Classical Mechanics
Lecture 10

Today's Concept:

Center of Mass

- Finding it
- Using it

Your exams will be returned in your tutorial.

We'll discuss the exam on Friday.

See me if you forgot to put your name on the exam.

Please make sure you've registered your iClicker.

Go to webct or http://www.sfu.ca/phys/120/1131/iclicker

Your Comments

Center of mass just got a little more difficult. What is r relative to? The origin? Do we define this origin? Also will we actually have to do 3D center of mass problems? If so that would make me sad, very sad.

Just because the prelecture guy tells me integration is easy and does it in a happy voice doesn't make it so. If we could go over how the CM velocity and CM acceleration formulas are found from the position that would be awesome

I want to do more number examples, especially with finding the center of mass of objects with integrals. We do too much variable stuff, I need to see some numbers.

Id like to have a doughnut in my center of mass

why doesn't the moon crash into the earth, causing widespread panic and destruction?

The prelecture was pretty easy. I found the triple integral to be a bit tedious. But I throughly enjoyed the doughnut problem and to solve it I had to go out and get a doughnut for it to make sense, altough I am looking forward to the lecture in hopes that there might be doughnuts :)

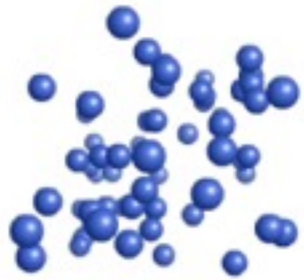
¡qué lástima!

Center of Mass

Center of Mass

(for Discrete Distributions)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} m_i \vec{r}_i$$

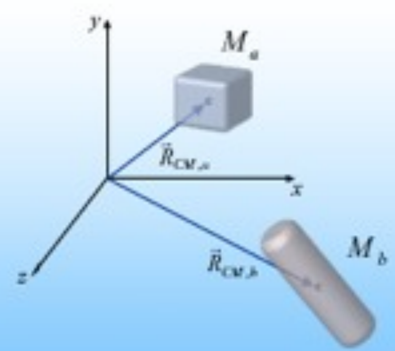


(for Continuous Distributions)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r}\, dm$$

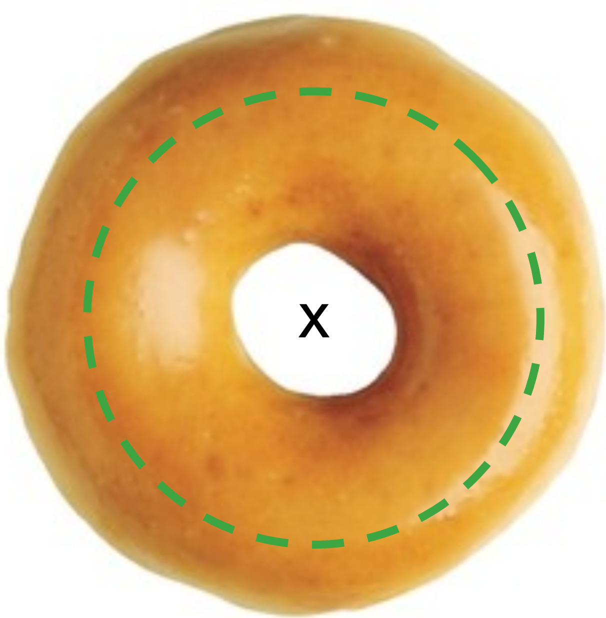
(for System of Solid Objects)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} M_i \vec{R}_{CM,i}$$



CheckPoint

A yummy glazed doughnut is shown below. Where is the center of mass of this fantastic culinary delight?

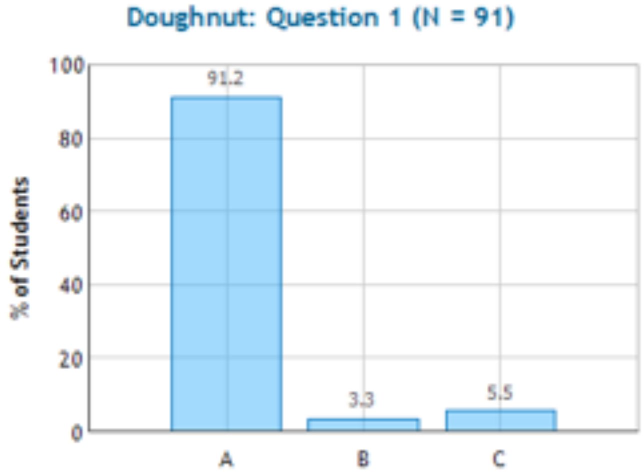


A) In the center of the hole.

B) Anywhere along the blue dashed line going through the solid part of the dough.

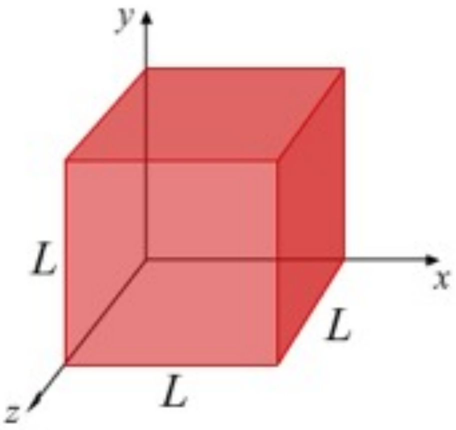
C) The center of mass is not defined in cases where there is missing mass.

