

Classical Mechanics

Lecture 8

Today's Concepts:

- a) Potential Energy
- b) Mechanical Energy

Stuff you asked about:

Gravity is the law. violators will be brought down.

How were these equations derived? I don't want to have to memorize, I would like a logical explanation of these equations.

Why is the potential energy of a spring hanging vertically $\frac{1}{2}ky^2$? I thought potential energy was always defined as mgh . What is the easiest way to understand these rates/ratios?

The whole thing with setting a point on a spring equal to $h=0$

the potential energy when the object is far from the surface of the earth. It is difficult, since the constant g is no longer the same as the one when the object is near the surface. How can I calculate the potential energy?

Too many letters and variables jumping around the screen

Show some examples of how to do a question, instead of giving all these notes and little experiment.

Summary

Lecture 7

$$\Delta K \equiv W_{total}$$

Work – Kinetic Energy theorem

Lecture 8

$$\Delta U \equiv -W$$

For springs & gravity (conservative forces)

$$E \equiv K + U$$

Total Mechanical Energy

E = Kinetic + Potential

$$\Delta E = W_{NC}$$

Work done by any force other than gravity and springs will change E

Relax. There is nothing new here

It's just re-writing the work-KE theorem:

$$\Delta K = W_{tot} = \boxed{W_{gravity} + W_{springs}} + \underbrace{W_{NC}}_{\text{everything except gravity and springs}}$$

$- \Delta U_{gravity} - \Delta U_{springs}$

$$\Delta K + \Delta U_{gravity} + \Delta U_{springs} = W_{NC}$$

$$\Delta K + \Delta U = W_{NC}$$

$$\Delta E = W_{NC}$$

$$\Delta E = 0 \quad \text{If other forces aren't doing work}$$

Finding the potential energy change:

Use formulas to find the magnitude

Check the sign by understanding the problem...

	Force \vec{F}	Work $W_{1 \rightarrow 2}$	Change in P.E. $\Delta U = U_2 - U_1$	P.E. Function U
Gravity (Near Earth)	$m\vec{g}$	$-mg(h_2 - h_1)$	$mg(h_2 - h_1)$	$mgh + U_o$
Gravity (General Expression)	$-G \frac{m_1 m_2}{r^2} \hat{r}$	$Gm_1 m_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$	$-Gm_1 m_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$	$G \frac{m_1 m_2}{r} + U_o$
Spring	$-k\vec{x}$	$-\frac{1}{2} k(x_2^2 - x_1^2)$	$\frac{1}{2} k(x_2^2 - x_1^2)$	$\frac{1}{2} kx^2 + U_o$

CheckPoint

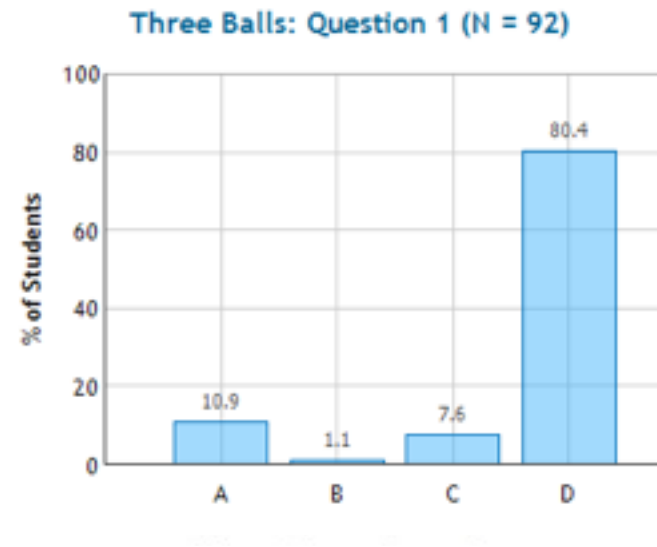
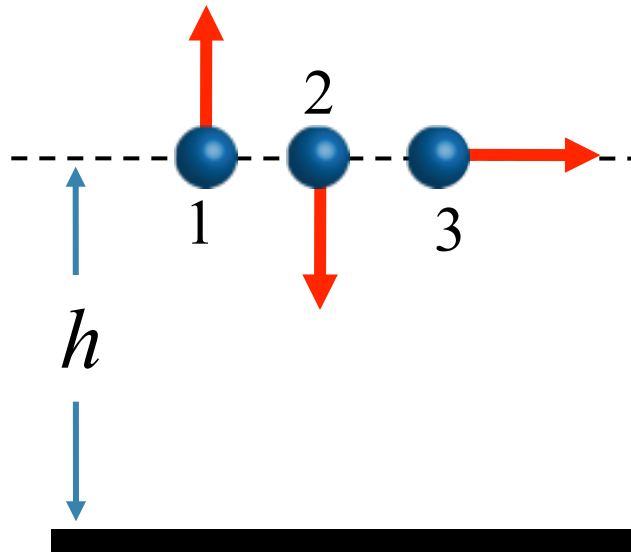
Three balls of equal mass are fired simultaneously with equal speeds from the same height h above the ground. Ball 1 is fired straight up, ball 2 is fired straight down, and ball 3 is fired horizontally. Rank in order from largest to smallest their speeds v_1 , v_2 , and v_3 just before each ball hits the ground.

