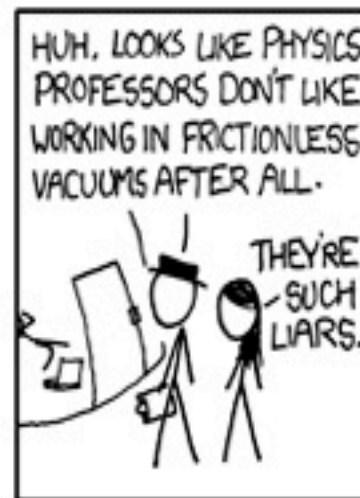
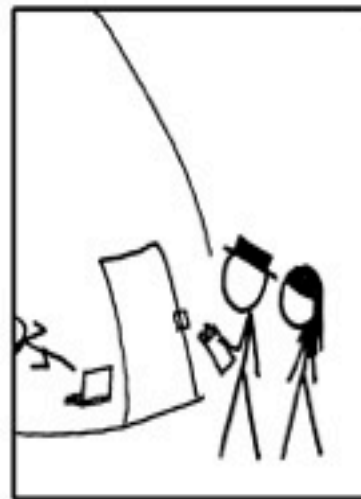
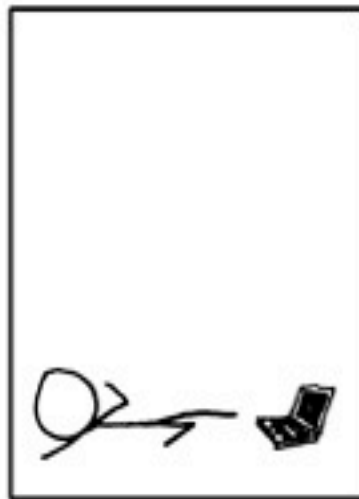
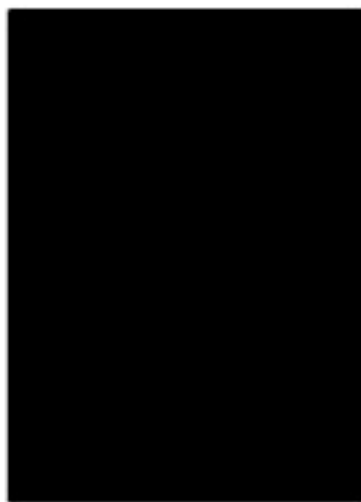


Classical Mechanics

Lecture 15

Today's Concepts:

- a) Parallel Axis Theorem
- b) Torque & Angular Acceleration



Your Comments

How the h3ll can torque, angular velocity and angular acceleration be pointing in a direction that they're not acting in?

I don't like the guy in the video

Try captions or the script.
(Use text-to-speech.)

Parallel-axis theorem, but I think that was already planned so do your thaaaaaang.

I hate rotational inertia. It is the bane of my physics existence. Is there any easy way to find it?

Explain the moment of inertia in the different rotational axis. Which one will have the largest, smallest etc...

If the mass is farther from the axis of rotation then moment of inertia is larger.

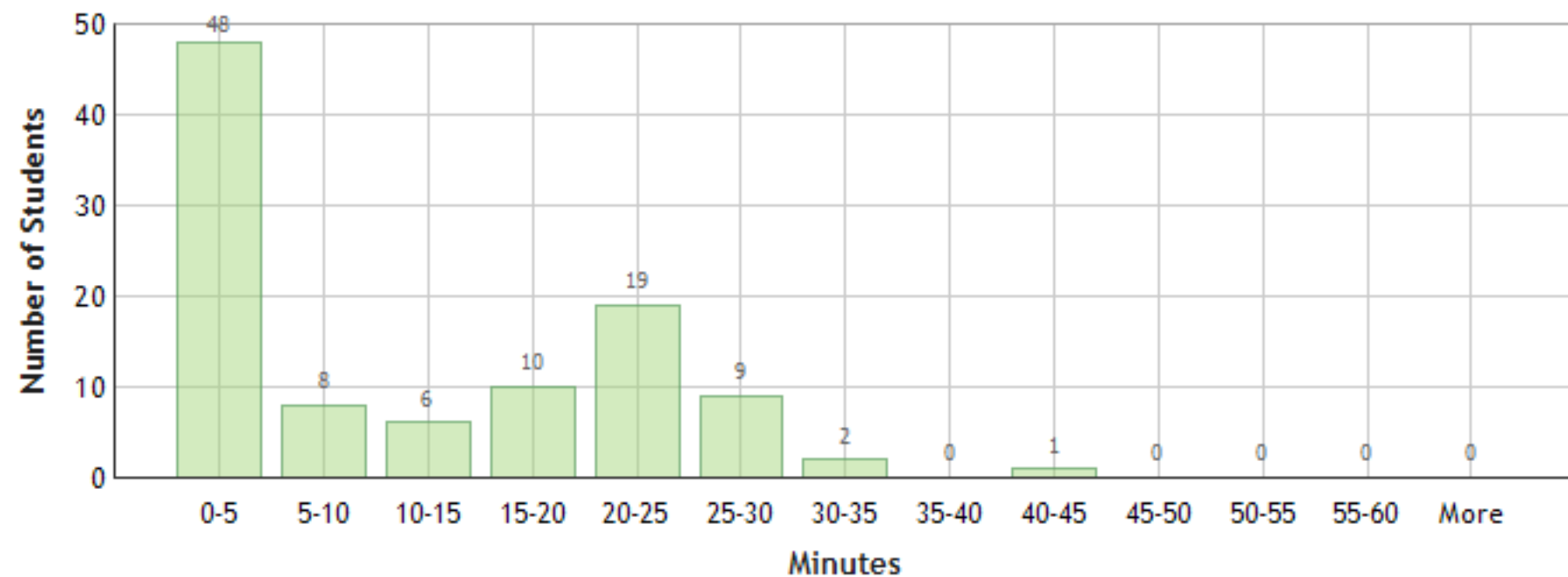
it is getting hard

Who ever said physics wasn't practical? I just learned the most efficient way to slam a door.