The yummy doughnut is round and has equal weight distribution all over the doughnut, therefore, the center of mass must be in the middle



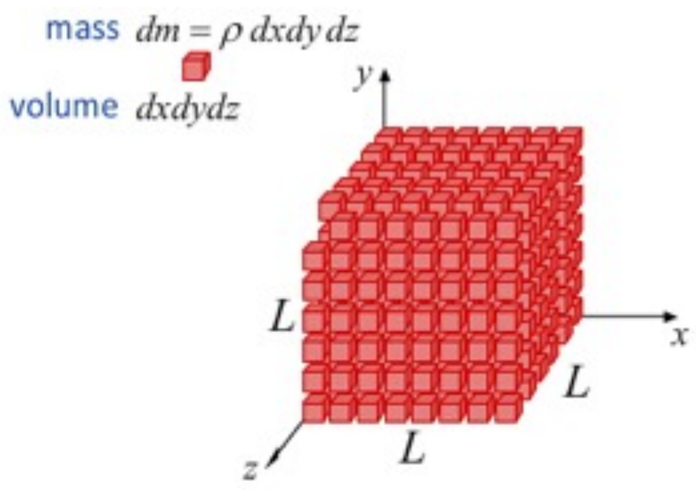
Cube Example

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r}\, dm$$

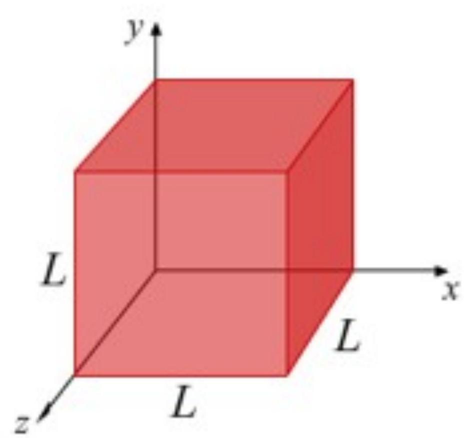


Cube Example: Using Volume Mass Density

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r}\, dm$$



$$\vec{R}_{CM} = \frac{1}{M_{Total}} \iiint \vec{r}\, \rho\, dxdydz$$



Cube Example: CM Calculation

Center of Mass
(for Continuous Distributions)

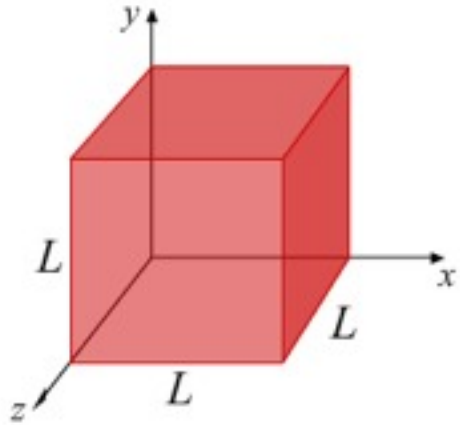
$$X_{CM} = \frac{1}{M_{Total}} \rho \int_0^L \int_0^L \int_0^L x\, dx dy dz$$

$$= \frac{1}{M_{Total}} \rho \int_0^L x\, dx \int_0^L dy \int_0^L dz$$

$$= \frac{1}{M_{Total}} \rho \left(\frac{1}{2} x^2 \Big|_0^L \right) \left(y \Big|_0^L \right) \left(z \Big|_0^L \right)$$

$$= \frac{1}{M_{Total}} \rho \left(\frac{1}{2} L^2 \right) (L)(L)$$

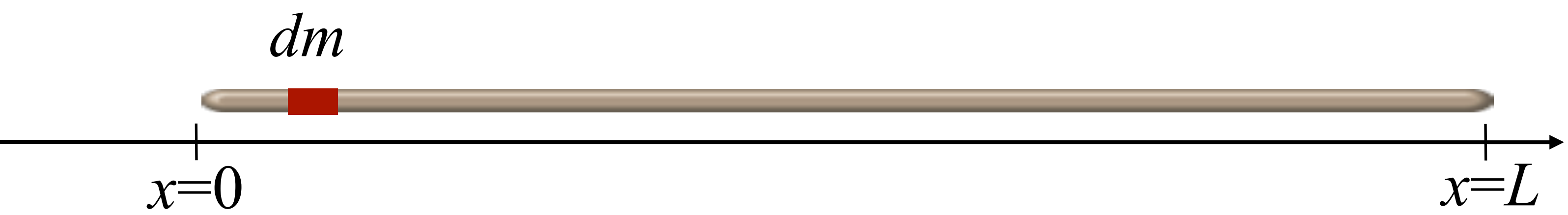
$$= \frac{1}{M_{Total}} \left(\frac{L}{2} \right) (\rho L^3)$$



Example

A uniform stick of length L and mass M.