A) $v_1 > v_2 > v_3$

B) $v_3 > v_2 > v_1$

C) $v_2 > v_3 > v_1$

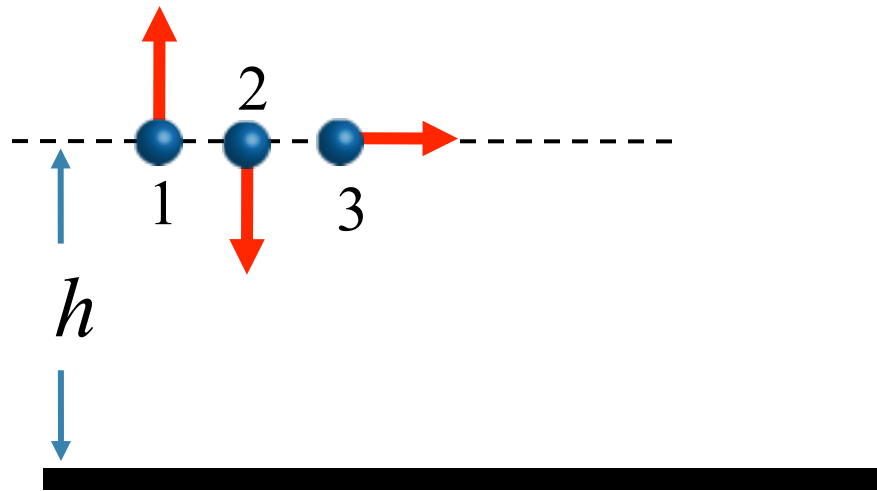
D) $v_1 = v_2 = v_3$



Clicker Question



Which of the following quantities are **NOT** the same for the three balls as they move from height h to the floor:



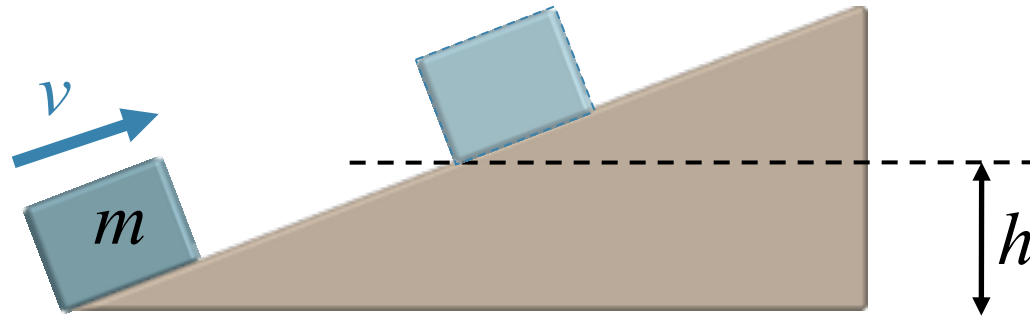
- A) The change in their kinetic energies
- B) The change in their potential energies
- C) The time taken to hit the ground

Clicker Question



A block of mass m is launched up a frictionless ramp with an initial speed v and reaches a maximum vertical height h . A second block having twice the mass ($2m$) is launched up the same ramp with the same initial speed (v). What is the maximum vertical height reached by the second block?

- A) h
- B) $\sqrt{2} h$
- C) $2h$
- D) $4h$

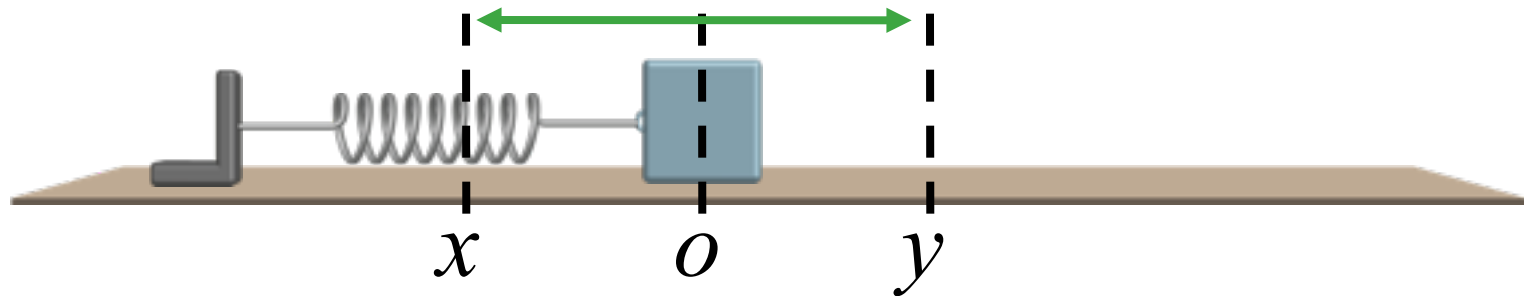


$$\cancel{m}gh = \frac{1}{2} \cancel{m}v^2 \quad \rightarrow \quad h = \frac{1}{2g} v^2$$

Clicker Question



A block attached to a spring is oscillating between point x (fully compressed) and point y (fully stretched). The spring is un-stretched at point o . At point o , which of the following quantities is at its maximum value?