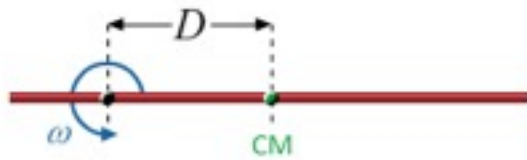
this is lots of fun!!!!

I'd love to give this course a thumbs up on the evaluation at the end of the semester, but if I can't figure out the direction of rotation it might as well become a thumbs down.

Time Spent Viewing Prelecture (N = 103)



Parallel Axis Theorem

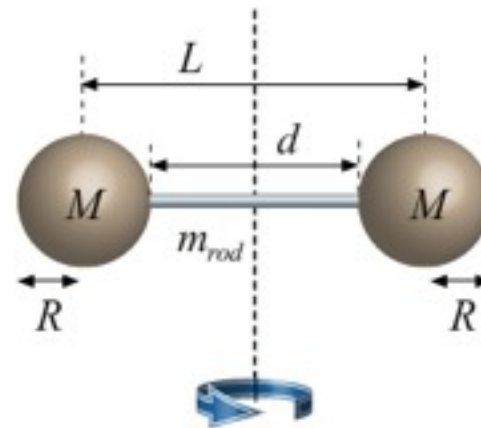


Parallel Axis Theorem

$$I_{Total} = MD^2 + I_{CM}$$

Smallest when $D = 0$

I honestly don't see the significance or use of the parallel axis theorem. I'm sure it's great and important but maybe you can convince me of its greatness. I feel this way because I feel like I can just calculate the moment of inertia in the same way we learned on Tuesday



$$I_{Dumbbell} = \frac{1}{12} m_{rod} d^2 + 2 \left[\frac{2}{5} MR^2 + M \left(\frac{L}{2} \right)^2 \right]$$

Clicker Question



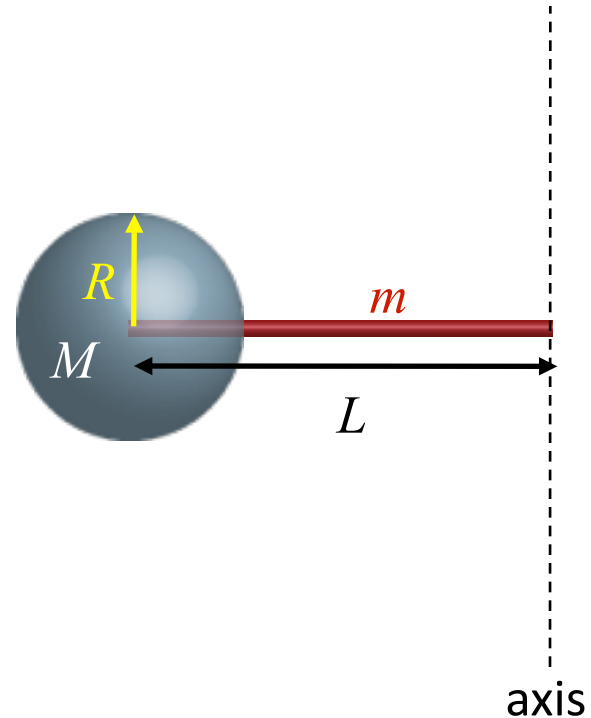
A solid ball of mass M and radius is connected to a rod of mass m and length L as shown. What is the moment of inertia of this system about an axis perpendicular to the other end of the rod?

A) $I = \frac{2}{5}MR^2 + ML^2 + \frac{1}{3}mL^2$

B) $I = \frac{2}{5}MR^2 + mL^2 + \frac{1}{3}ML^2$

C) $I = \frac{2}{5}MR^2 + \frac{1}{3}mL^2$

D) $I = ML^2 + \frac{1}{3}mL^2$

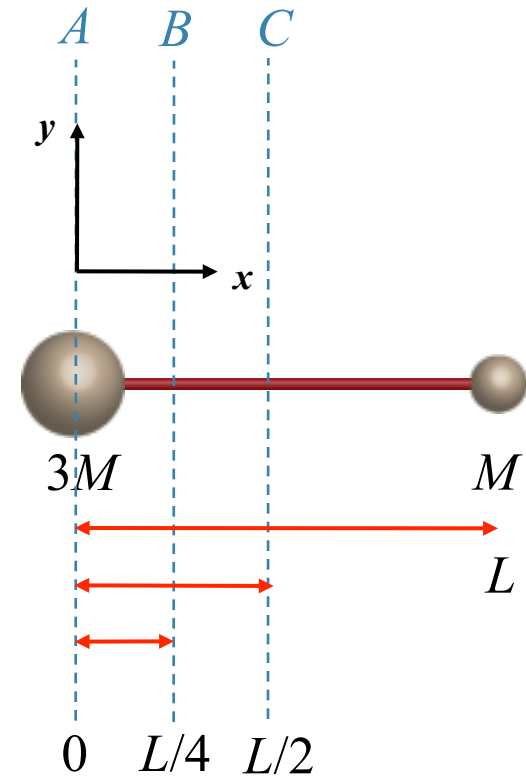


CheckPoint



A ball of mass $3M$ at $x = 0$ is connected to a ball of mass M at $x = L$ by a massless rod. Consider the three rotation axes A , B , and C as shown, all parallel to the y axis.