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r}\, dm$$



$$dm = M\frac{dx}{L} = \frac{M}{L}dx$$

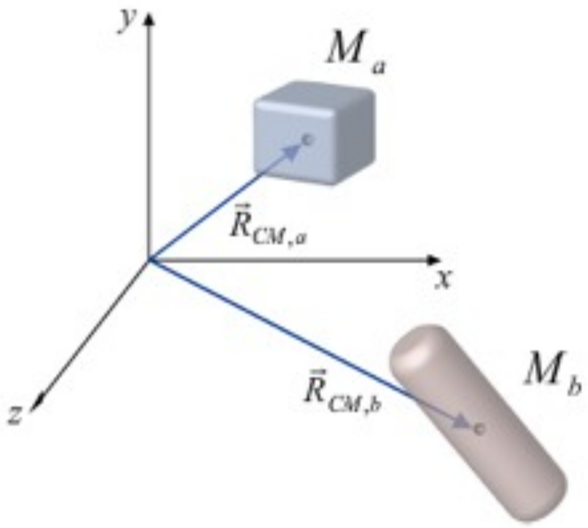
$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\frac{M}{L}dx = \frac{1}{L}\int_0^L x\,dx$$

$$= \frac{1}{L}\left(\frac{1}{2}x^2\right)_0^L = \frac{1}{L}\frac{1}{2}\left(L^2 - 0^2\right) = \frac{L}{2}$$

Center of Mass for System of Objects

Center of Mass
(for System of Solid Objects)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} M_i \vec{R}_{CM,i}$$



Meter Stick Demo

Where is the center of mass? (in cm)

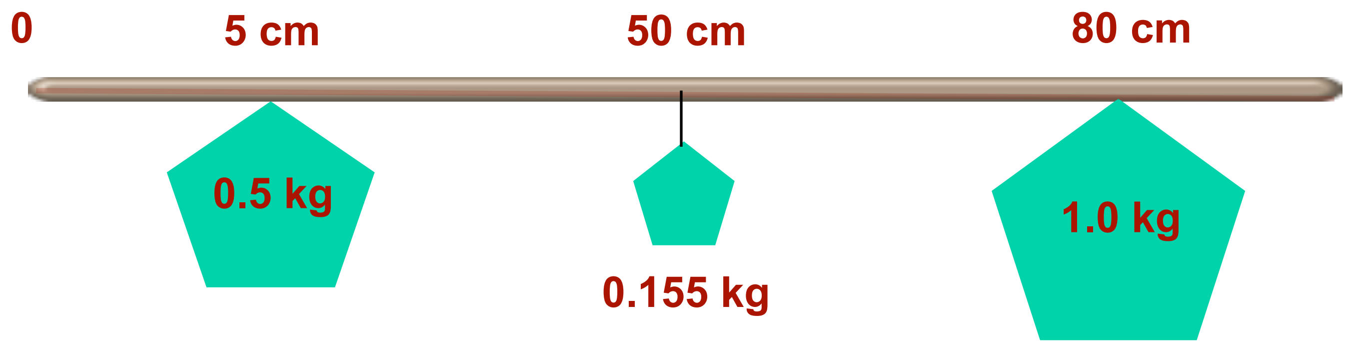
Estimate (or guess).

$$x_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

=(.5*5 + 1*80)/1.5 = 55.0 cm

Try it

Center of mass including meter stick



Meter stick is 0.155 kg. This is not quite negligible.

Consider it to be a point object put at its c.m.

$$x_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

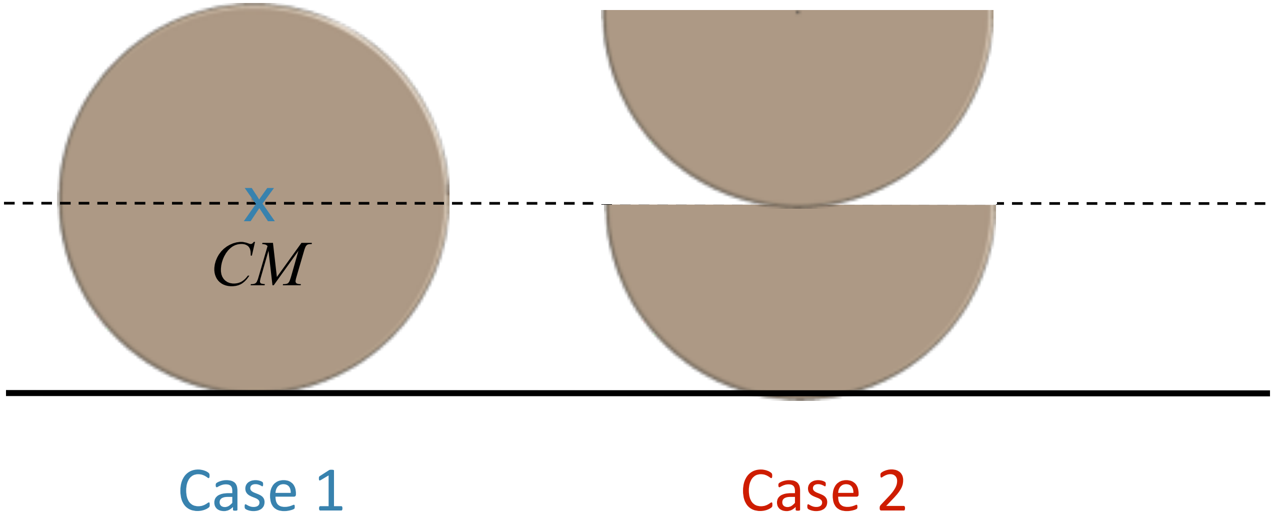
= (.5*5 + 1*80 + 0.155*50)/(1.5+0.155) = 54.5 cm

