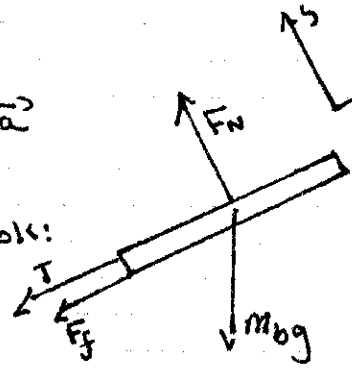
Ch. 8 #38

Phys 120 Assignment #3

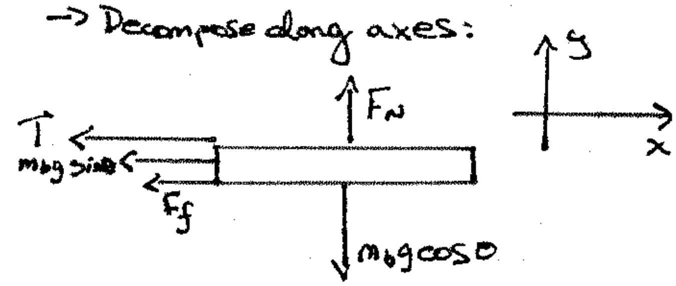
Define coordinate systems so they lie along direction of $\vec{v_i}$

a) <u>Dynamics:</u> $\vec{F} = m\vec{a}$

Forces on 1kg book:



→ Decompose along axes:



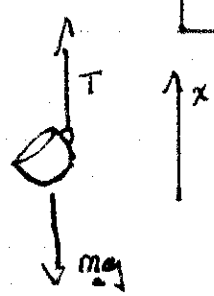
$$\sum F_y = m_b a_y = 0 \Rightarrow F_N - m_b g\cos\theta = 0 \Rightarrow F_N = m_b g\cos\theta$$

$$\sum F_x = m_b a_{bx} \Rightarrow -T - m_b g\sin\theta - F_f = m_b a_{bx}$$

$$F_f = \mu_k F_N = \mu_k m_b g\cos\theta$$

$$\Rightarrow \boxed{m_b a_{bx} = -T - m_b g(\sin\theta + \mu_k\cos\theta)} \quad (1)$$

Forces on 0.5kg cup:



$$\sum F = m_c a_{cx}$$

$$\Rightarrow \boxed{T - m_c g = m_c a_{cx}} \quad (2)$$

Constraint: Fixed string length (inextensible string)

$$\Rightarrow a_{bx} = a_{cx} = a$$

→ 2 equations, 2 unknowns (a and T)

$$m_b a = -T - m_b g(\sin\theta + \mu_k\cos\theta) \quad (1a)$$
$$m_c a = T - m_c g \quad (2a)$$

(1a)+(2a) $\Rightarrow (m_b + m_c)a = -m_b g(\sin\theta + \mu_k\cos\theta) - m_c g$

$$\Rightarrow a = -\left\{\frac{m_b}{m_b+m_c}(\sin\theta + \mu_k\cos\theta) + \frac{m_c}{m_b+m_c}\right\}g$$

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$$a = -\frac{g}{1+m_b/m_c}\left\{1 + \frac{m_b}{m_c}\left(\sin\theta + \mu_k\cos\theta\right)\right\}$$

Kinematics: Now with the Dynamics done, we know $\vec{a}$. Thus, at least in principle, we can figure out anything we want to know about the motion.

Since a doesn't depend on time or position,

$a = \text{constant}$

So we can use our constant-a Formulae of kinematics

In particular

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Rightarrow \quad \Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{v_f^2 - v_i^2}{2} \cdot \frac{-(1+m_b/m_c)}{1+m_b/m_c(\sin\theta + \mu_k\cos\theta)}$$

$$\Delta x = \frac{v_i^2 - v_f^2}{2g}\ \frac{(1+m_b/m_c)}{1+m_b/m_c(\sin\theta + \mu_k\cos\theta)}$$

For $v_i = 3\,m/s$
$v_f = 0$ (book comes to rest)