For which rotation axis is the moment of inertia of the object smallest? (It may help you to figure out where the center of mass of the object is.)



75% got this right

CheckPoint

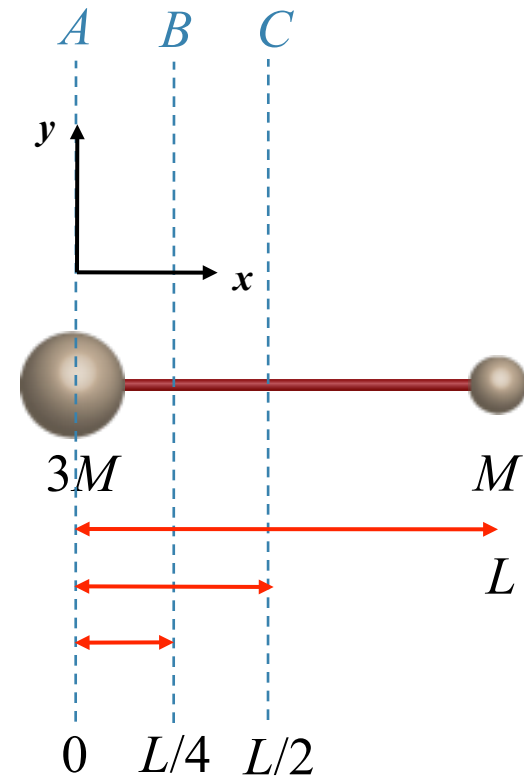
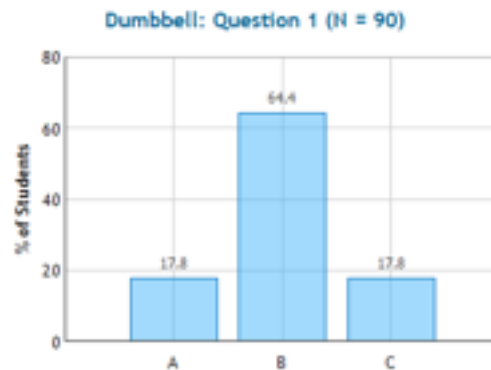


For which rotation axis is the moment of inertia of the object smallest?

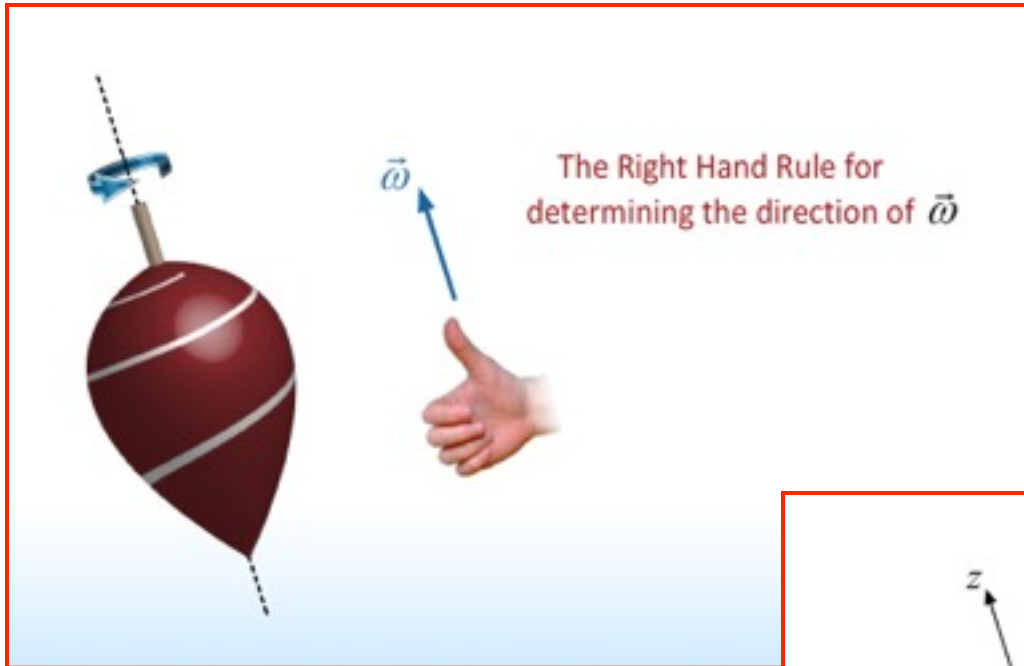
A) Less mass is rotating at point a.

B) The smallest moment of inertia is at the centre of mass which is closest to the most massive ball ($3M$).

