particle undergoing asymmetric periodic motion.

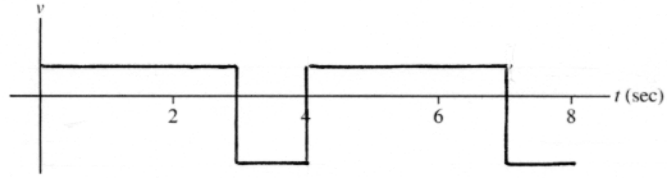


   c. A particle undergoing simple harmonic motion.



3. Consider the particle whose motion is represented by the x-versus-t graph below.





a. Is this periodic motion? yes b. Is this motion SHM? no

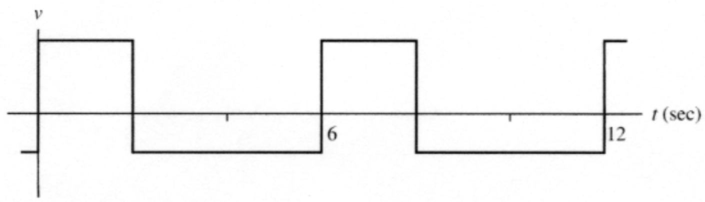
c. What is the period? 4 sec d. What is the frequency? $f = \frac{1}{T} = 0.25$ Hz

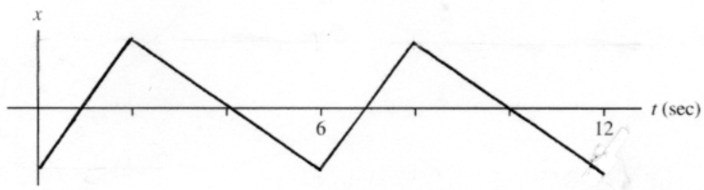
e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

4. Shown below is the velocity-versus-time graph of a particle.

a. What is the period of the motion? 6 s

b. Draw the particle's position-versus-time graph, starting from $x = 0$ at $t = 0$ s.





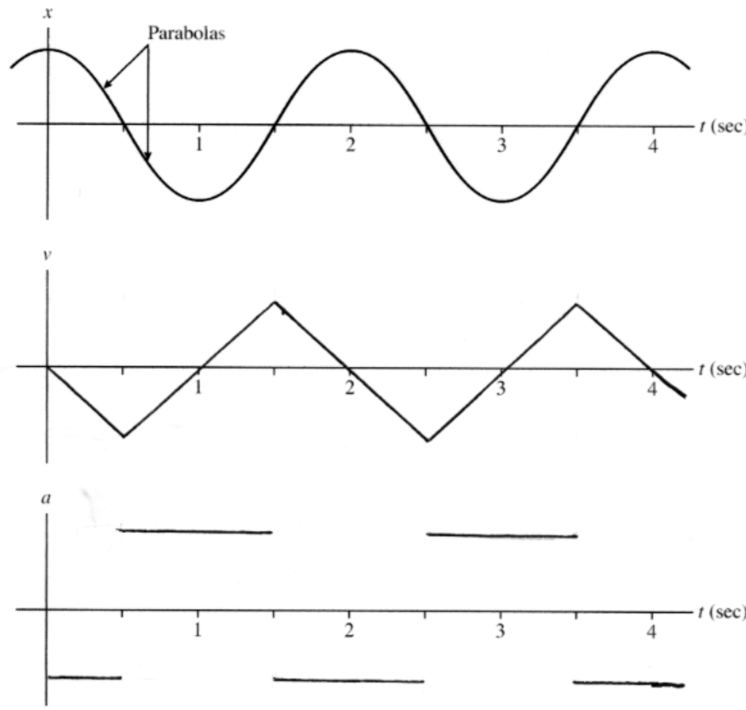
5. The graph on the next page is the position-versus-time graph of an oscillating particle. It is constructed of *parabolic* segments that are joined at $x = 0$.

a. Is this simple harmonic motion? Why or why not?

