Classical Mechanics
Lecture 17

Today's Concepts:

a) Torque Due to Gravity

b) Static Equilibrium

Midterm 2 is on March 15, 9:30 to 10:20 am. We will try to start on time, but please don't enter until you're asked to.

Bring a standard scientific calculator. (casio, sharp, TI...)

You will be given the same formula sheet you had before, no other notes or crib sheets allowed.

Write your answers in pen if you'd in case you might need to ask for a regrade.

Midterm exam 2 covers material introduced in Lecture and smartPhysics up to and including Mar. 1 (to and including Unit 13). (Material before Midterm 1 is not excluded.)

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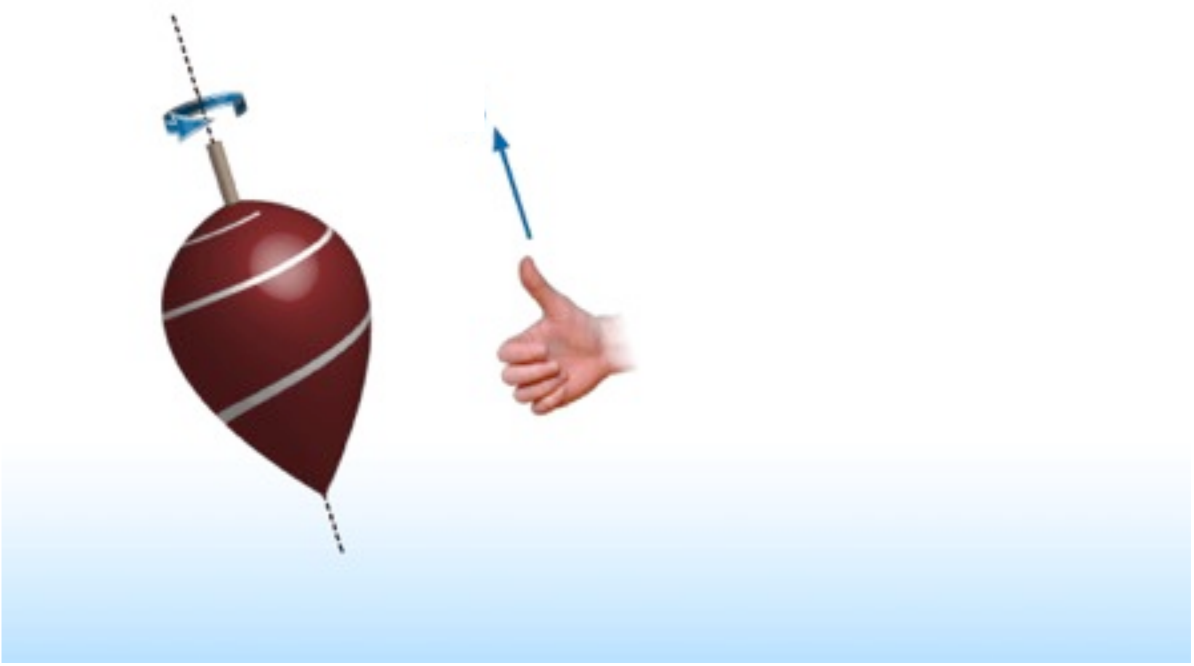
For practice you can try "second hour exams" here: http://courses.physics.illinois.edu/phys211/practice-exams.asp

On the course schedule at http://www.sfu.ca/phys/120/1131/Phys120Schedule.pdf you'll find references to the sections in Tipler&Mosca which are relevant for each lecture.

If you go to the end of the chapters you will also find problems for all the sections.

What happens if you are tightening a reverse threaded bolt by turning it counter clockwise? the torque would be going away from the bolt, but the bolt would screw in? explain please? is the direction of torque really this arbitrary or is there some sense to why it acts as it does?



New Topic, Old Physics:

Statics:

As the name implies, "statics" is the study of systems that don't move.
Ladders, sign-posts, balanced beams, buildings, bridges...

The key equations are familiar to us: $\sum \vec{F} = m\vec{a}$ $\sum \vec{\tau} = I\vec{\alpha}$

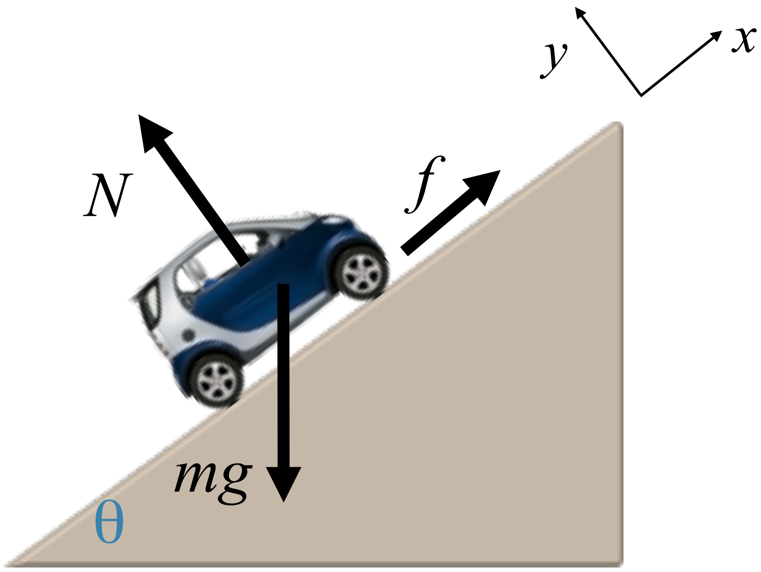
If an object doesn't move: $\vec{a} = 0$ $\vec{\tau} = 0$