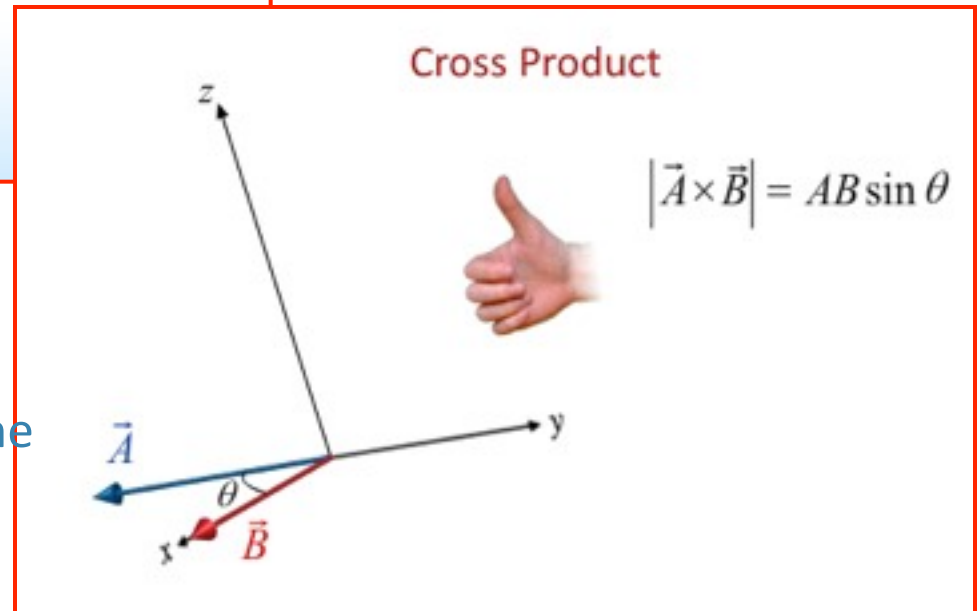
C) I did the equation on paper. This one was actually kind of tricky.



Right Hand Rule for finding Directions



Why do the angular velocity and acceleration point perpendicular to the plane of rotation?

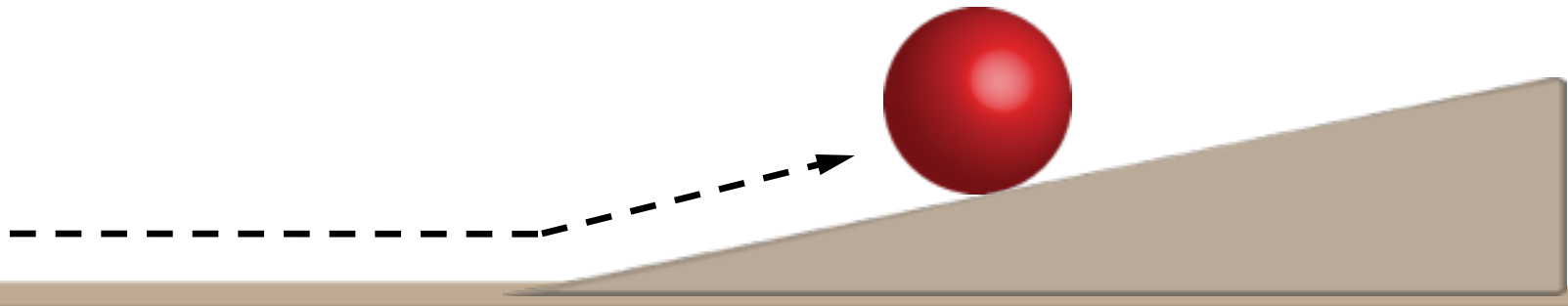


Clicker Question



A ball rolls across the floor, and then starts up a ramp as shown below. In what direction does the **angular velocity** vector point when the ball is rolling up the ramp?

- A) Into the page
- B) Out of the page
- C) Up
- D) Down

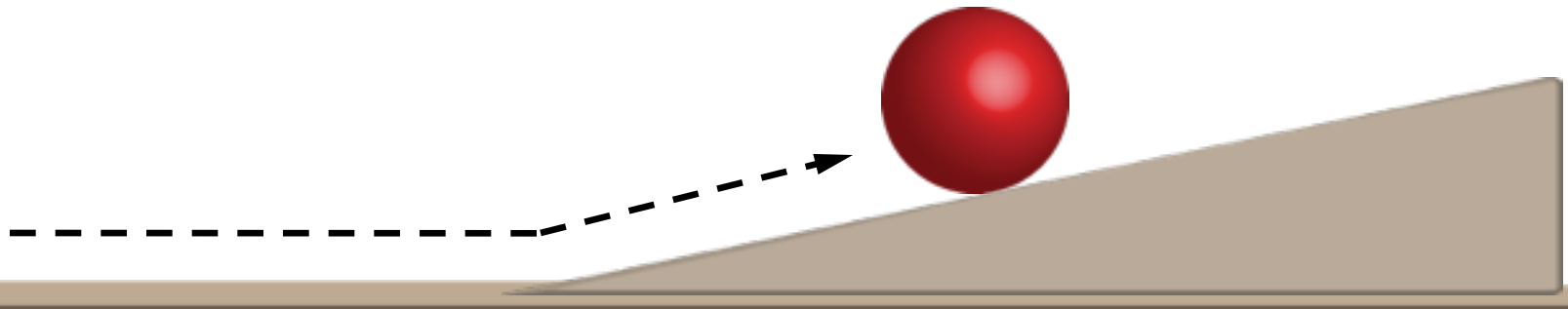


Clicker Question



A ball rolls across the floor, and then starts up a ramp as shown below. In what direction does the **angular acceleration** vector point when the ball is rolling up the ramp?