No. Sinusoidal graphs are not the same as parabolic segments. Derivatives of sinusoids are also sinusoidal.

b. Draw the corresponding velocity-versus-time graph.
Hint: What is the derivative of a parabolic function? linear function

c. Draw the corresponding acceleration-versus-time graph.



d. At what times is the position a maximum? 0, 2, 4 s

At those times, is the velocity a maximum, a minimum, or zero? 0

At those times, is the acceleration a maximum, a minimum, or zero? minimum

e. At what times is the position a minimum (most negative)? 1, 3, 5 s

At those times, is the velocity a maximum, a minimum, or zero? 0

At those times, is the acceleration a maximum, a minimum, or zero? maximum

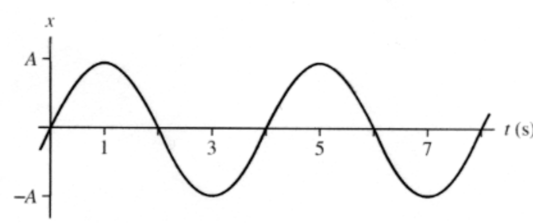
f. At what times is the velocity a maximum? 1.5 , 3.5 s

At those times, where is the particle? 0

g. Can you find a simple relationship between the *sign* of the position and the *sign* of the acceleration at the same instant of time? If so, what is it?

Signs are opposite.

6. The figure shows the position-versus-time graph of a particle in SHM.



a. At what time or times is the particle moving to the right at maximum speed?

0, 4 s

b. At what time or times is the particle moving to the left at maximum speed?

2, 6 s

c. At what time or times is the particle instantaneously at rest?

1, 3, 5, 7 s

14.2 Simple Harmonic Motion and Circular Motion

7. A particle goes around a circle 5 times at constant speed, taking a total of 2.5 seconds.

   a. Through what angle *in degrees* has the particle moved? $5 \times 360° = 1800°$

   b. Through what angle *in radians* has the particle moved? $10\pi = 31.4$ rads

   c. What is the particle's frequency f?

   $$\frac{5\text{cycles}}{2.5\text{ s}} = 2.0\text{ Hz}$$

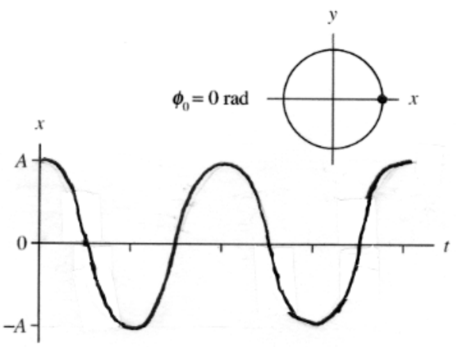
   d. Use your answer to part b to determine the particle's angular frequency ω.

   $$\omega = 2\pi f = 4\pi = 12.6\text{ rad/s}$$

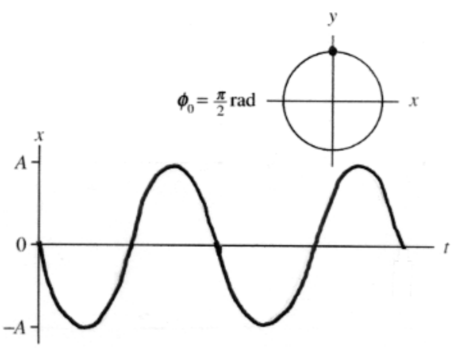
   e. Does ω (in rad/s) $= 2\pi f$ (in Hz)? yes

8. A particle moves counterclockwise around a circle at constant speed. For each of the phase constants given below:
   - Show with a dot *on the circle* the particle's starting position.
   - Sketch two cycles of the particle's x-versus-t graph.

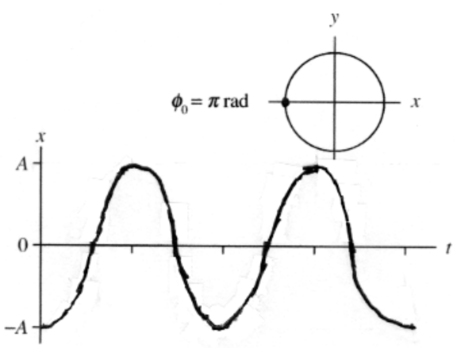
a.