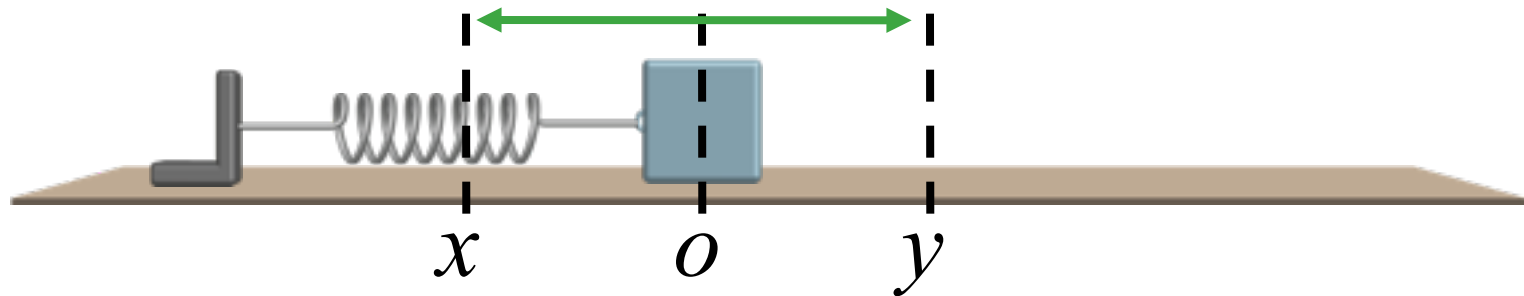


- A) The block's kinetic energy
- B) The spring potential energy
- C) Both A and B

Clicker Question



A block attached to a spring is oscillating between point x (fully compressed) and point y (fully stretched). The spring is un-stretched at point o . At point x , which of the following quantities is at its maximum value?

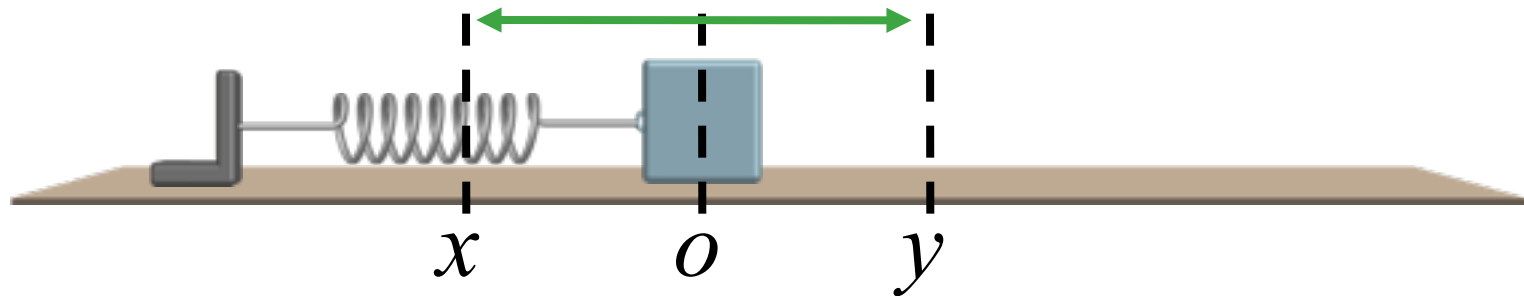


- A) The block's kinetic energy
- B) The spring potential energy
- C) Both A and B

Clicker Question



A block attached to a spring is oscillating between point x (fully compressed) and point y (fully stretched). The spring is un-stretched at point o . At which point is the acceleration of the block zero?



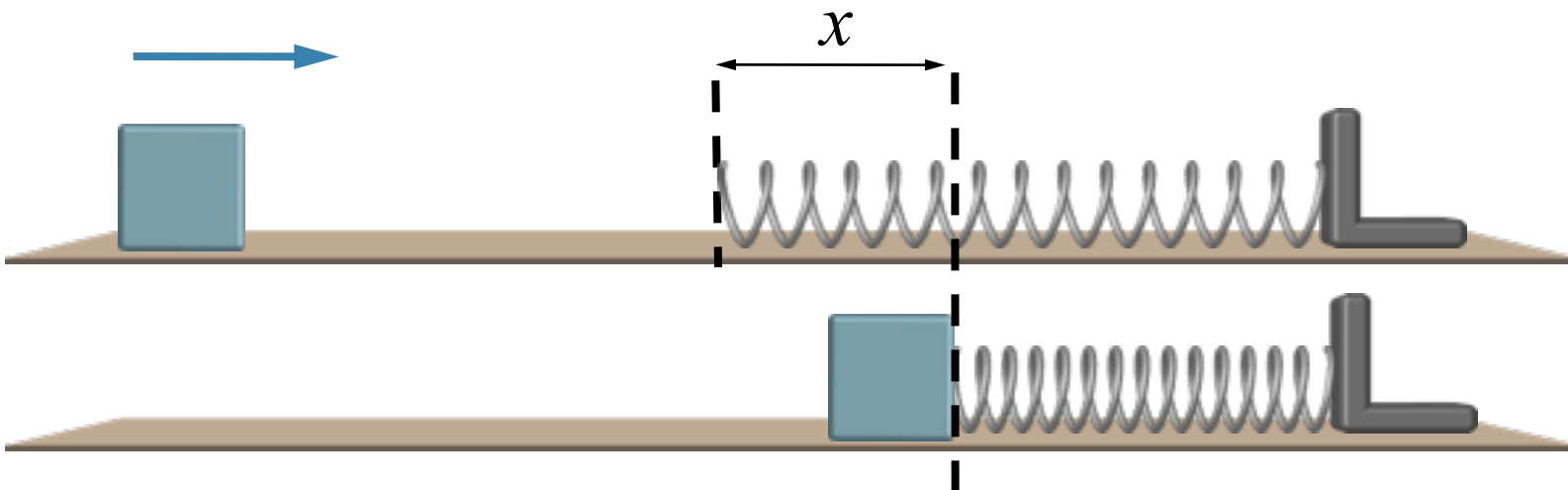
- A) At x
- B) At o
- C) At y

CheckPoint

A box sliding on a horizontal frictionless surface runs into a fixed spring, compressing it a distance x_1 from its relaxed position while momentarily coming to rest.