- A) Into the page
- B) Out of the page

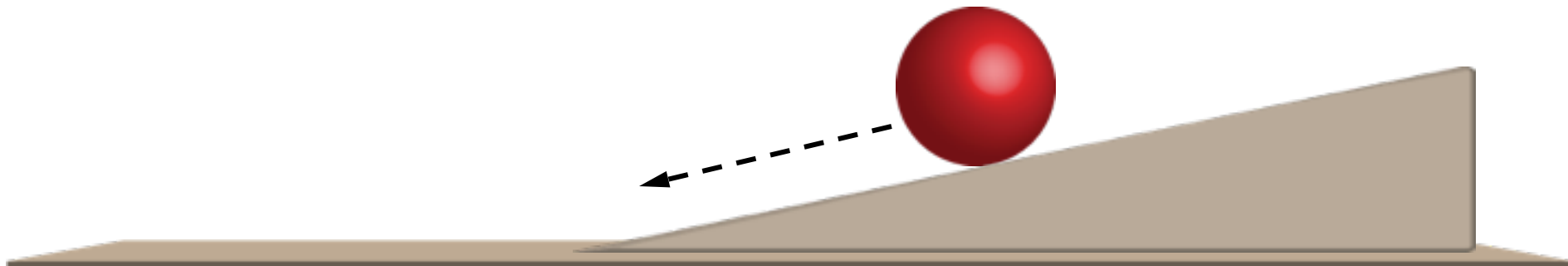


Clicker Question



A ball rolls across the floor, and then starts up a ramp as shown below. In what direction does the **angular acceleration** vector point when the ball is rolling **back down** the ramp?

- A) into the page
- B) out of the page



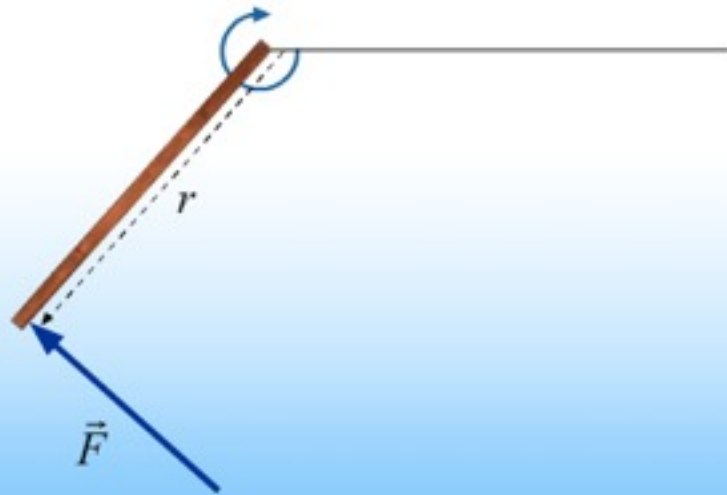
Torque

$$\tau = rF \sin(\theta)$$

Torque $\tau = rF_{\theta} = I\alpha$

Ways to close the door more quickly

1. Push harder
2. Push further from the hinge
3. Apply the push perpendicular to the door



CheckPoint

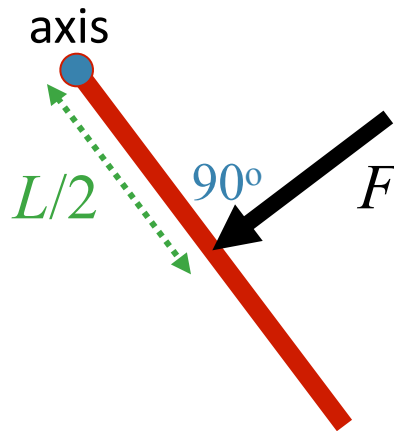
In **Case 1**, a force F is pushing perpendicular on an object a distance $L/2$ from the rotation axis. In **Case 2** the same force is pushing at an angle of 30 degrees a distance L from the axis.

In which case is the torque due to the force about the rotation axis biggest?