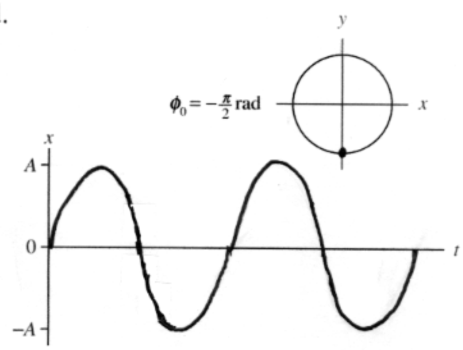
b.



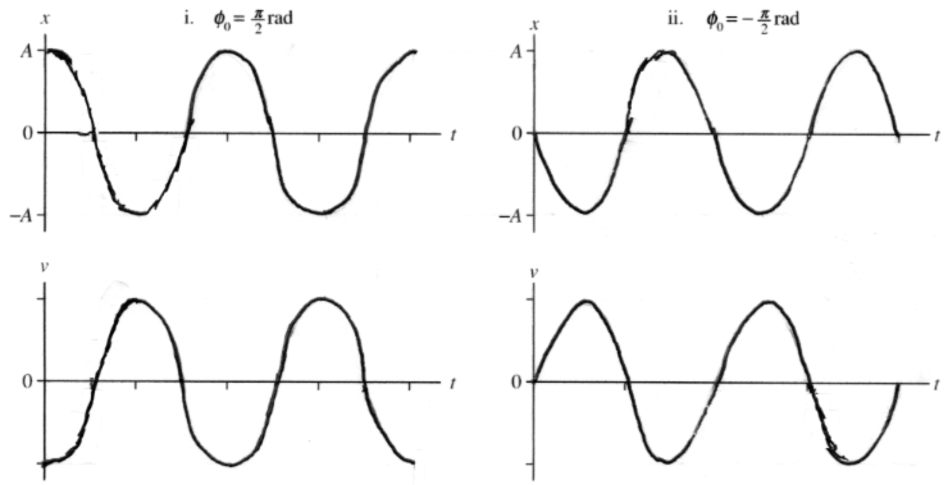
c.



d.

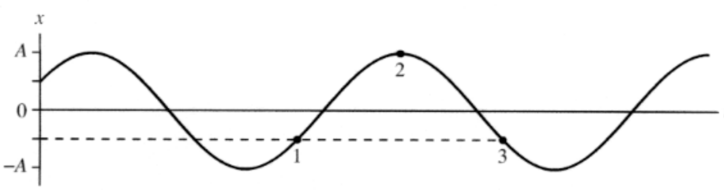


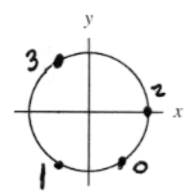
9. a. On the top set of axes below, sketch two cycles of the x-versus-t graphs for a particle in simple harmonic motion with phase constants i) $\phi_0 = \pi/2$ rad and ii) $\phi_0 = -\pi/2$ rad.
   b. Use the bottom set of axes to sketch velocity-versus-time graphs for the particles. Make sure each velocity graph aligns vertically with the correct points on the x-versus-t graph.



Redo

10. The graph below represents a particle in simple harmonic motion.





   a. What is the phase constant ϕ_0? Explain how you determined it.