Clicker Question

The disk shown in Case 1 clearly has its CM at the center. Suppose the disk is cut in half and the pieces arranged as shown in Case 2

In which case is the center of mass highest?

A) Case 1 B) Case 2 C) same



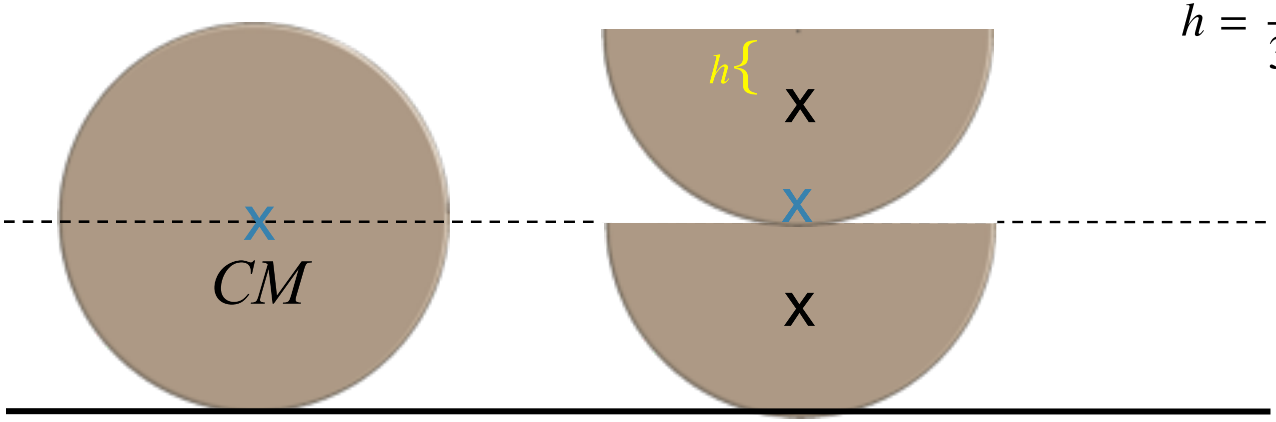
Clicker Question

The disk shown in Case 1 clearly has its CM at the center. Suppose the disk is cut in half and the pieces arranged as shown in Case 2

In which case is the center of mass highest?

A) Case 1 B) Case 2 C) same

$$h = \frac{4r}{3\pi} < \frac{r}{2}$$



Case 1 Case 2

Kinematic Quantities

Displacement of Center of Mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M_{Total}}$$

Velocity of Center of Mass

$$\vec{V}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M_{Total}}$$

Acceleration of Center of Mass

$$\vec{A}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M_{Total}}$$

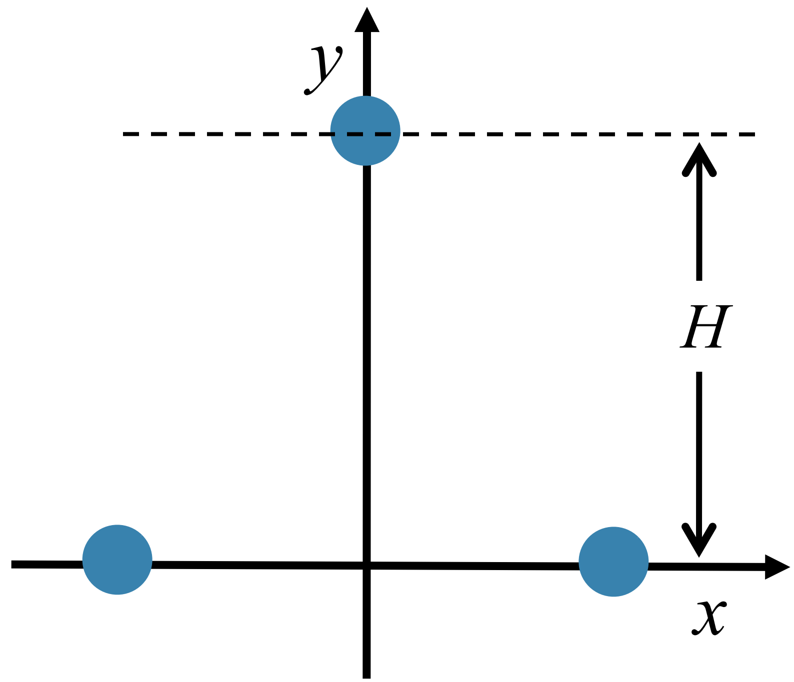
$$\vec{A}_{CM} = \frac{\vec{F}_{Net,External}}{M_{Total}}$$