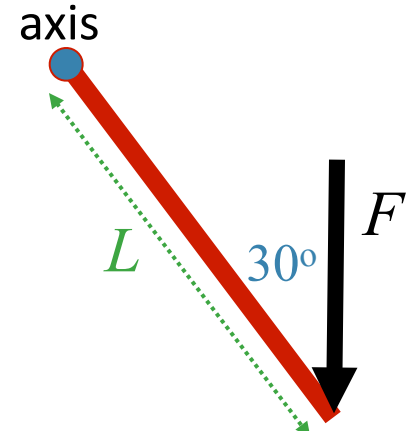
A) Case 1

B) Case 2

C) Same



Case 1



Case 2

64% got this right

CheckPoint

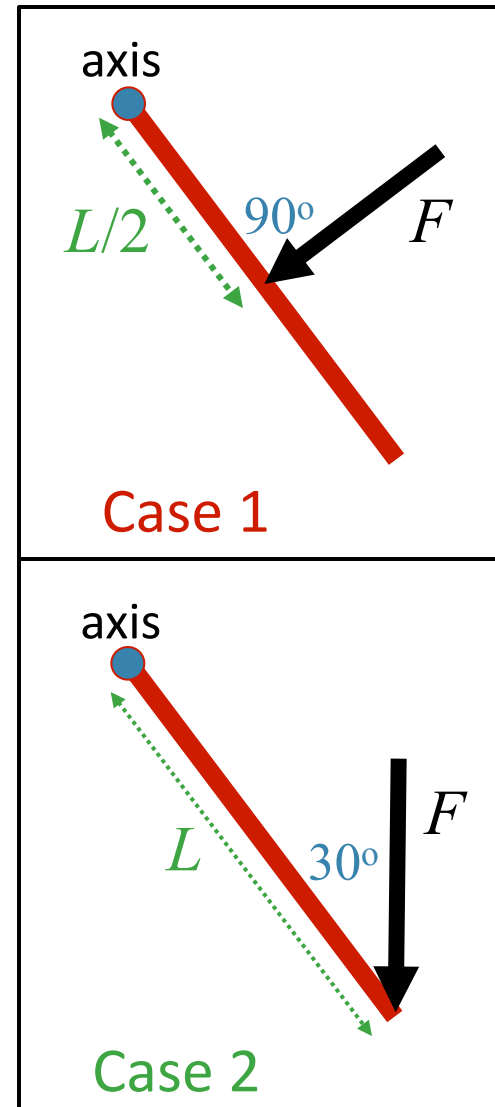
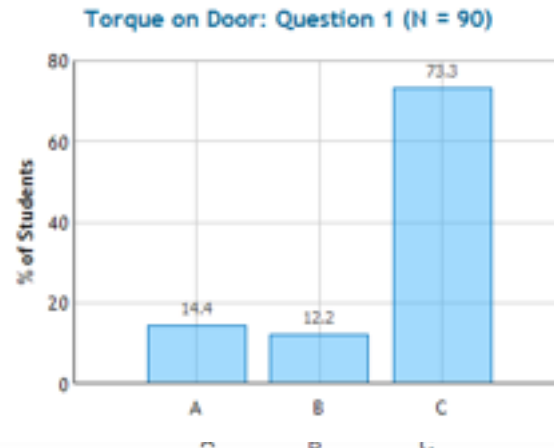
In which case is the torque due to the force about the rotation axis biggest?

A) Case 1 B) Case 2 C) Same

A) perpendicular force acting multiplied by perpendicular distance from the axis of rotation is torque

B) More of the force is parallel

C) For case 1, the magnitude of the torque is $F \cdot (L/2)$. For case 2, the magnitude of the torque is $F \cdot \sin(30 \text{ degrees}) \cdot L = F \cdot L/2$, which is exactly the same as the one in case 1.



Similarity to 1D motion

Linear Motion

Rotational Motion

$$x = x_o + v_o t + \frac{1}{2} a t^2 \longleftrightarrow \theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$v = v_o + a t \longleftrightarrow \omega = \omega_o + \alpha t$$

$$m \longleftrightarrow I$$

$$\frac{1}{2} m v^2 \longleftrightarrow \frac{1}{2} I \omega^2$$

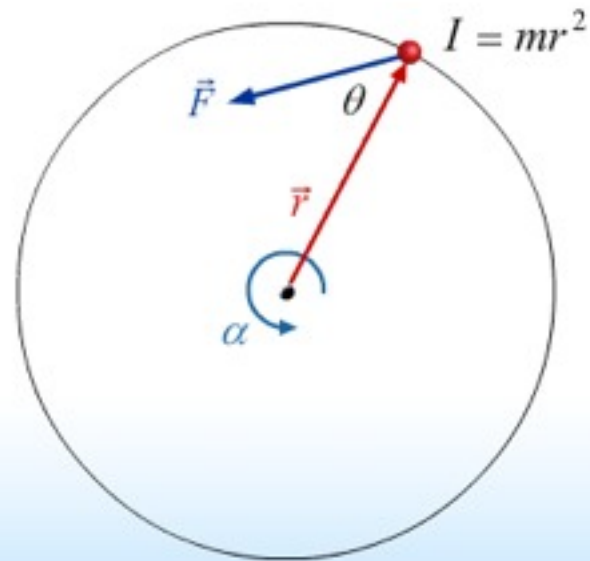
$$\vec{F} = m \vec{a} \longleftrightarrow \vec{\tau} = I \vec{\alpha}$$

Torque

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}| |\vec{F}| \sin \theta$$

$$\vec{\tau}_{\text{Net}} = I \vec{\alpha}$$



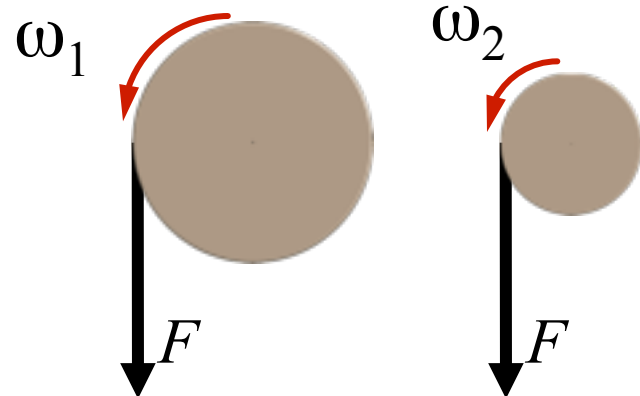
Clicker Question



Strings are wrapped around the circumference of two solid disks and pulled with identical forces. Disk 1 has a bigger radius, but **both have the same moment of inertia**.