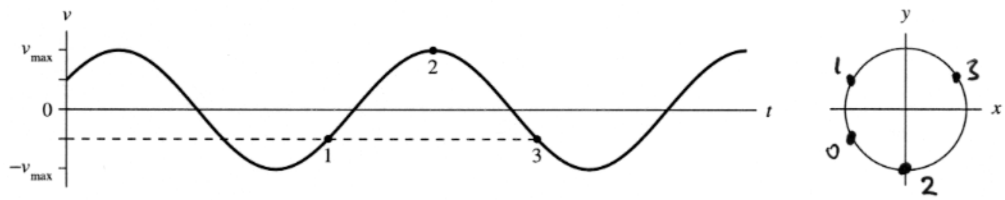
$X_0 = \frac{A}{2}$ $\cos\phi_0 = \frac{1}{2}$ $\phi_0 = \cos^{-1}\left(\frac{1}{2}\right) = -60°$

   b. What is the phase of the particle at each of the three numbered points on the graph?

   Phase at 1: -120° (or 240°) Phase at 2: 0° Phase at 3: 120°

   c. Place dots on the circle above to show the position of a circular-motion particle at the times corresponding to points 1, 2, and 3. Label each dot with the appropriate number.

11. The graph shows the *velocity* versus time for a particle in simple harmonic motion.



a. What is the phase constant ϕ_0? Explain how you determined it.

$$\frac{d(\cos \omega t)}{dt} = -\sin \omega t$$

$$-\sin \phi_0 = \frac{1}{2} \quad \text{so} \quad \phi_0 = \sin^{-1}\left(-\frac{1}{2}\right) \qquad \phi_0 = 210°$$

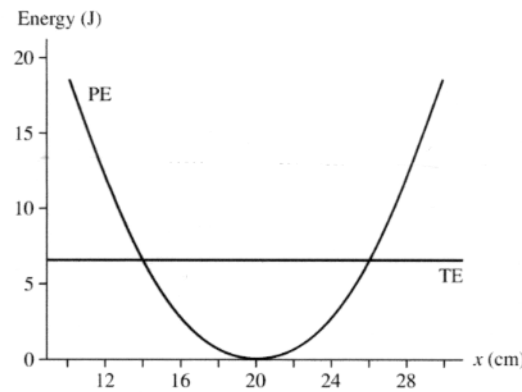
b. What is the phase of the particle at each of the three labeled points on the graph?

Phase at 1: 150° Phase at 2: 270° Phase at 3: 30°

c. Place dots on the circle to show the position of a circular-motion particle at the times corresponding to points 1, 2, and 3. Label each dot with the appropriate number.

14.3 Energy in Simple Harmonic Motion

12. The figure shows the potential-energy diagram and the total energy line of a particle oscillating on a spring.



a. What is the spring's equilibrium length?

20 cm

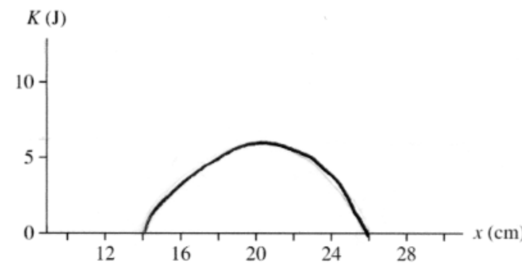
b. Where are the turning points of the motion? Explain how you identify them.

PE=TE

14cm, 26cm

c. What is the particle's maximum kinetic energy?

~7J



d. Draw a graph of the particle's kinetic energy as a function of position.

e. What will be the turning points if the particle's total energy is doubled?

12cm, 28cm

13. A block oscillating on a spring has an amplitude of 20 cm. What will be the block's amplitude if its total energy is doubled? Explain.

The energy varies as the amplitude squared so to double the energy requires a $\sqrt{2}$ increase in the amplitude.

$$E_{max} = \frac{1}{2}kX_0^2$$

$$\frac{E_2}{E_1} = \frac{\frac{1}{2}kX_2^2}{\frac{1}{2}kX_1^2} = \frac{X_2^2}{X_1^2}$$

$$\frac{E_2}{E_1} = 2 = \frac{X_2^2}{(20cm)^2}$$

$$X_2 = 20cm \times \sqrt{2} = 28.3cm$$

14. A block oscillating on a spring has a maximum speed of 20 cm/s. What will be the block's maximum speed if its total energy is doubled? Explain.