If the initial speed of the box were doubled, how far x_2 would the spring compress?

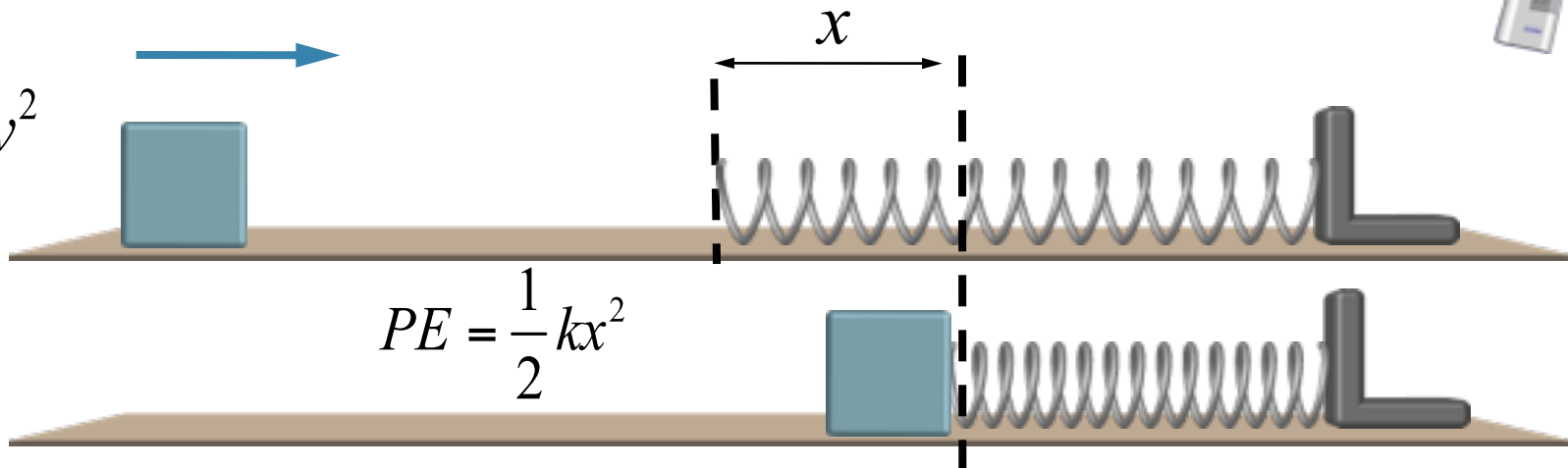
A) $x_2 = \sqrt{2}x_1$ B) $x_2 = 2x_1$ C) $x_2 = 4x_1$