$\mu_k = 0.2$

$m_b = 1\,kg$
$m_c = 0.5\,kg$
$\theta = 20°$
$g = 9.81\,m/s^2$

$$\Delta x = \frac{(3\,m/s)^2}{2(9.81\,m/s^2)} \cdot \frac{1+1\,kg/0.5\,kg}{1+\frac{1\,kg}{0.5\,kg}\left(\sin(20°) + 0.2\cos(20°)\right)}$$

$$= \frac{9\,m^2/s^2}{2\cdot 9.81\,m/s^2} \cdot \frac{3}{1+2\left(\sin(20°)+0.2\cos(20°)\right)}$$

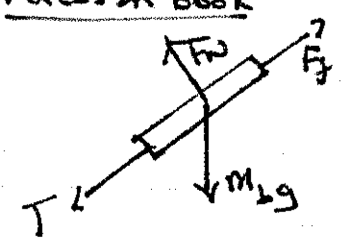
$$= 0.67\,m$$

b) Does it stick or slip

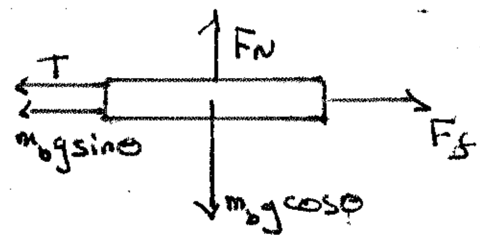
→ Recall: Static friction takes on whatever value it needs to to keep the book from slipping, up to a maximum value of $F_{fs\,max} = \mu_s F_N$.

Let's assume the book sticks, and then check that static friction is strong enough to hold it,

Forces on book



→ Decompose along axes



$$\Sigma F_y = m_b a_y = 0 \Rightarrow F_N - m_b g\cos\theta = 0 \Rightarrow F_N = m_b g\cos\theta$$

$$\Sigma F_x = m_b \underbrace{a_x = 0}_{\text{assumed Static}} \Rightarrow F_f - T - m_b g\sin\theta = 0$$

$$\Rightarrow \boxed{F_f = T + m_b g\sin\theta} \quad (3)$$

Forces on Cup

$$\rightarrow \Sigma F = T - m_c g = \cancel{m_c a}^{\,0} \Rightarrow T - m_c g = 0 \Rightarrow \boxed{T = m_c g}$$

(if book is stationary, so is the cup) (4)

Plug (4) into (3) → The frictional force required to hold the book in place is

$$F_f = m_c g + m_b g\sin\theta = g\{m_c + m_b\sin\theta\}$$

$$= 9.81\,\mathrm{m/s^2}\left(0.5\,\mathrm{kg} + 1\,\mathrm{kg}\cdot\sin(20°)\right)$$

$$F_f = 8.26\,\mathrm{N}$$

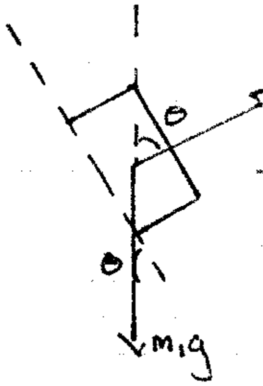
The maximum force static friction can exert is

$$F_{fs\,max} = \mu_s F_N = \mu_s \cdot m_b g\cos\theta = (0.5)(1\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\cos(20°)$$

$$= 4.61\,\mathrm{N}$$

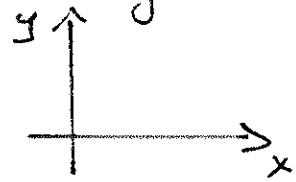
$\Rightarrow F_f > F_{fs\,max}$ ⇒ Static friction cannot hold book from moving
⇒ It slips.