→ $\sum \vec{F} = 0$ The net force on the object is zero

$\sum \vec{\tau} = 0$ The net torque on the object is zero (for any axis)

Example:

What are all of the forces acting on a car parked on a hill?



Car on Hill:

Use Newton's 2nd Law: $F_{NET} = MA_{CM} = 0$

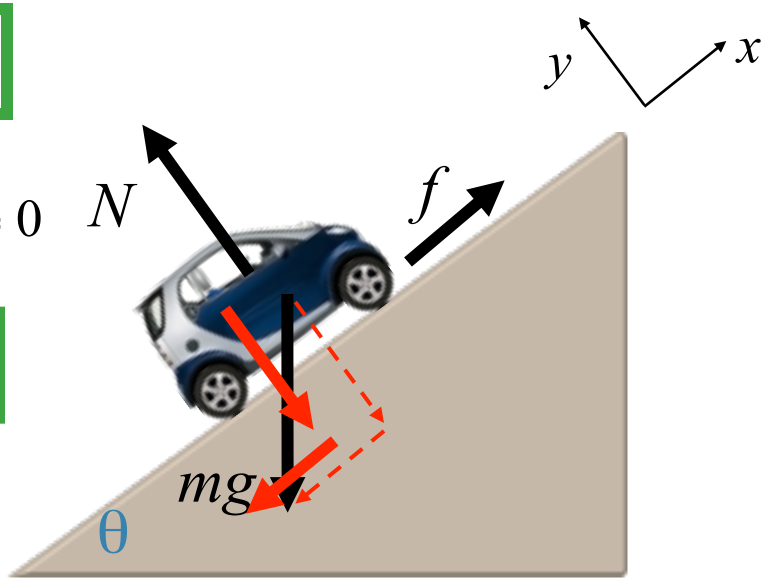
Resolve this into x and y components:

x: $f - mg\sin\theta = 0$

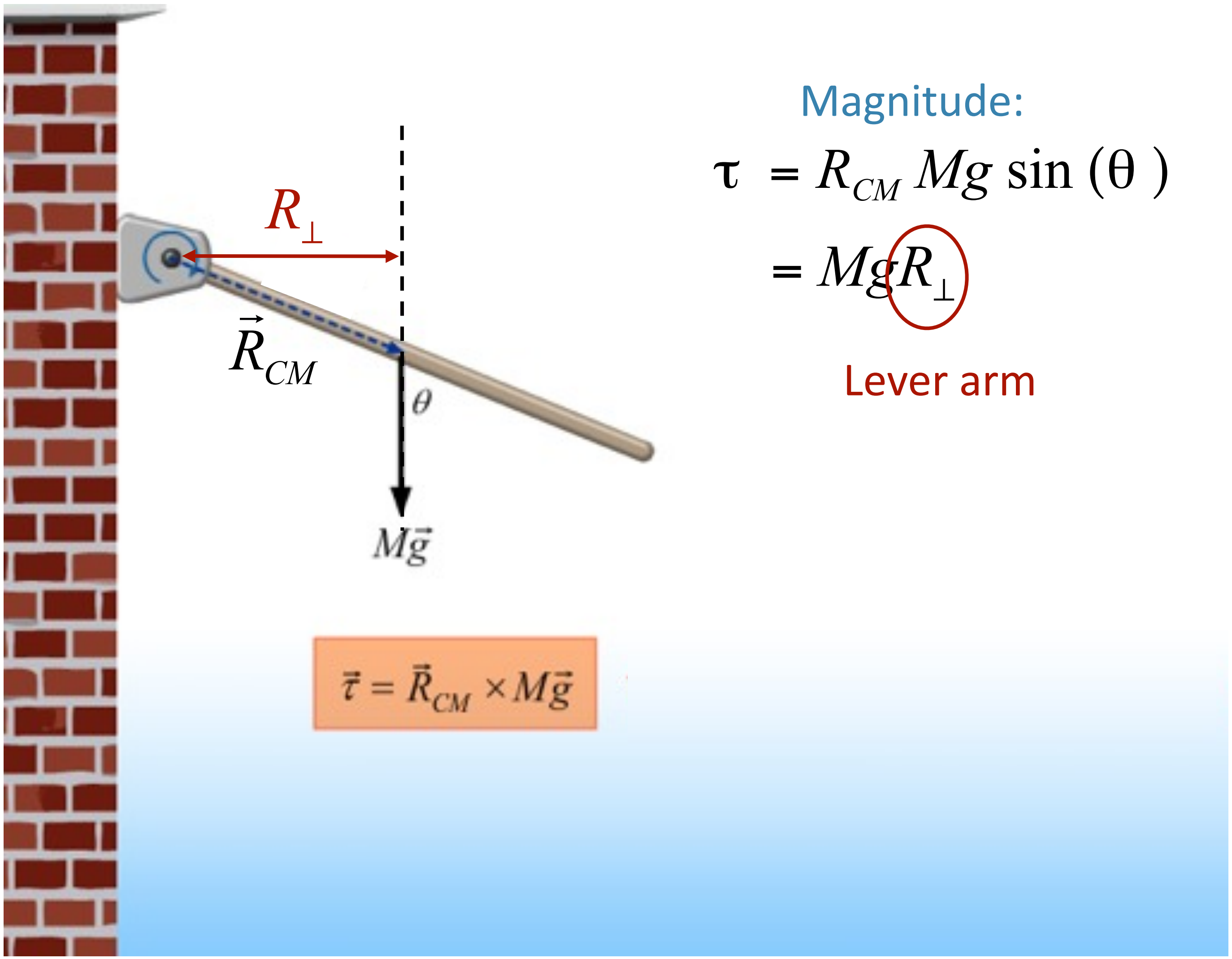
→ $f = mg\sin\theta$

y: $N - mg\cos\theta = 0$

→ $N = mg\cos\theta$



Torque Due to Gravity



$$\vec{\tau} = \vec{R}_{CM} \times M\vec{g}$$

Magnitude:

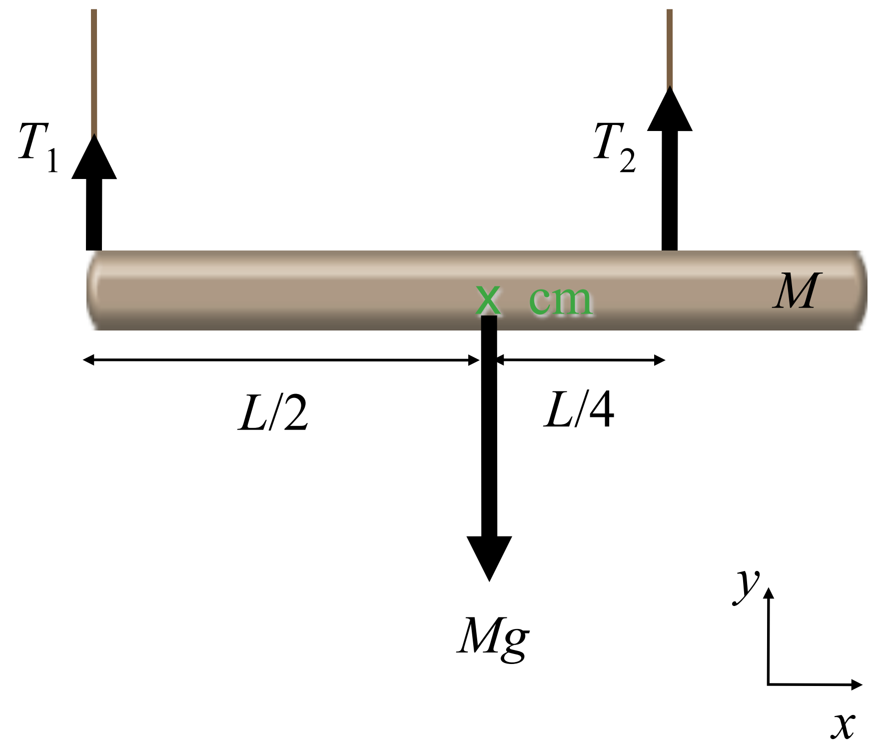
$$\tau = R_{CM} Mg \sin(\theta)$$

$$= MgR_\perp$$

Lever arm

Example:

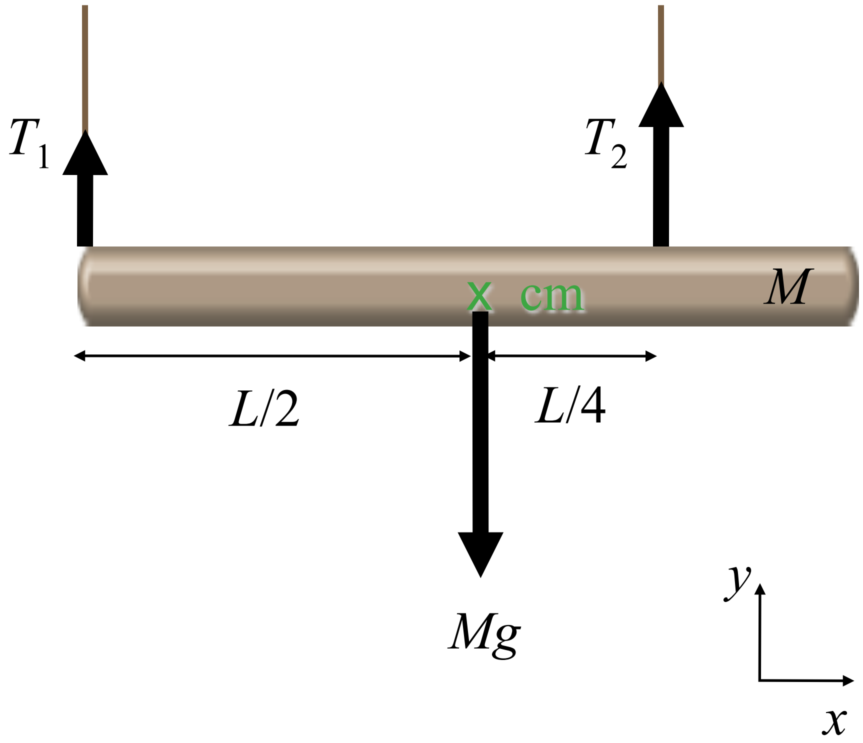
Now consider a plank of mass M suspended by two strings as shown. We want to find the tension in each string:



Balance forces:

$$\sum \vec{F} = 0$$

$$T_1 + T_2 = Mg$$



Balance Torques

$$\sum \vec{\tau} = 0$$

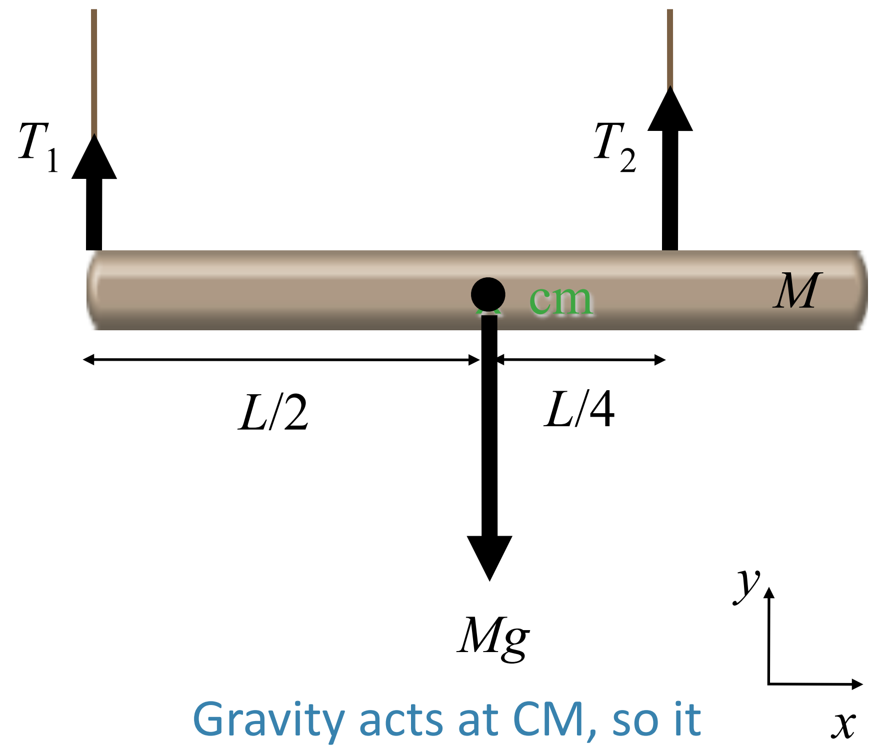
Choose the rotation axis to be out of the page through the CM:

The torque due to the string on the right about this axis

$$\tau_2 = T_2\, L/4$$

The torque due to the string on the left about this axis is:

$$\tau_1 = T_1\, L/2$$



Gravity acts at CM, so it exerts no torque about the CM

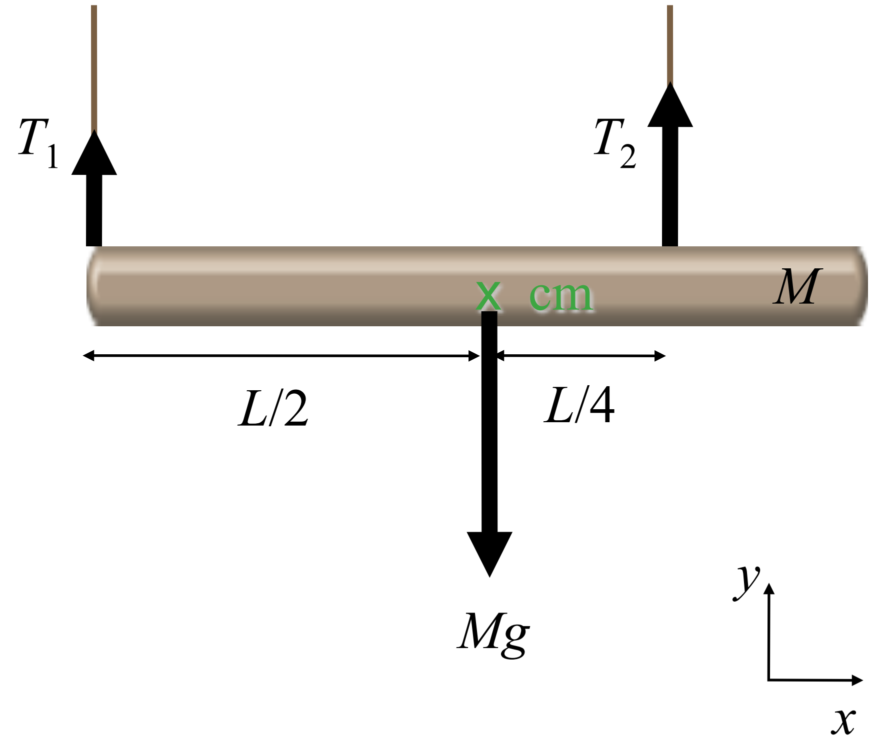
Finish the problem

The sum of all torques must be 0:

$$\rightarrow \tau_1 + \tau_2 = 0$$

$$\rightarrow -T_1 \frac{L}{2} + T_2 \frac{L}{4} = 0$$

$$\rightarrow T_2 = 2T_1$$



We already found that

$T_1 + T_2 = Mg$

$$\rightarrow \boxed{T_1 = \frac{1}{3}Mg \qquad T_2 = \frac{2}{3}Mg}$$

What if you choose a different axis?

$$\sum \vec{\tau} = 0$$

Choose the rotation axis to be out of the page at the left end of the beam:

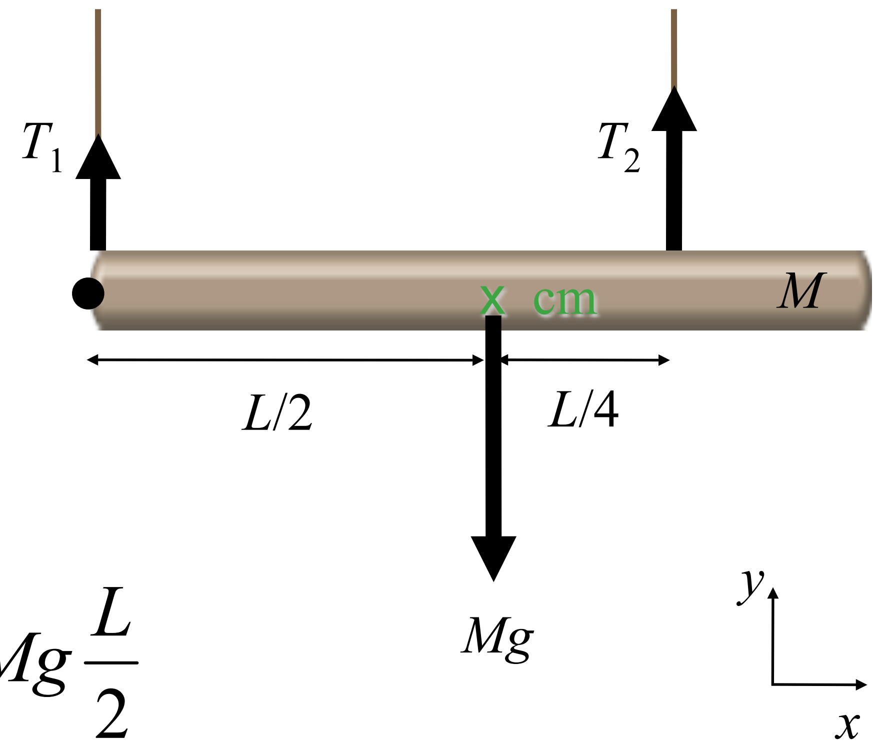
The torque due to the string on the right about this axis is:

$$\tau_2 = T_2 \frac{3L}{4}$$

The torque due to the string on the left is zero

$$\tau_1 = 0$$

Torque due to gravity: $\tau_g = -Mg\dfrac{L}{2}$



You end up with the same answer!