CheckPoint



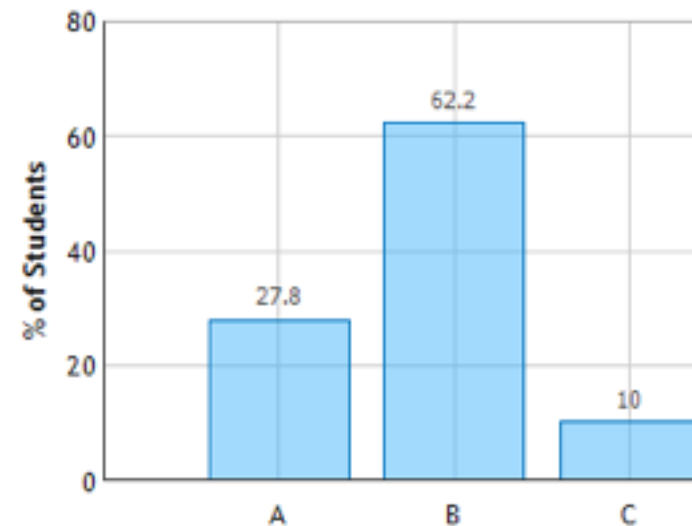
$$KE = \frac{1}{2}mv^2$$



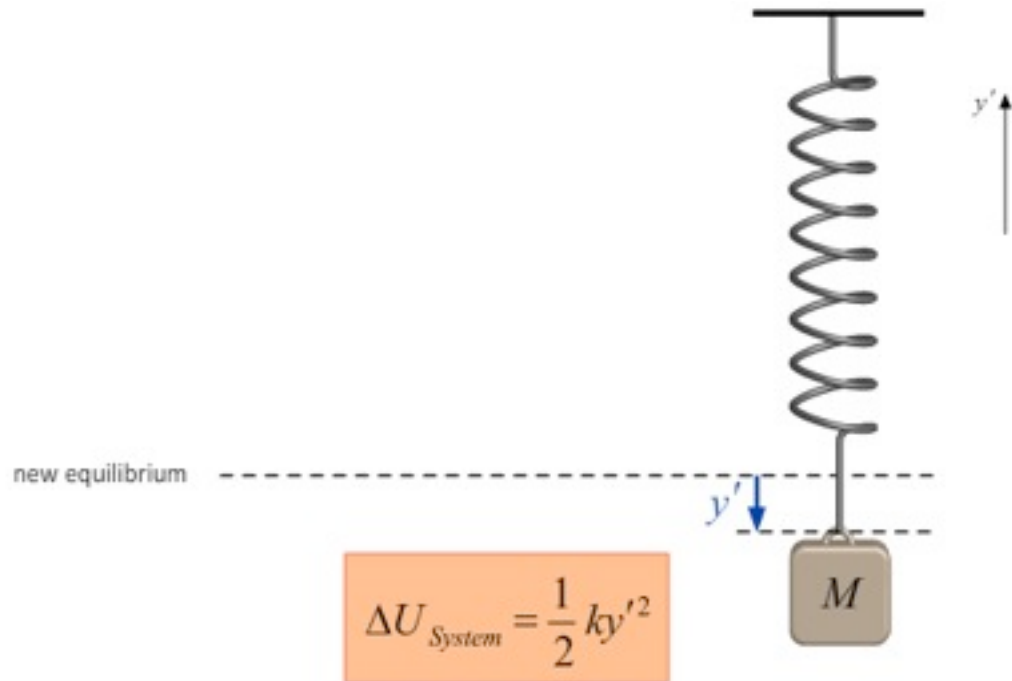
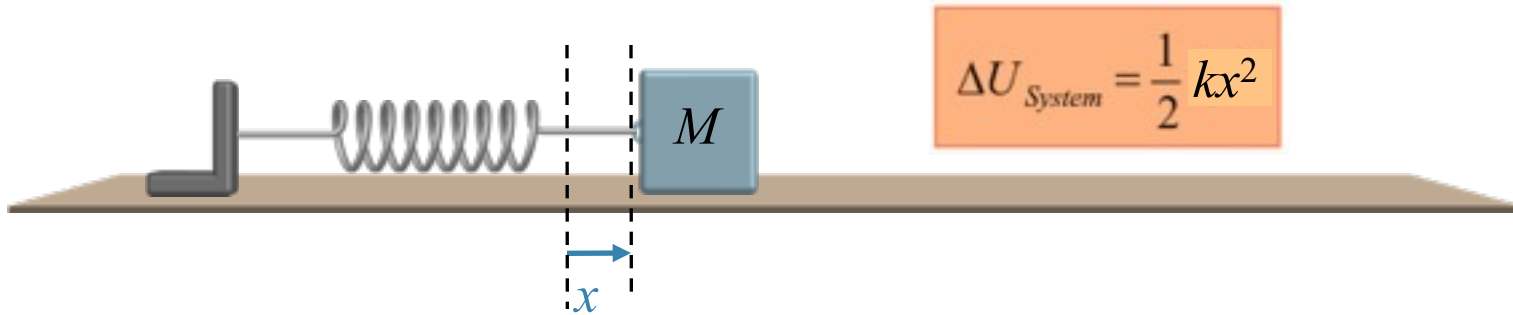
A) $x_2 = \sqrt{2}x_1$ B) $x_2 = 2x_1$ C) $x_2 = 4x_1$

- A) the formula is $\frac{1}{2}kx^2$ so it would be the square root two when the equation is rearranged
- B) Since both the velocity and distance variables are squared in the kinetic energy and spring potential energy equation, double velocity also doubles extension.
- C) The velocity is squared so it will be 4 times more distance.

Box and Spring: Question 1 (N = 90)



Spring Summary



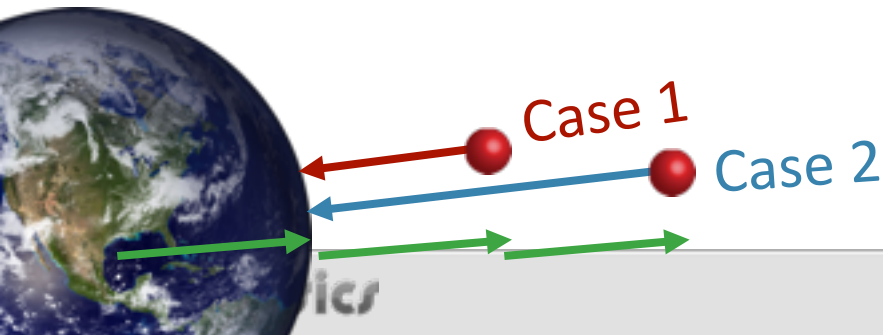
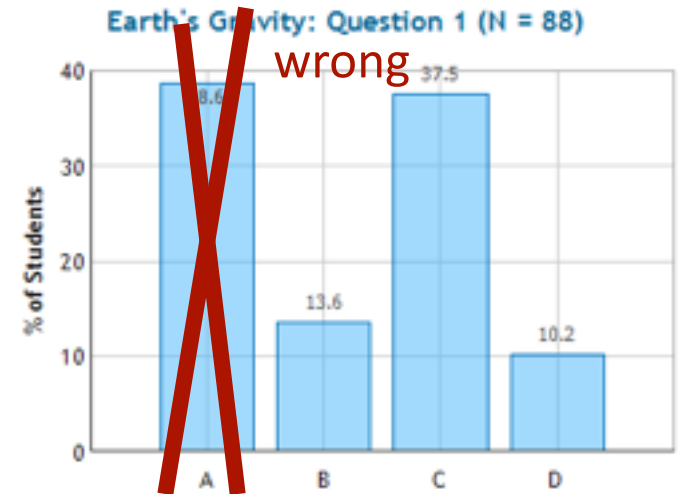
CheckPoint

In **Case 1** we release an object from a height above the surface of the earth equal to **1 earth radius**, and we measure its kinetic energy just before it hits the earth to be K_1 .