$$E = \frac{1}{2}mv^2$$

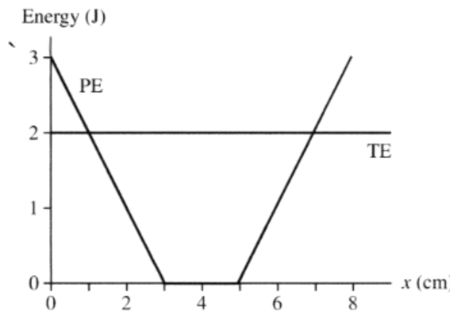
$$\frac{E_2}{E_1} = \left(\frac{V_2}{V_1}\right)^2$$

$$V_2^2 = V_1^2\left(\frac{E_2}{E_1}\right)$$

$$V_2 = V_1\sqrt{\frac{E_2}{E_1}}$$

$$V_2 = 20\,\frac{cm}{s}\sqrt{2} = 28.3\,\frac{cm}{s}$$

15. The figure shows the potential energy diagram of a particle.



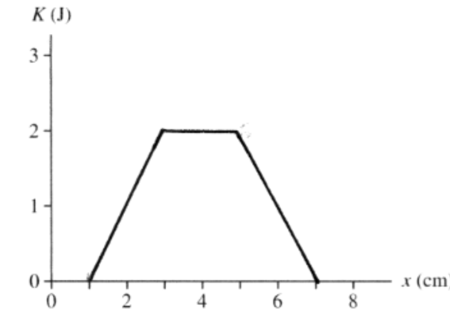
a. Is the particle's motion periodic? How can you tell?

Yes, there are turning points at 1cm and at 7cm and the particle oscillates between these two points.

b. Is the particle's motion simple harmonic motion? How can you tell? No. The PE curve is not quadratic.

c. What is the amplitude of the motion?

3 cm.



d. Draw a graph of the particle's kinetic energy as a function of position.

16. Equation 14.25 in the textbook states that $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$. What does this mean? Write a couple of sentences explaining how to interpret this equation.

Energy is transferred back and forth between all potential energy at the extremes ($\frac{1}{2}kA^2$) and all kinetic energy at the equilibrium point(s) ($\frac{1}{2}mV_{max}^2$). The equation does not say that the particle ever has amplitude A and speed V_{max}. The equation relates expressions for the energy at two different times.

14.4 The Dynamics of Simple Harmonic Motion

14.5 Vertical Oscillations

17. A block oscillating on a spring has period $T = 2$ s.

a. What is the period if the block's mass is doubled? Explain.

Note: You do not know values for either m or k. Do *not* assume any particular values for them. The required analysis involves thinking about ratios.

$T = 2\pi\sqrt{\frac{m}{k}}$ so if the mass is doubled, the period will increase by $\sqrt{2}$ $T' = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \sqrt{2}\,T$

$T' = 2.83\text{s}$

b. What is the period if the value of the spring constant is quadrupled?

If the spring constant is quadrupled, the period is decreased by a factor of 2. $T' = 2\pi\sqrt{\frac{m}{4k}} = \frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{2}T$

$T' = 1\text{s}.$

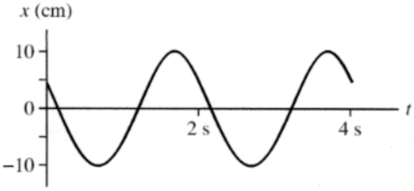
c. What is the period if the oscillation amplitude is doubled while m and k are unchanged?

The period is unchanged. The amplitude does not affect the period. $T' = 2\text{s}$

18. For graphs a and b, determine:

- The angular frequency ω.
- The oscillation amplitude A.
- The phase constant ϕ_0.

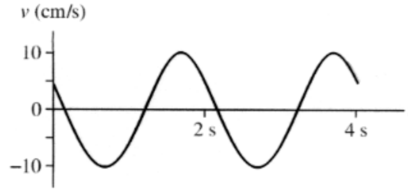
Note: Graphs a and b are independent. Graph b is *not* the velocity graph of a.

a. 