The sum of all torques must be 0:

$$\tau_1 + \tau_2 + \tau_g = 0$$

$$T_2 \frac{3L}{4} - Mg\frac{L}{2} = 0$$

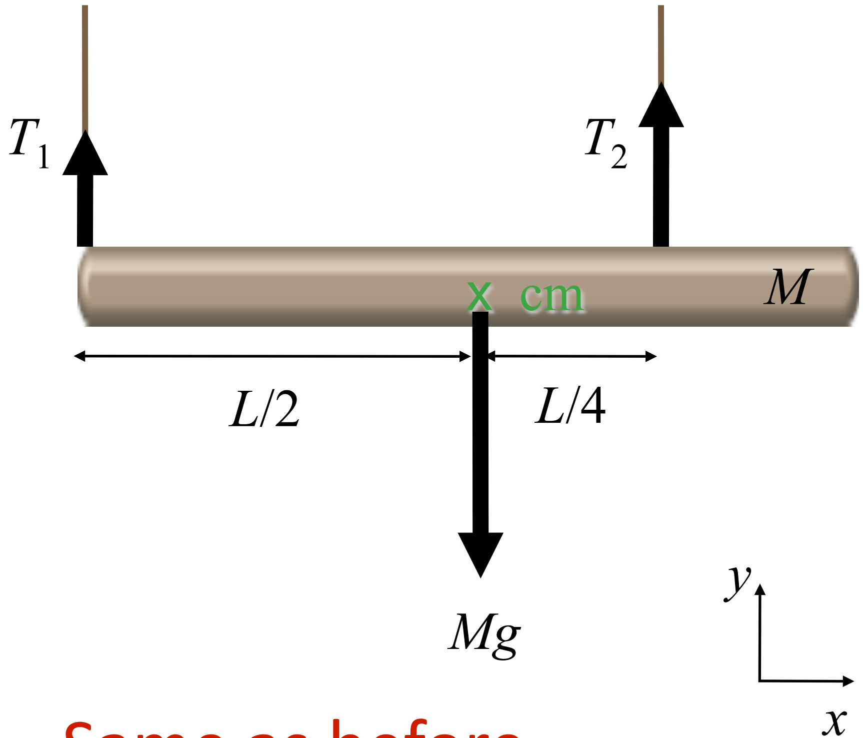
$$T_2 = \frac{2}{3}Mg$$

We already found that

$T_1 + T_2 = Mg$

$$T_1 = \frac{1}{3}Mg$$

$$T_2 = \frac{2}{3}Mg$$



Same as before

Approach to Statics: Summary

In general, we can use the two equations

$$\sum \vec{F} = 0 \qquad \sum \vec{\tau} = 0$$

to solve any statics problem.

When choosing axes about which to calculate torque, we can sometimes be clever and make the problem easier....