Which disk has the biggest angular acceleration?

- A) Disk 1
- B) Disk 2
- C) same

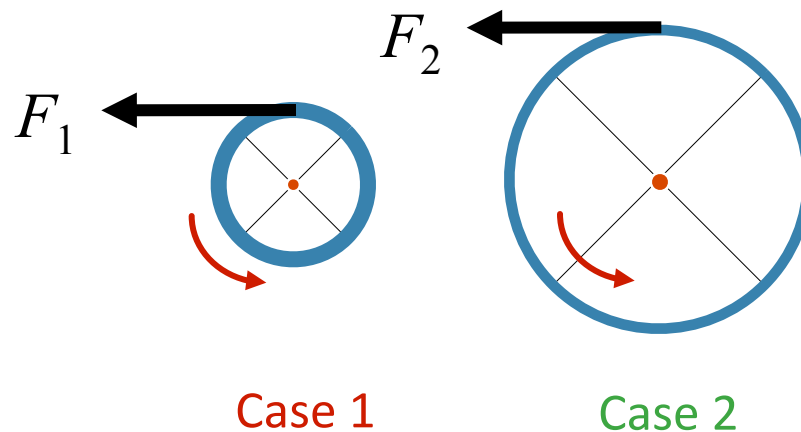


CheckPoint

Two hoops can rotate freely about fixed axles through their centers. The hoops have the **same mass**, but one has **twice the radius** of the other. Forces F_1 and F_2 are applied as shown.

How are the magnitudes of the two forces related if the angular acceleration of the two hoops is the same?

- A) $F_2 = F_1$
- B) $F_2 = 2F_1$
- C) $F_2 = 4F_1$



CheckPoint

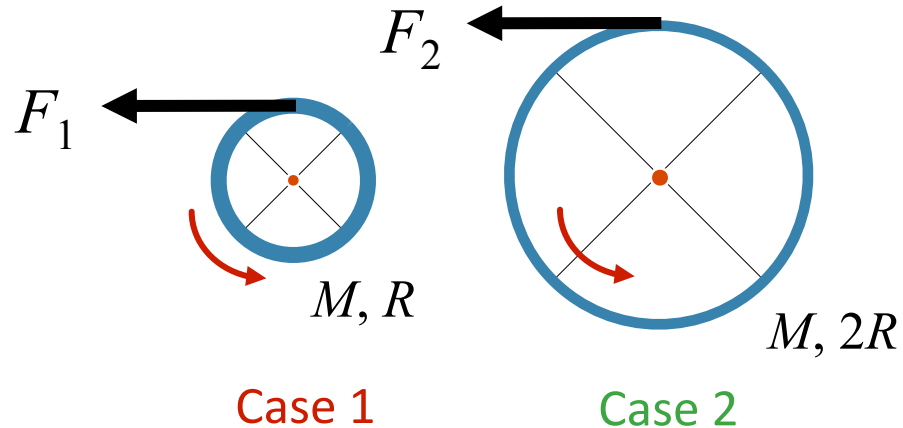


How are the magnitudes of the two forces related if the angular accelerations of the two hoops are the same?

A) $F_2 = F_1$

B) $F_2 = 2F_1$

C) $F_2 = 4F_1$



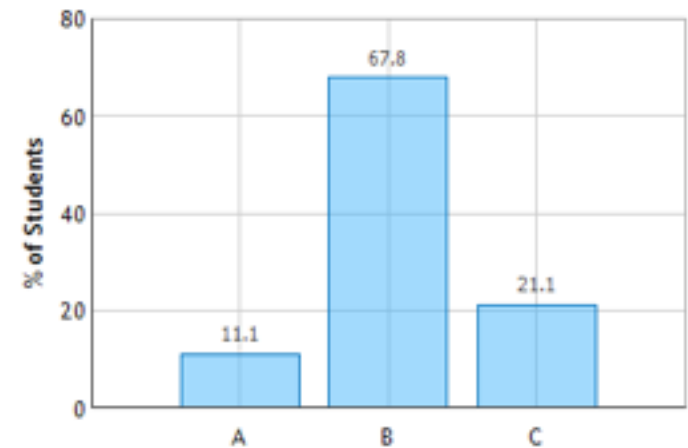
A) eh

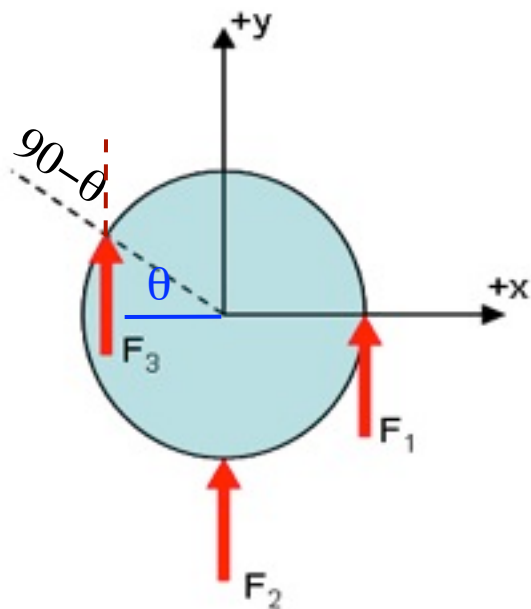
B) $F = mra$. $F_1 = mra$. However, $F_2 = 2mra$.
Therefore $F_2 = 2F_1$

C) $I = mr^2$ for F_2 . And $r = 2$ squared. That makes
 $F = 4m \cdot \alpha$.