$\omega =$ 3.14 rad/s

$A =$ 10 cm

$\phi_0 =$ 60°

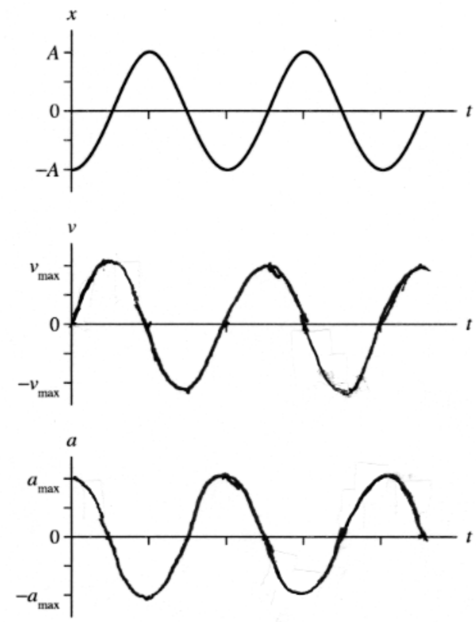
b. 

$\omega =$ 3.14 rad/s

$A =$ $10/\pi$ cm = 3.18cm ($v_{max} = \omega A$)

$\phi_0 =$ −30°

19. The graph on the right is the position-versus-time graph for a simple harmonic oscillator.



a. Draw the v-versus-t and a-versus-t graphs.

b. When x is greater than zero, is a ever greater than zero? If so, at which points in the cycle?

No

c. When x is less than zero, is a ever less than zero? If so, at which points in the cycle?

No

d. Can you make a general conclusion about the relationship between the sign of x and the sign of a? The signs of x and a are opposite.

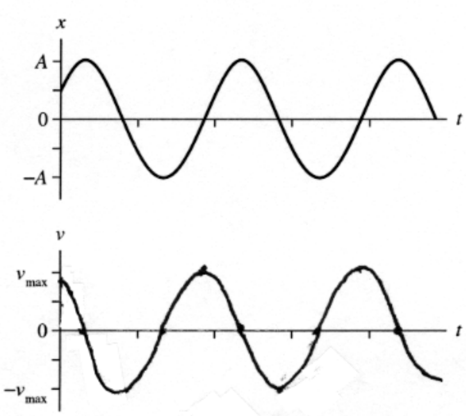
e. When x is greater than zero, is v ever greater than zero? If so, how is the oscillator moving at those times? Yes. If $x > 0$ and $v > 0$, the object is slowing down as it approaches a turning point.

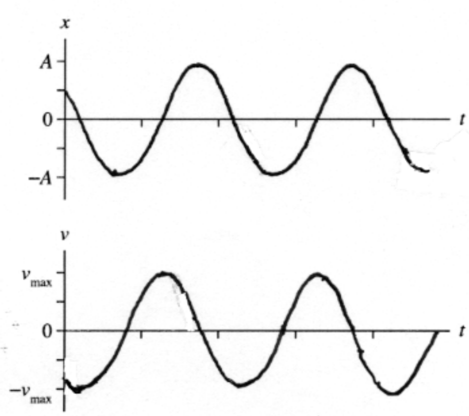
20. For the oscillation shown on the left below:

a. What is the phase constant ϕ_0? $\cos\phi_0 = \frac{1}{2}$, $\phi_0 = -60^\circ$

b. Draw the corresponding v-versus-t graph on the axes below the x-versus-t graph.

c. On the axes on the right, sketch two cycles of the x-versus-t and the v-versus-t graphs if the value of ϕ_0 found in part a is replaced by its negative, $-\phi_0$.

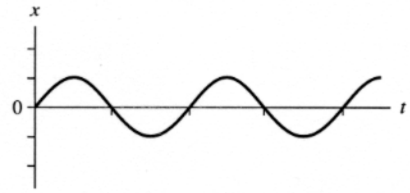




d. Describe *physically* what is the same and what is different about the initial conditions for two oscillators having "equal but opposite" phase constants ϕ_0 and $-\phi_0$.