If the total force is zero, the CM won't accelerate

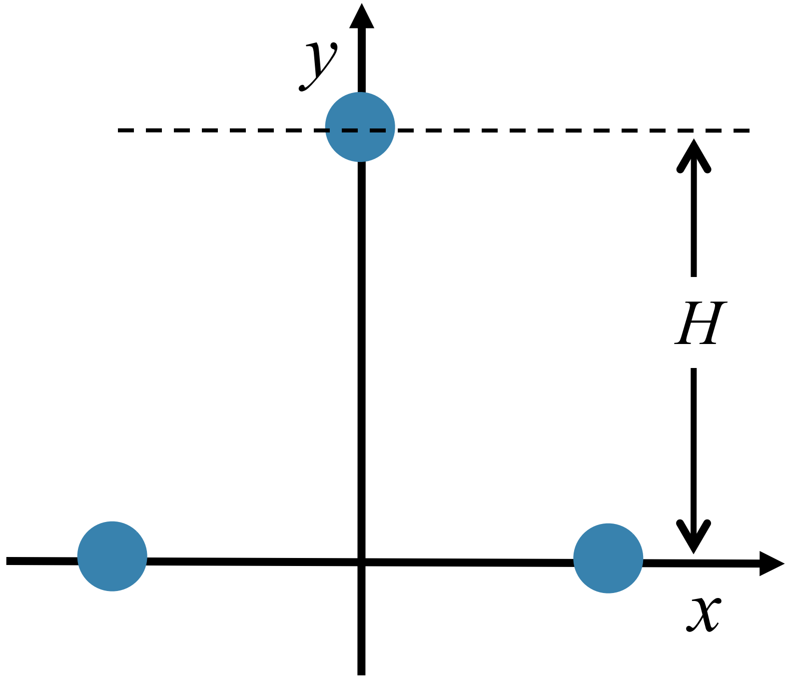
Clicker Question

Three tiny equal-mass objects are placed in interstellar space at the corners of an equilateral triangle. When the masses are released, they attract and quickly slide to a single point. What are the coordinates of that point?

	X_{CM}	Y_{CM}
A)	0	0
B)	0	$H/2$
C)	0	$H/3$
D)	$H/4$	$H/4$
E)	$H/4$	0



Three tiny equal-mass objects are placed in interstellar space at the corners of an equilateral triangle. When the masses are released, they attract and quickly slide to a single point. What are the coordinates of that point?



	X_{CM}	Y_{CM}
C)	0	$H/3$

$X_{CM} = 0$ (symmetry)

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{m \times 0 + m \times 0 + m \times H}{3m} = \frac{H}{3}$$

Homework Problem

System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

	m	x	y	v_x	v_y
1	7.8 kg	-2.8 m	-4.7 m	3.2 m/s	-4.2 m/s
2	7.8 kg	-3.7 m	3.7 m	-5.2 m/s	5.2 m/s
3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

1) What is the x position of the center of mass? m Submit

2) What is the y position of the center of mass? m Submit

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4} = 0.975 \text{ m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4} = -1 \text{ m}$$

Homework Problem

System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

	m	x	y	v_x	v_y
1	7.8 kg	-2.8 m	-4.7 m	3.2 m/s	-4.2 m/s
2	7.8 kg	-3.7 m	3.7 m	-5.2 m/s	5.2 m/s
3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

3) What is the speed of the center of mass? m/s Submit

$$v_{cm,x} = \frac{m_1 v_{1,x} + m_2 v_{2,x} + m_3 v_{3,x} + m_4 v_{4,x}}{m_1 + m_2 + m_3 + m_4}$$

$$v_{cm,y} = \frac{m_1 v_{1,y} + m_2 v_{2,y} + m_3 v_{3,y} + m_4 v_{4,y}}{m_1 + m_2 + m_3 + m_4}$$

$$v_{cm} = \sqrt{v_{cm,x}^2 + v_{cm,y}^2}$$

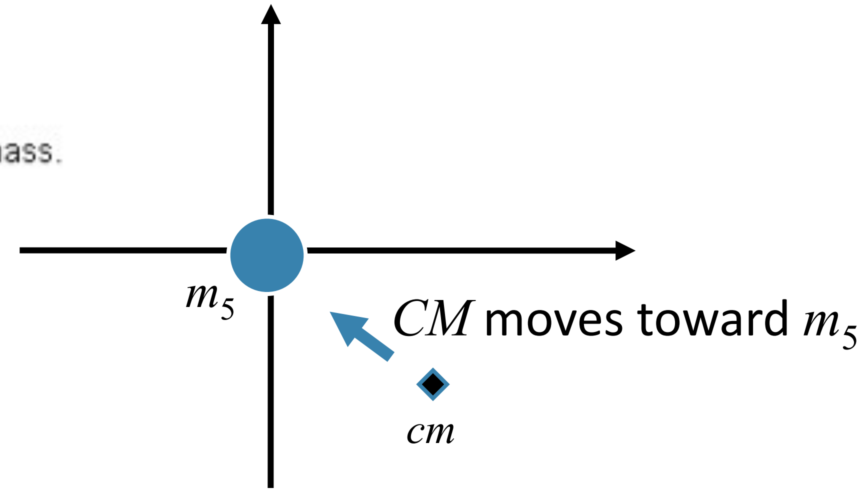
System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

	m	x	y	v_x	v_y
1	7.8 kg	-2.8 m	-4.7 m	3.2 m/s	-4.2 m/s
2	7.8 kg	-3.7 m	3.7 m	-5.2 m/s	5.2 m/s
3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