In **Case 2** we release an object from a height above the surface of the earth equal to **2 earth radii**, and we measure its kinetic energy just before it hits the earth to be K_2 .

Compare K_1 and K_2 .

- A) $K_2 = 2K_1$
- B) $K_2 = 4K_1$
- C) $K_2 = 4K_1/3$
- D) $K_2 = 3K_1/2$



Clicker Question



For gravity: $U(r) = -\frac{GM_em}{r} + \cancel{U_0}$

What is the potential energy of an object of mass m on the earth's surface:

A) $U_{surface} = -\frac{GM_em}{0}$

B) $U_{surface} = -\frac{GM_em}{R_E}$

C) $U_{surface} = -\frac{GM_em}{2R_E}$



Clicker Question

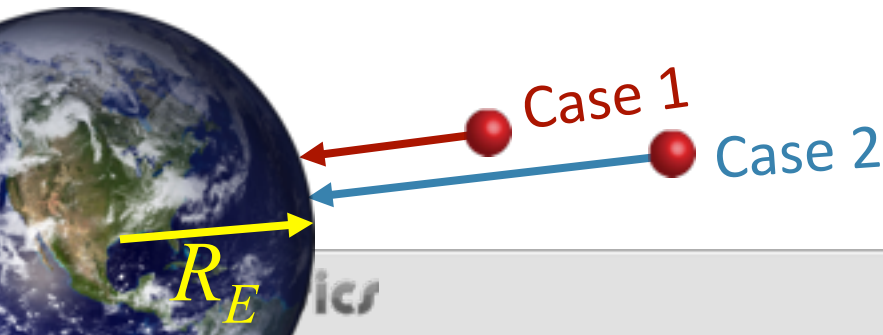
$$U(r) = -\frac{GM_em}{r}$$

What is the potential energy of a object starting at the height of **Case 1**?

A) $U_1 = -\frac{GM_em}{R_E}$

B) $U_1 = -\frac{GM_em}{2R_E}$

C) $U_1 = -\frac{GM_em}{3R_E}$



Clicker Question

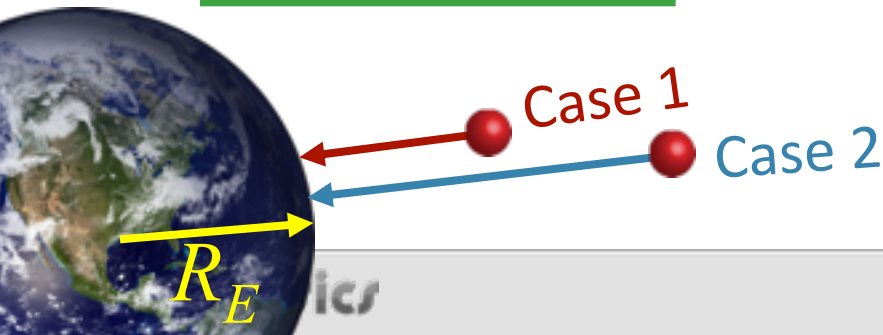
$$U(r) = -\frac{GM_em}{r}$$

What is the potential energy of a object starting at the height of **Case 2**?

A) $U_2 = -\frac{GM_em}{R_E}$

B) $U_2 = -\frac{GM_em}{2R_E}$

C) $U_2 = -\frac{GM_em}{3R_E}$

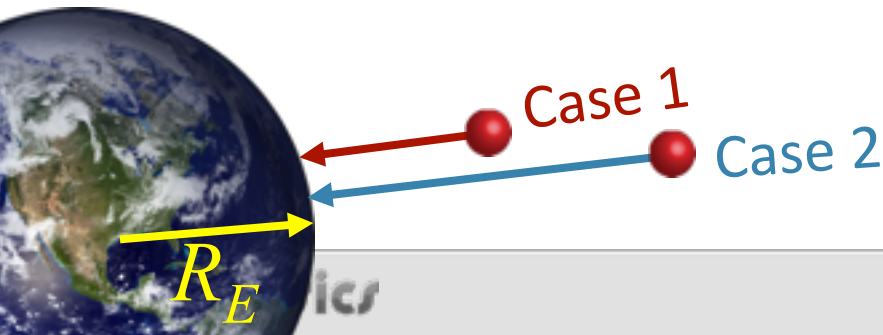


$$U_{\text{surface}} = -\frac{GM_em}{R_E} \quad U_1 = -\frac{GM_em}{2R_E} \quad U_2 = -\frac{GM_em}{3R_E}$$

What is the change in potential in **Case 1**?

$$\text{A) } \Delta U_{\text{case1}} = -GM_em \left(\frac{1}{2R_e} - \frac{1}{R_e} \right) = \frac{1}{2} \frac{GM_em}{R_e}$$

$$\text{B) } \Delta U_{\text{case1}} = -GM_em \left(\frac{1}{R_e} - \frac{1}{2R_e} \right) = \frac{-1}{2} \frac{GM_em}{R_e}$$