Forces on m_1:

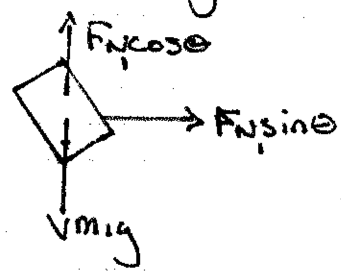


~~Force~~ Force of contact with m_2 → requires "equal and opposite" reaction force on m_2

→ Convenient to set up axes along direction of acceleration



→ Decompose forces along axes



$$\sum F_y = F_N\cos\theta - m_1 g = m_1 \cancel{a_y} = 0$$

$$\Rightarrow F_N\cos\theta = m_1 g$$

$$\Rightarrow \boxed{F_{N_1} = \frac{m_1 g}{\cos\theta}} \quad (1)$$

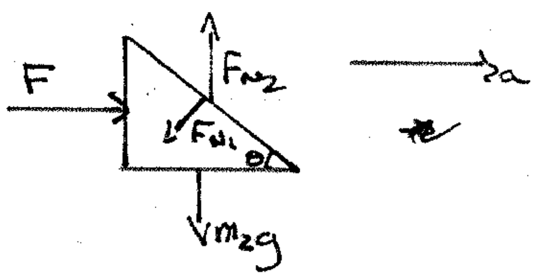
$$\sum F_x = F_N\sin\theta = m_1 a$$ use (1) to eliminate F_N

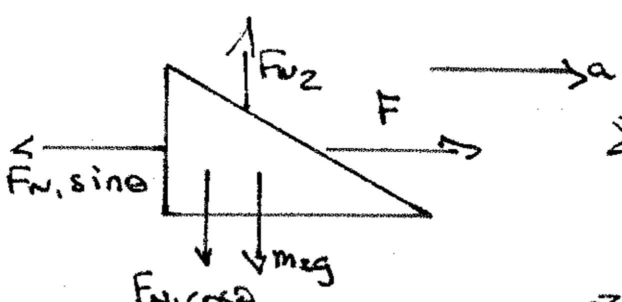
$$\Rightarrow \frac{m_1 g}{\cos\theta}\cdot\sin\theta = m_1 a$$

$$\Rightarrow$$

$$\boxed{a = g\tan\theta} \quad (2)$$

Forces on m_2





$$\sum F_y = F_{N_2} - F_{N_1}\cos\theta - m_2 g = 0$$

→ Not really interesting

$$\sum F_x = F - F_{N_1}\sin\theta = m_2 a$$

$$\Rightarrow F = m_2 a + F_{N_1}\sin\theta$$

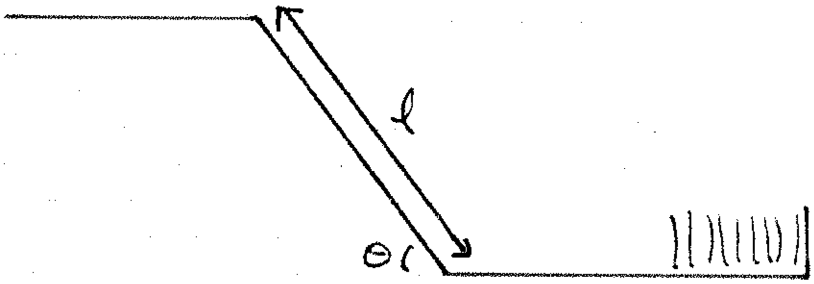
from (2), $a = g\tan\theta$

$$F = m_2 g\tan\theta + F_{N_1}\sin\theta$$

from (1) $F_{N_1} = \frac{m_1 g}{\cos\theta}$ $\Rightarrow \boxed{F = (m_1 + m_2) g\tan\theta}$

Ch. 10 #56

a)



m = mass of athlete
M = mass of bobsled.
x = maximal spring compression.

initial energy: $E_i = \frac{1}{2} m v_i^2 + (m+M)gh = \frac{1}{2} m v_i^2 + (m+M) g \ell \sin\theta$

$\downarrow$

no friction $\rightarrow$ No energy losses $\Rightarrow \Delta E = 0$

$\downarrow$

final energy: $E_f = \frac{1}{2} k x^2$ $\rightarrow$ no motion $\rightarrow$ all energy is stored in spring.