4) When a fifth mass is placed at the origin, what happens to the horizontal (x) location of the center of mass?

A ○ It moves to the right.
B ○ It moves to the left.
C ○ It does not move.
D ○ It can not be determined unless you know the mass.



System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

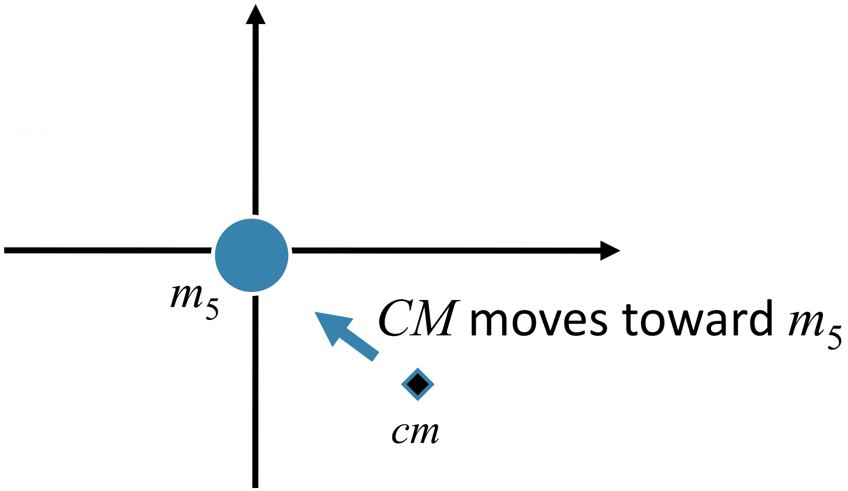
	m	x	y	v_x	v_y
1	7.8 kg	-2.8 m	-4.7 m	3.2 m/s	-4.2 m/s
2	7.8 kg	-3.7 m	3.7 m	-5.2 m/s	5.2 m/s
3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

5) When a fifth mass is placed at the center of mass, what happens to the vertical (y) location of the center of mass?

A ○ It moves up.

B ○ It moves down.

C ○ It does not move.



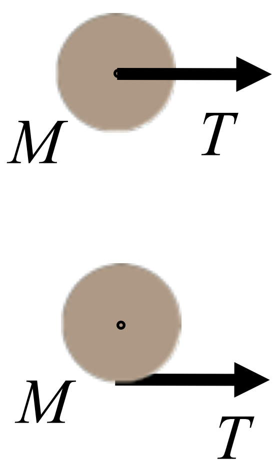
Clicker Question

Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?

A) Puck 1 B) Puck 2 C) Same

1)

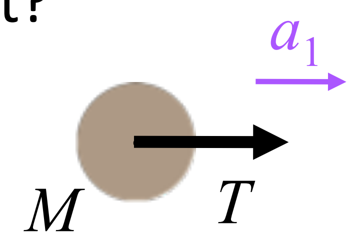
2)



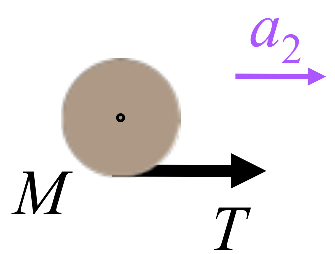
Clicker Question

Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?

1)



2)



C) Same

$$\vec{a}_{CM} = \frac{\vec{F}_{Net,External}}{M_{Total}}$$

SAME

CheckPoint

Two objects, one having twice the mass of the other, are initially at rest. Two forces, one twice as big as the other, act on the objects in opposite directions as shown.

$$\vec{a}_{CM} = \frac{\vec{F}_{Net,External}}{M_{Total}}$$

Which of the following statements about the acceleration of the center of mass of the system is true:

A) $a_{CM} = F/M$ to the right

B) $a_{CM} = F/(3M)$ to the right

C) $a_{CM} = 0$

D) $a_{CM} = F/(3M)$ to the left

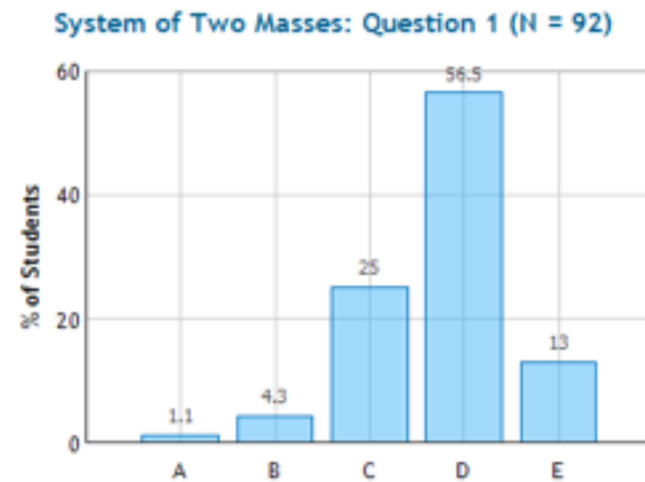
E) $a_{CM} = F/M$ to the left

CheckPoint

$$\vec{a}_{CM} = \frac{\vec{F}_{Net,External}}{M_{Total}}$$

Which of the following statements about the acceleration of the center of mass of the system is true:

C) $a_{CM} = 0$
D) $a_{CM} = F/(3M)$ to the left
E) $a_{CM} = F/M$ to the left



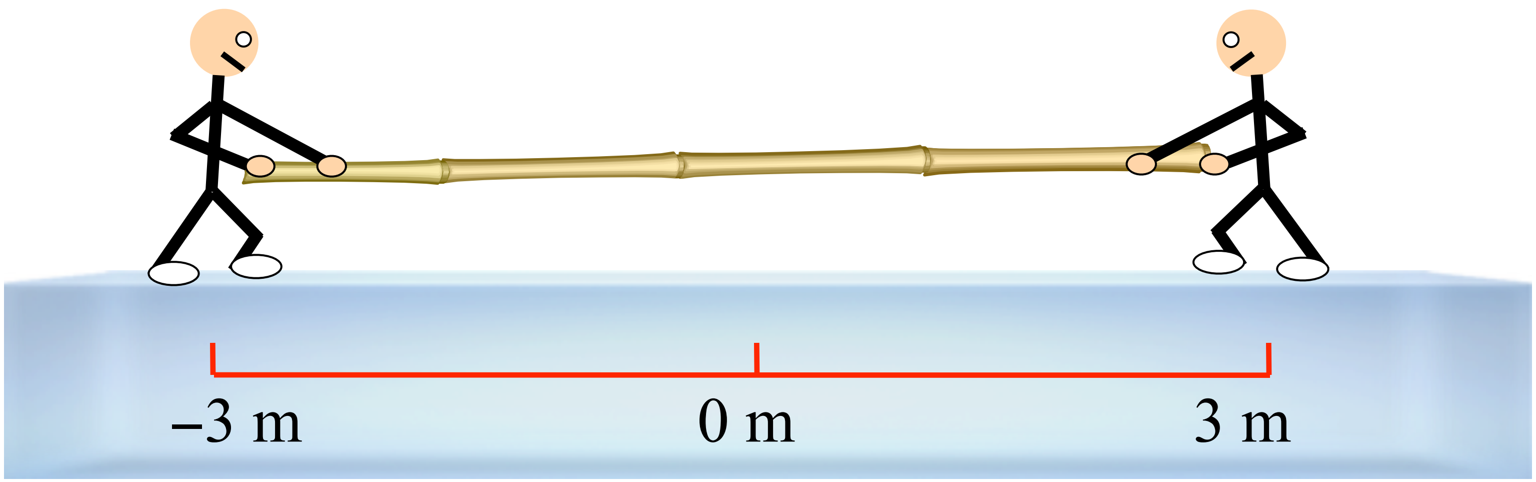
C) $F=Ma$. The $2M$ and the $2F$ would cancel each other out, making the bigger object accelerate at the same rate as the smaller ball. Therefore, the acceleration would be zero.

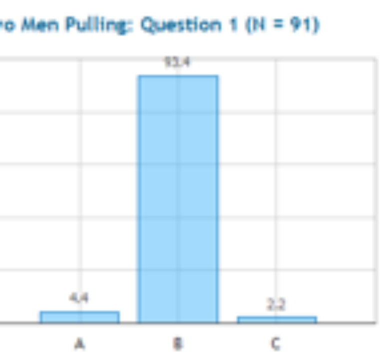
D) Acceleration is equal to the net force divided by the total mass. The net force is F to the left and the total mass is $3M$

E) $A_{CM} = F\,\text{total}/M\,\text{total} = 3F/3M = F/M$

CheckPoint

Two guys who weight the same are holding onto a massless pole while standing on horizontal frictionless ice. If the guy on the left starts to pull on the pole, where do they meet?