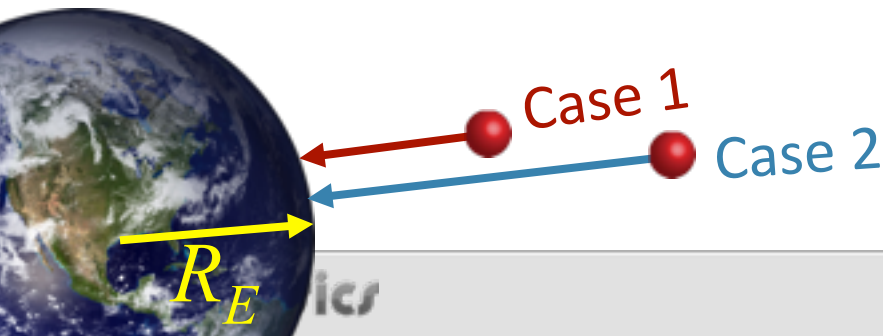


$$U_{\text{surface}} = -\frac{GM_e m}{R_E} \quad U_1 = -\frac{GM_e m}{2R_E} \quad U_2 = -\frac{GM_e m}{3R_E}$$

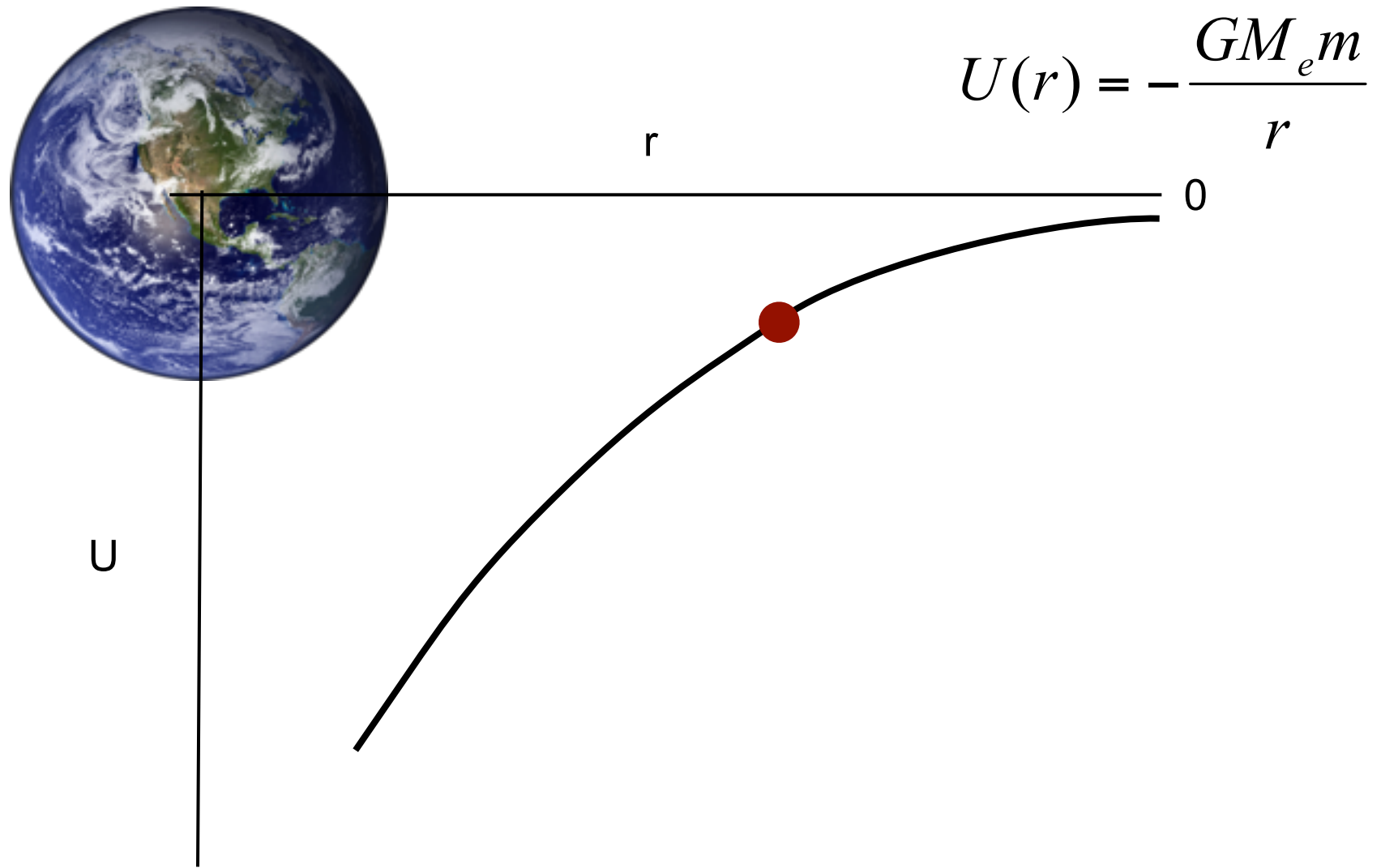
What is the change in potential in Case 2?