$$E_i + \Delta E = E_f \quad \Rightarrow \quad E_i = E_f$$

$$\frac{1}{2} K x^2 = \frac{1}{2} m v_i^2 + (m+M) g \ell \sin\theta$$

$$\Rightarrow x = \sqrt{\frac{m v_i^2 + 2(m+M) g \ell \sin\theta}{K}}$$

$M = 20\,kg$
$\ell = 50\,m$
$\theta = 20°$
$K = 2000\,N/m$
$g = 9.81\,m/s^2$

Lisa: $m = 40\,kg$
$v_i = 12\,m/s$

$$x = \sqrt{\frac{40\,kg\,(12\,m/s)^2 + 2\,(40\,kg + 20\,kg) \cdot 9.81\,m/s^2 \cdot 50\,m \cdot \sin(20°)}{2000\,N/m}}$$

$$x = 3.60\,m$$

$\Rightarrow$ $\boxed{\text{Lisa compresses the spring 3.60 m}}$

Ch 10 #56 (cont'd)

b) In the absence of friction, the only term involving the track is the $2(m+M)gl\sin\theta$

$\rightarrow$ since $l\sin\theta = h$ = height above which the athlete starts.

$\Rightarrow$ Under the conditions of frictionless snow, the shape of the track does not matter.

If we really want to analyze the problem, we must include friction, and account for athlete safety and fairness of the competition.

$\rightarrow$ <u>Frictional effects</u> $\quad F_f = \mu F_N = \mu(m+M)g\cos\theta$

$$\Rightarrow W_f = -\mu(m+M)g\cos\theta\, l. = \Delta E$$

$$\Rightarrow E_f = \frac{1}{2}kx^2 = E_i + \Delta E$$

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv_i^2 + (m+M)gl\{\sin\theta - \mu\cos\theta\}.$$

$$\Rightarrow x = \sqrt{\frac{2\cdot\frac{1}{2}mv_i^2 + 2(m+M)gl\{\sin\theta-\mu\cos\theta\}}{K}}$$

Let's factor out the athlete's energy

$$x = \sqrt{\frac{2\cdot\frac{1}{2}mv_i^2}{K}\left\{1 + \frac{2(m+M)gl\{\sin\theta-\mu\cos\theta\}}{mv_i^2}\right\}}.$$

$$= \sqrt{\frac{2E}{K}}\sqrt{1 + 2\frac{(m+M)}{m}\frac{gl}{v_i^2}\{\sin\theta-\mu\cos\theta\}}$$

If we want this to be a "fair" competition, then x should only be sensitive to the energy the athlete is able to generate → in effect, the second term should be smaller than 1, otherwise effects that have nothing to do with the athlete dominate, and the competition is unfair

$$\Rightarrow \quad \frac{2(m+M)}{m} \cdot \frac{g\,l}{v_i^2} \left\{ \sin\theta - \mu\cos\theta \right\} \ll 1$$

$$\Rightarrow \quad l \ll \frac{1}{2} \cdot \frac{1}{1+M/m} \cdot \frac{v_i^2}{g} \cdot \frac{1}{\sin\theta - \mu\cos\theta}$$

~~⇒ However, we want l to be~~

$$l \ll \frac{1}{2} \cdot \frac{1}{1+M/m} \cdot \frac{v_i^2}{g} \cdot \frac{1}{\sin\theta - \mu\cos\theta}$$

So if we take, for instance, θ such that

$$\sin(\theta) - \mu\cos\theta = 0 \Rightarrow \mu = \tan\theta$$

$$\Rightarrow \theta = \arctan(\mu)$$

then we can have a track of any length, and still have a fair competition.

$\rightarrow$ let $\theta_0 = \arctan(\mu)$

Now for safety: The human body has limits on the accelerations it can withstand.

For instance, sustained accelerations of 10g's (that's ~ 100 m/s²) cause unconciousness after several seconds

Instantaneous accelerations of about 100g's (~ 1000 m/s²) breaks bones, squashes organs, and is generally unsafe.