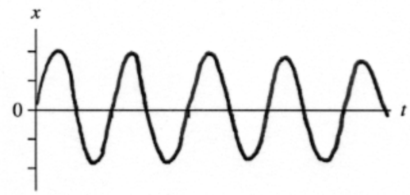
The initial starting point is the same for ϕ_0 and $-\phi_0$. In the first case, however, the motion is moving towards maximum displacement, while in the second case it is moving towards the equilibrium point. For ϕ, the initial velocity is positive. For $-\phi_0$, it is negative.

21. The top graph shows the position versus time for a mass oscillating on a spring. On the axes below, sketch the position-versus-time graph for this block for the following situations:



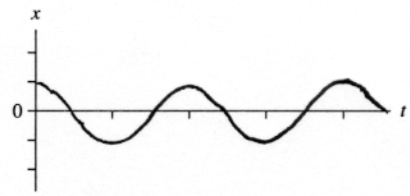
Note: The changes described in each part refer back to the original oscillation, not to the oscillation of the previous part of the question. Assume that all other parameters remain constant. Use the same horizontal and vertical scales as the original oscillation graph.

a. The amplitude and the frequency are doubled.

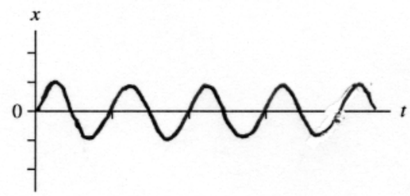


b. The amplitude is halved and the mass is quadrupled.

c. The phase constant is increased by $\pi/2$ rad.



d. The maximum speed is doubled while the amplitude remains constant.



14.6 The Pendulum

22. A pendulum on planet X, where the value of g is unknown, oscillates with a period of 2 seconds. What is the period of this pendulum if:

a. Its mass is doubled?

Note: You do not know the values of m, L, or g, so do not assume any specific values.

$T = \sqrt{\frac{L}{g}}$ is independent of the mass. $\boxed{T = 2\,s}$

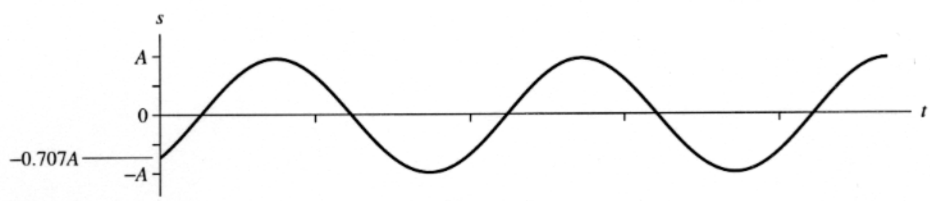
b. Its length is doubled?

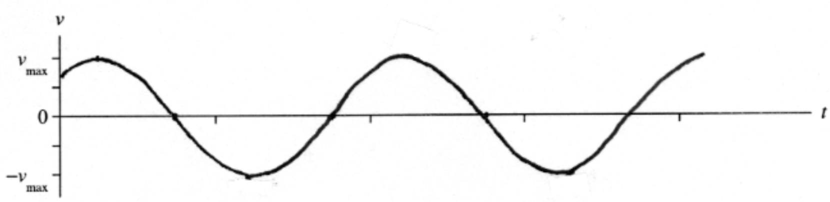
$$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{2L_1}{L_1}} \quad \text{or} \quad T_2 = \sqrt{2}\,T_1 = \boxed{2.8\,s}$$

c. Its oscillation amplitude is doubled?

The period is independent of the amplitude for small angles. $\boxed{T = 2\,s}$

23. The graph shows the displacement s versus time for an oscillating pendulum.





a. Draw the pendulum's velocity-versus-time graph.