$$\text{A) } \Delta U_{\text{case2}} = -GM_e m \left(\frac{1}{3R_e} - \frac{1}{R_e} \right) = \frac{2}{3} \frac{GM_e m}{R_e}$$

$$\text{B) } \Delta U_{\text{case2}} = -GM_e m \left(\frac{1}{R_e} - \frac{1}{3R_e} \right) = \frac{-2}{3} \frac{GM_e m}{R_e}$$



Draw U



$$\Delta U_{case1} = -\frac{GM_em}{2R_e} \quad \Delta U_{case2} = -\frac{2GM_em}{3R_e}$$

What is the ratio $\frac{\Delta K_2}{\Delta K_1} = \frac{\Delta U_2}{\Delta U_1}$

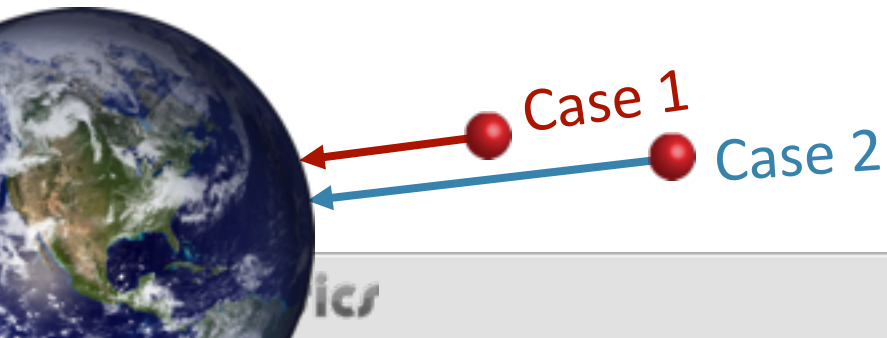
$$= \frac{-2/3}{-1/2} = \frac{4}{3}$$

A) 2

B) 4

C) 4/3

D) 3/2



CheckPoint

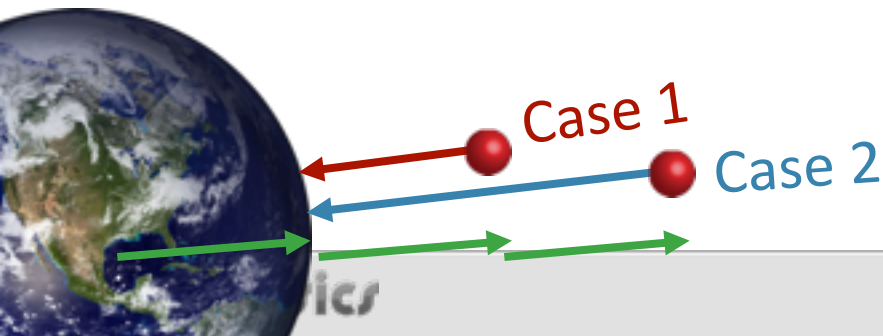
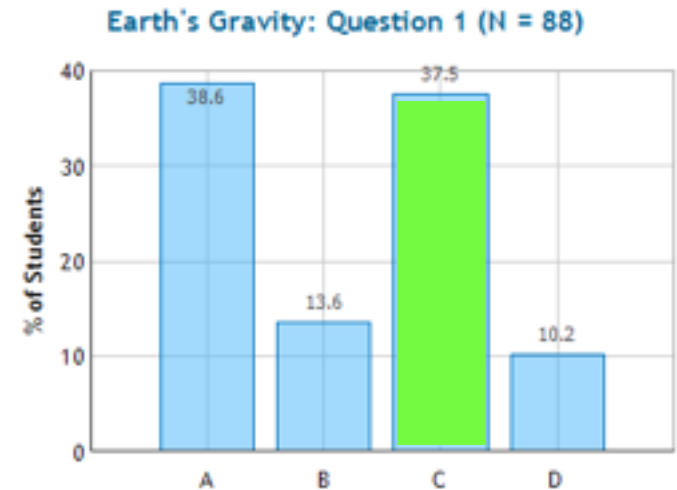
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Jason's Explanation

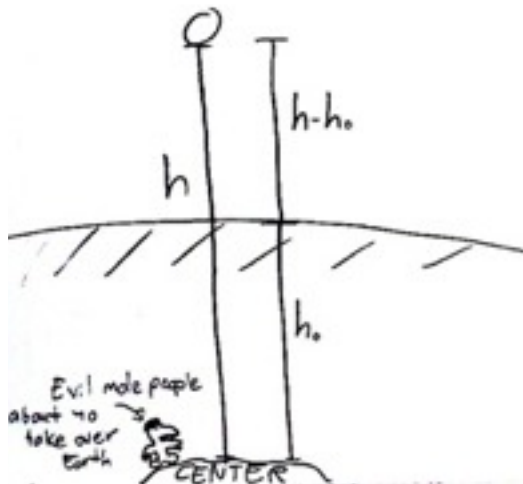
Earth's Surface

$$U_g = -mgh + U_0$$

say $U_0 = mgh_0$

$$U_g = -mgh + mgh_0$$

$$U_g = -mg(h - h_0)$$



General

$$U_g = -G \frac{Mm}{r} + U_0$$

say $U_0 = G \frac{Mm}{r_0}$

$$U_g = -G \frac{Mm}{r} + G \frac{Mm}{r_0}$$

$$U_g = -GMm \left(\frac{1}{r} - \frac{1}{r_0} \right) \neq -GMm \left(\frac{1}{r-r_0} \right)$$