Ch. 10 #56 (cont'd)

What is the acceleration of a body in contact with a spring?

$$F_{spring} = -kx$$

$$\Rightarrow (m+M)\,a = -kx$$

$$\Rightarrow a = \frac{-kx}{m+M}. \quad \Rightarrow |a| = \frac{kx}{m+M}$$

→ to estimate this, take the x we've calculated

for $\theta = \theta_0 = \arctan(\mu)$

$$x = \sqrt{\frac{2E}{K}} \Rightarrow |a| = \frac{kx}{m+M} \cdot \sqrt{\frac{m v_i^2}{K}} \ll 10 \cdot g$$

$$\Rightarrow |a| = \sqrt{\frac{m\, v_i^2\, K}{(m+M)^2}} \ll 10g$$

$$\Rightarrow \quad K \ll 100 g^2 \cdot \frac{(m+M)^2}{m} \cdot \frac{1}{v_i^2}$$

for $v_i \sim 15\,m/s$, $m \simeq 40kg$

$$K \ll 100\,(9.81\,m/s^2)^2 \cdot \frac{(60kg)^2}{40kg} \cdot \frac{1}{(15\,m/s)^2}$$

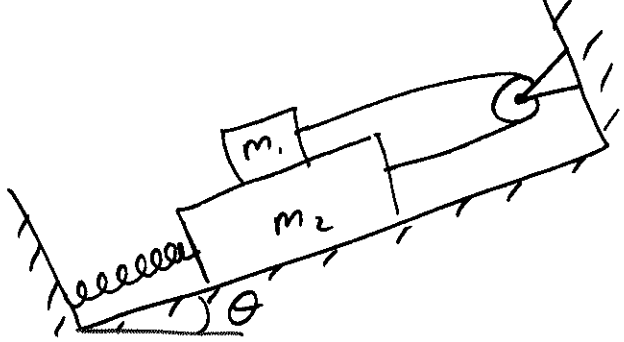
$$K \ll 3800\ N/m$$

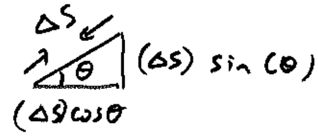
→ This (2000 N/m) is not an ureasonable spring constant.

→ Summary: In order for the energy gained from falling down the slope to exactly cancel frictional effects, the angle of the slope should be chosen such that $\theta = \arctan(\mu)$, μ = coefficient of kinetic friction

for safety, the spring constant needs to be less than ~ 3800 N/m

→ Since the track largely cancels out, neither the height nor track length really matter





let ΔS = distance blocks slide

Define the system: Everything - Earth, blocks, spring platform.

$$W_{ext} = \Delta KE_1 + \Delta KE_2 + \Delta U_{g_1} + \Delta U_{g_2} + \Delta U_{sp} + \Delta E_{Th} = 0$$

$\Delta KE_1 = \Delta KE_2 = 0 \longrightarrow$ system starts at rest & ends at rest.

$\Delta U_{g_1} = m_1 g (\Delta S) \sin\theta \quad \leftarrow$ gain in gravitational Potential energy

$\Delta U_{g_2} = - m_2 g (\Delta S) \sin\theta \quad \longleftarrow$ loss in gravitational Potential Energy

$\Delta U_{sp} = \frac{1}{2} K (\Delta S)^2$

$$\Delta E_{Th} = \Big[\underbrace{\mu_{k_1} m_1 g \cos\theta}_{\text{friction on top surface}} + \underbrace{\mu_{k_2} (m_1 + m_2) g \cos\theta}_{\text{friction on bottom surface}}\Big] \Delta S$$

$$= g\cos\theta \left[\mu_{k_1} m_1 + \mu_{k_2}(m_1 + m_2)\right] \Delta S$$

solve for ΔS

$$\Delta s\, g \sin\theta \left[m_1 - m_2\right] + \frac{1}{2} k (\Delta s)^2 + g\cos\theta \left[\mu_{k_1} m_1 + \mu_{k_2}(m_1+m_2)\right]\Delta s = 0$$

($\Delta s = 0$ is the trivial answer, this tells us $\Delta KE = 0$ the moment we release the system)

divide by Δs

$$g \sin\theta \left[m_1 - m_2\right] + \frac{1}{2} k \,\Delta s + g\cos\theta\left[\mu_{k_1} m_1 + \mu_{k_2}(m_1+m_2)\right] = 0$$

or

$$\boxed{\Delta s = \frac{2}{k}\left[-g\sin\theta\left[m_1 - m_2\right] - g\cos\theta\left[\mu_{k_1} m_1 + \mu_{k_2}(m_1+m_2)\right]\right]}$$