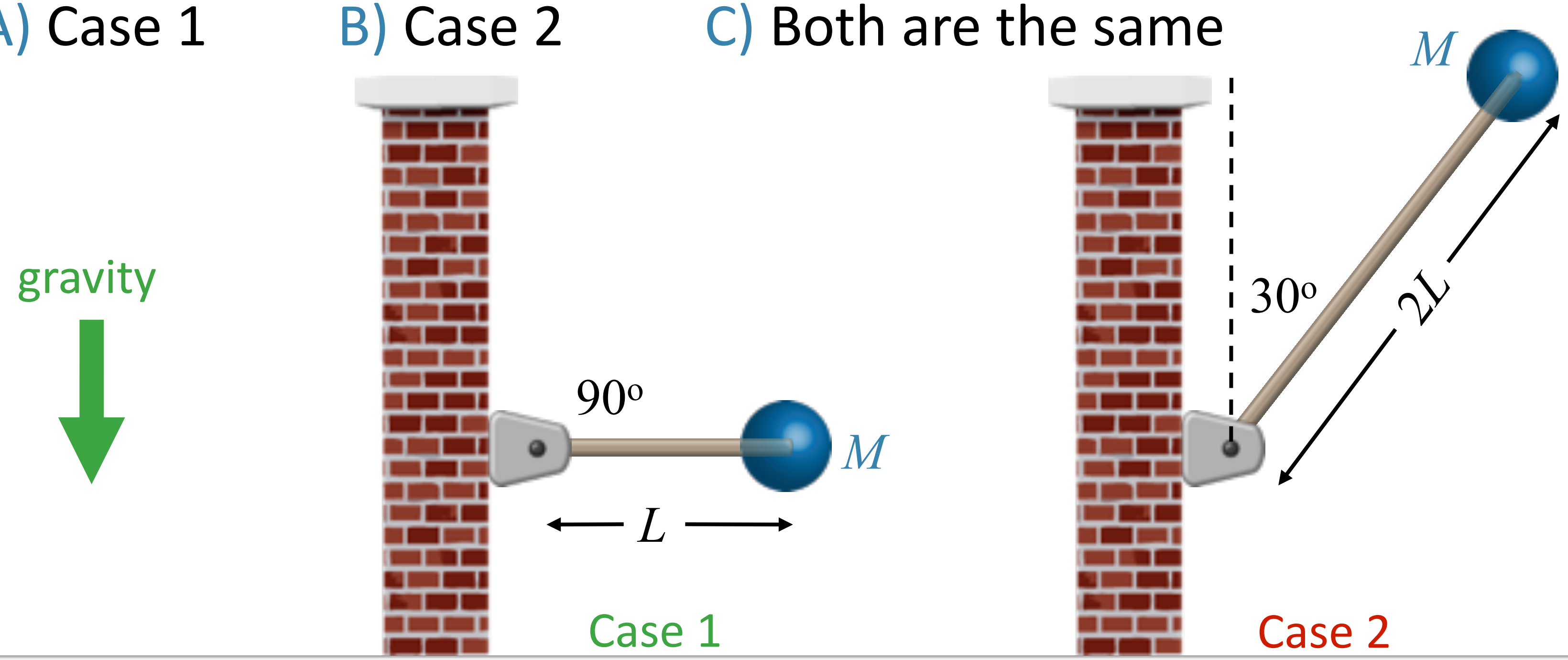
CheckPoint

In Case 1 one end of a horizontal massless rod of length L is attached to a vertical wall by a hinge, and the other end holds a ball of mass M.

In Case 2 the massless rod holds the same ball but is twice as long and makes an angle of 30^o with the wall as shown.

In which case is the total torque about the hinge biggest?

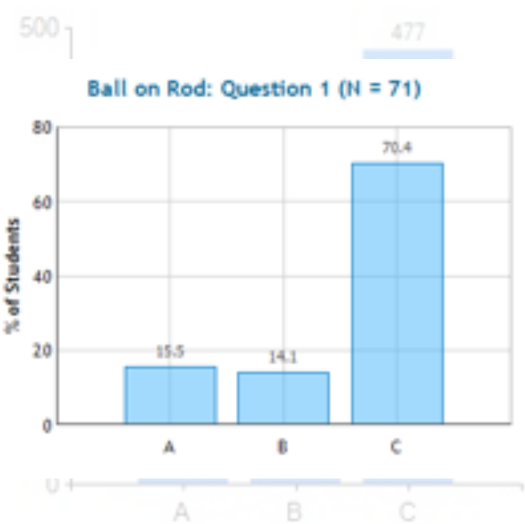
A) Case 1 B) Case 2 C) Both are the same

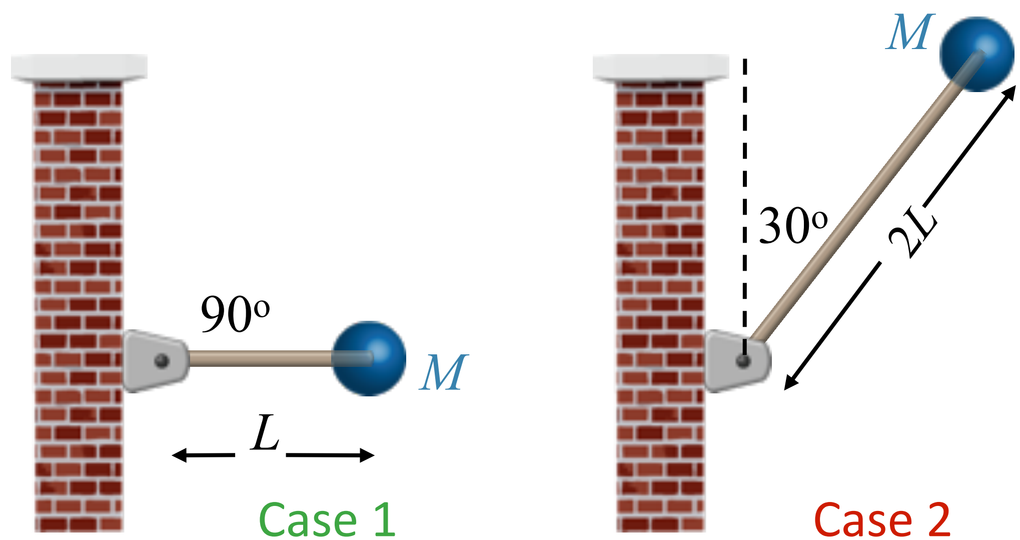


CheckPoint Response

In which case is the total torque about the hinge biggest?

A) Case 1 B) Case 2 C) Both are the same





A) The force is perpendicular to the radius, meaning that cos(theta) is greatest, so torque is greater.

B) In case 2 the lever arm is larger than in case 1 so the torque would be larger.