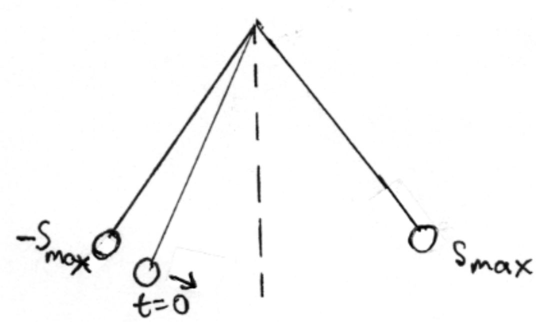
b. What is the value of the phase constant ϕ_0?

$$\cos\phi_0 = -.707$$

$$\phi_0 = 225°$$

c. In the space at the right, draw a *picture* of the pendulum that shows (and labels!)

- The extremes of its motion.
- Its position at $t = 0$ s.
- Its direction of motion (using an arrow) at $t = 0$ s.



14.7 Damped Oscillations

24. If the damping constant b of an oscillator is increased,

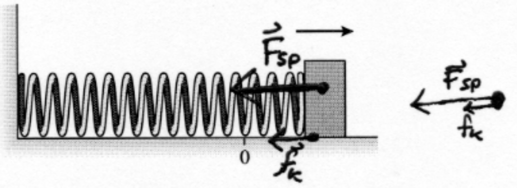
    a. Is the medium more resistive or less resistive? more

    b. Do the oscillations damp out more quickly or less quickly? more

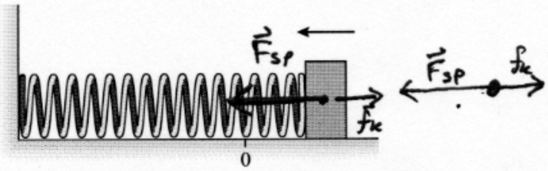
    c. Is the time constant τ increased or decreased? decreased

25. A block on a spring oscillates horizontally on a table with friction. Draw and label force vectors on the block to show all *horizontal* forces on the block.

    a. The mass is to the right of the equilibrium point and moving away from it.



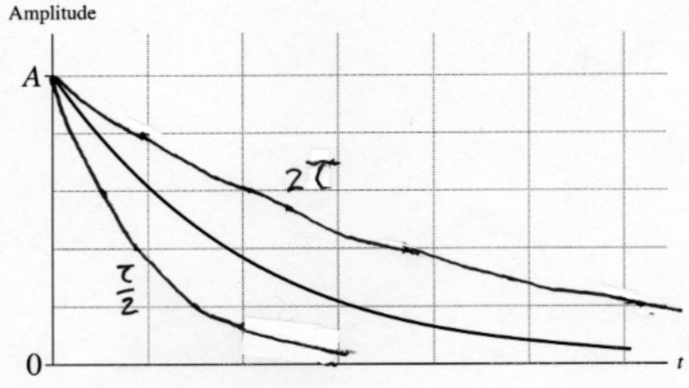
b . The mass is to the right of the equilibrium point and approaching it.



26. The figure below shows the envelope of the oscillations of a damped oscillator. On the same axes, draw the envelope of oscillations if

    a. The time constant is doubled.

    b. The time constant is halved.



27. a. Describe the difference between τ and T. Don't just *name* them; say what is different about the physical concepts that they represent.

T, the period, is the time for each cycle of the motion, the time required for the motion to repeat itself.

τ, the damping time constant, is the time required for the energy of the oscillator to drop by a factor of e^{-1}.

b. Describe the difference between τ and $t_{1/2}$.

τ is the time required for the energy to decay by $e^{-1} \sim 37\%$.

$t_{1/2}$ is the time required for the energy to decay by $\frac{1}{2}$.

14.8 Driven Oscillations and Resonance

28. What is the difference between the driving frequency and the natural frequency of an oscillator?

The driving frequency is the frequency of an external force that is applied to the system. The natural frequency is the frequency of oscillations due to the system's restoring force when displaced from equilibrium. The driving frequency and natural frequency are independent

29. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).

b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn out shock absorbers (small damping coefficient).

Draw both graphs on the same axes, and label them as to which is which.