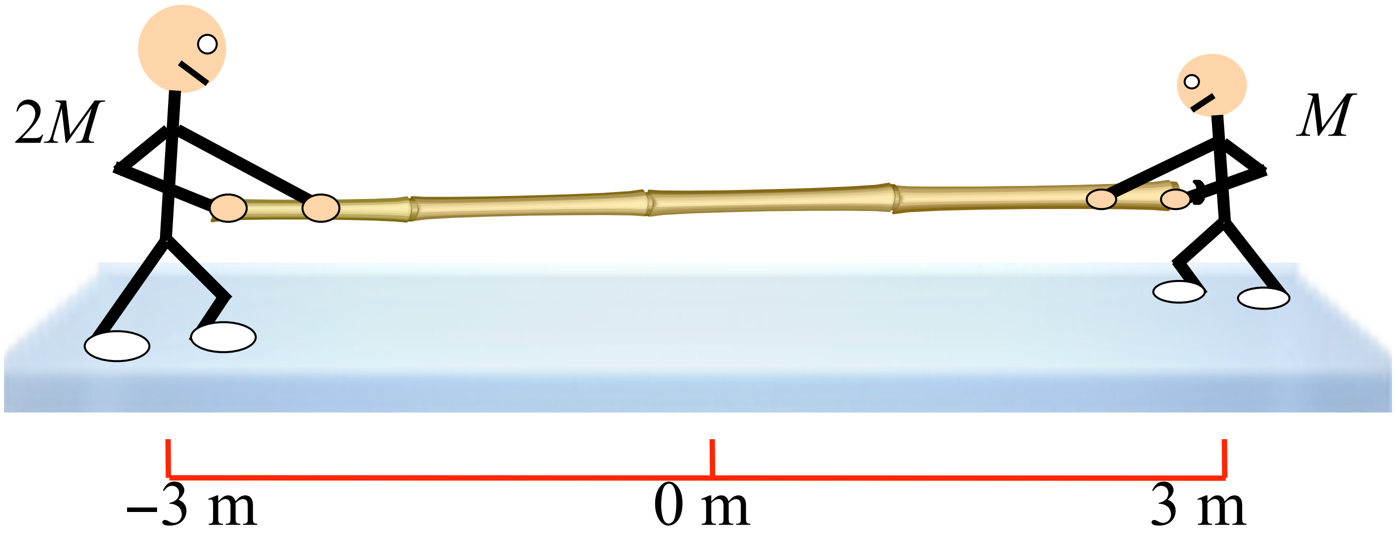
A) –3 m

B) 0 m

C) 3 m

Clicker Question

A large skinny guy with mass $2M$ and a smaller guy with mass M are holding onto a massless pole while standing on frictionless ice, as shown below. If the little guy pulls himself toward the big guy, where would they meet?



A) –3 m

B) –1 m

C) 0 m

D) 1 m

E) 3 m

A large skinny guy with mass $2M$ and a smaller guy with mass M are holding onto a massless pole while standing on frictionless ice, as shown below. If the little guy pulls himself toward the big guy, where would they meet?

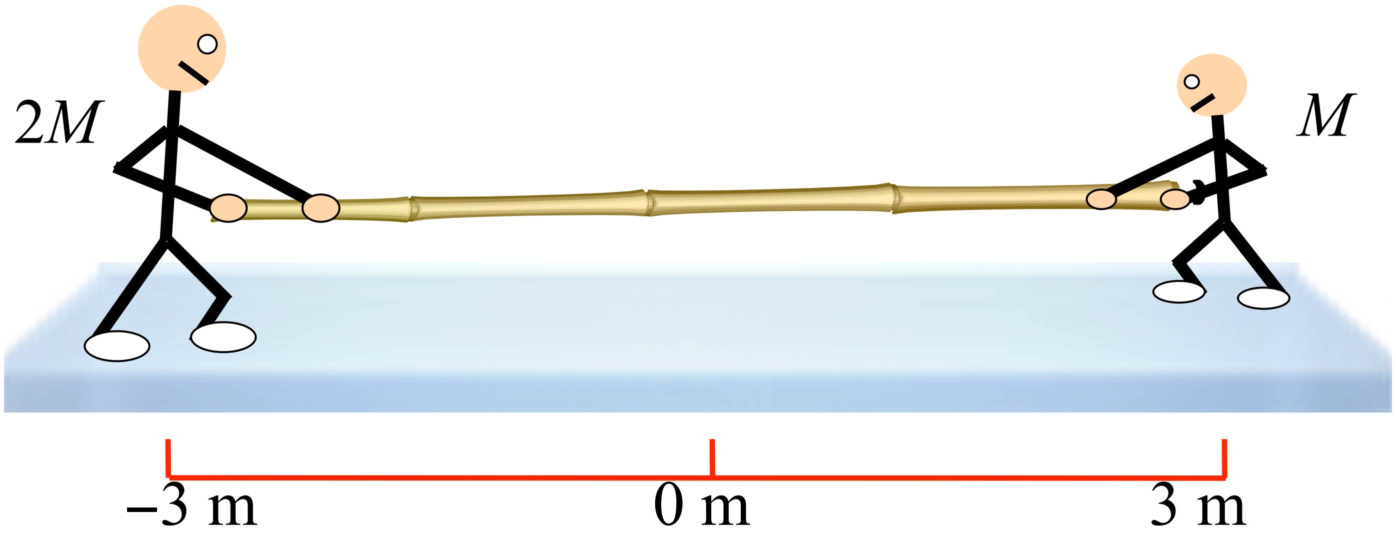
A) –3 m

B) –1 m

C) 0 m

D) 1 m

E) 3 m



$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2M(-3m) + M(3m)}{3M} = -1 \text{ m}$$