- just kus

Two Wheels: Question 1 (N = 90)





$$\tau = RF \sin \theta$$

A uniform disk with mass $m = 8.96 \text{ kg}$ and radius $R = 1.35 \text{ m}$ lies in the x - y plane and centered at the origin. Three forces act in the $+y$ -direction on the disk: 1) a force 337 N at the edge of the disk on the $+x$ -axis, 2) a force 337 N at the edge of the disk on the $-y$ -axis, and 3) a force 337 N acts at the edge of the disk at an angle $\theta = 36^\circ$ above the $-x$ -axis.

1) What is the magnitude of the torque on the disk due to F_1 ?

 N-m

$$\theta = 90^\circ$$

2) What is the magnitude of the torque on the disk due to F_2 ?

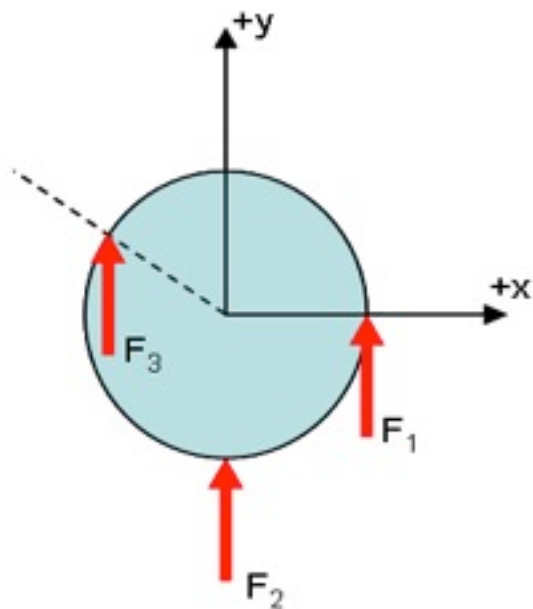
 N-m

$$\theta = 0^\circ$$

3) What is the magnitude of the torque on the disk due to F_3 ?

 N-m

$$\theta = 90 - 36 = 54$$



$$\tau = RF \sin \theta$$

Direction is perpendicular to both R and F , given by the *right hand rule*

A uniform disk with mass $m = 8.96$ kg and radius $R = 1.35$ m lies in the x - y plane and centered at the origin. Three forces act in the $+y$ -direction on the disk: 1) a force 337 N at the edge of the disk on the $+x$ -axis, 2) a force 337 N at the edge of the disk on the $-y$ -axis, and 3) a force 337 N acts at the edge of the disk at an angle $\theta = 36^\circ$ above the $-x$ -axis.

4) What is the x -component of the net torque on the disk?

 N-m

$$\tau_x = 0$$

5) What is the y -component of the net torque on the disk?

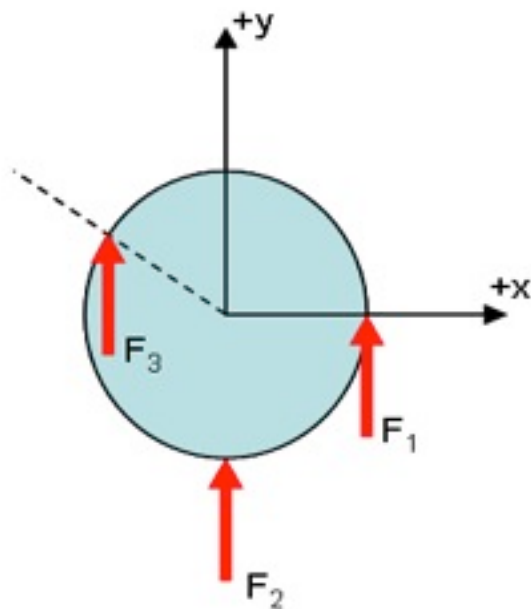
 N-m

$$\tau_y = 0$$

6) What is the z -component of the net torque on the disk?

 N-m

$$\tau_z = \tau_{F_1} + \tau_{F_2} + \tau_{F_3}$$




$$(i) \quad I_{DISK} = \frac{1}{2}MR^2$$

$$(ii) \quad \tau = I\alpha$$

$$(iii) \quad K = \frac{1}{2}I\omega^2$$

A uniform disk with mass $m = 8.96 \text{ kg}$ and radius $R = 1.35 \text{ m}$ lies in the x - y plane and centered at the origin. Three forces act in the $+y$ -direction on the disk: 1) a force 337 N at the edge of the disk on the $+x$ -axis, 2) a force 337 N at the edge of the disk on the $-y$ -axis, and 3) a force 337 N acts at the edge of the disk at an angle $\theta = 36^\circ$ above the $-x$ -axis.

7)  What is the magnitude of the angular acceleration of the disk?

 rad/s²

Use (i) & (ii)

8)  If the disk starts from rest, what is the rotational energy of the disk after the forces have been applied for $t = 1.8 \text{ s}$?

 J

Use (iii)