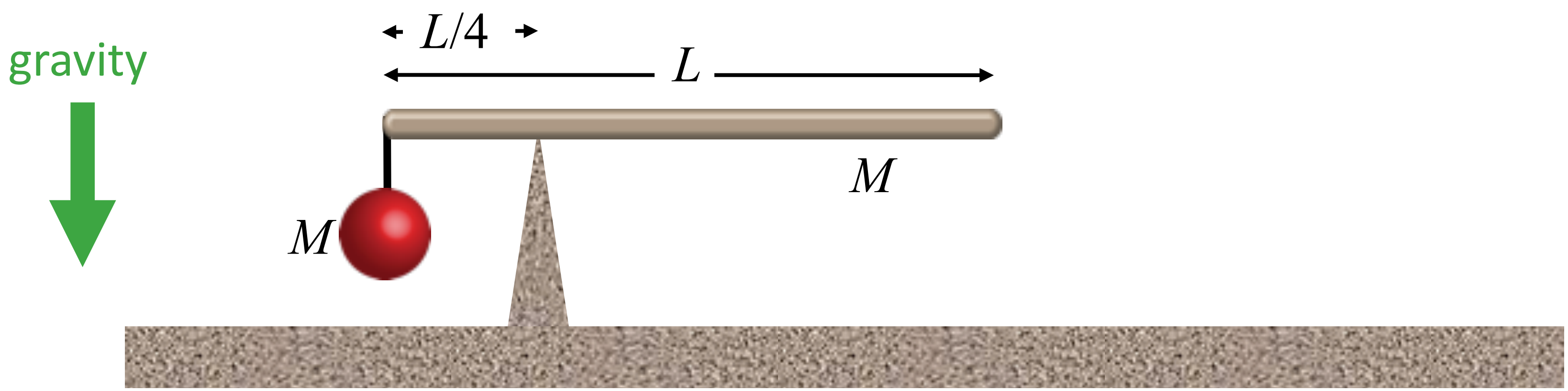
C) Because sin(30) equals to 0.5. This makes up for the difference in the length of the rod.

CheckPoint

An object is made by hanging a ball of mass M from one end of a plank having the same mass and length L. The object is then pivoted at a point a distance $L/4$ from the end of the plank supporting the ball, as shown below.

Is the object balanced?

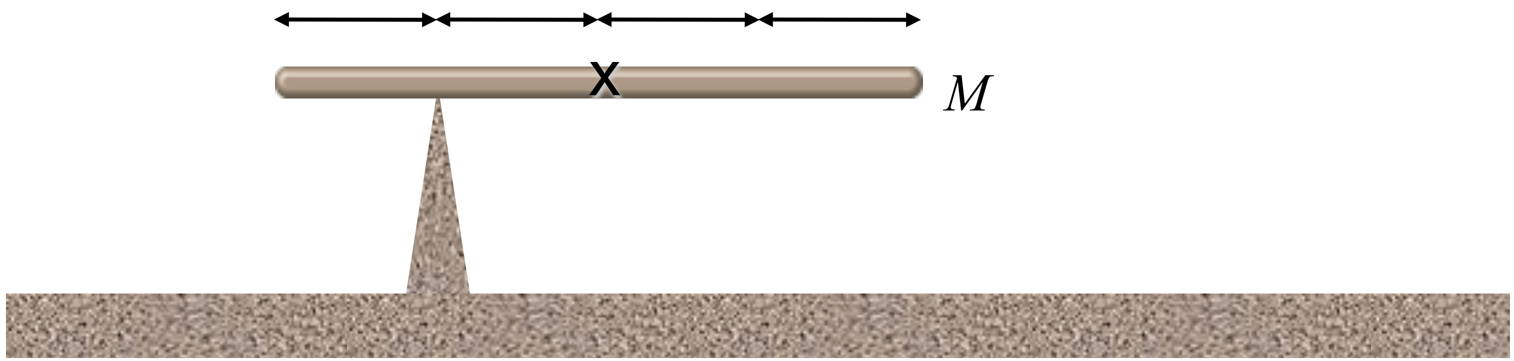
A) Yes B) No, it will fall left C) No, it will fall right



Clicker Question

How far to the right of the pivot is the center of mass of just the plank.

A) $L/4$ B) $L/2$ C) $3L/4$



Clicker Question

Is the object balanced?

A) Yes

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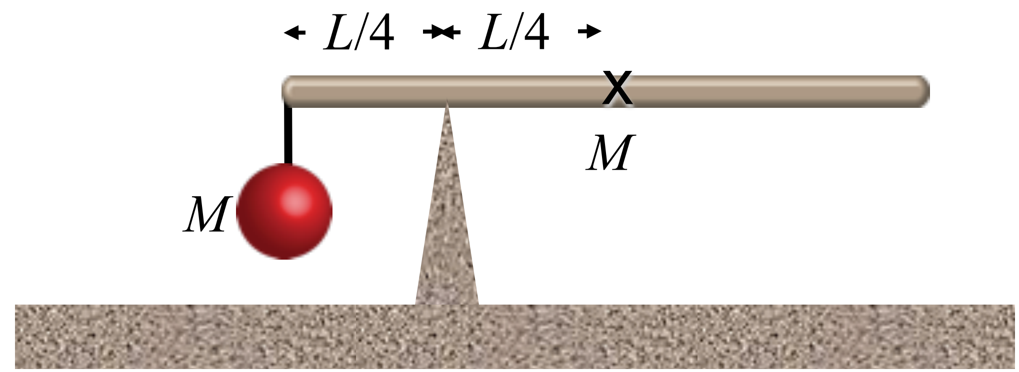
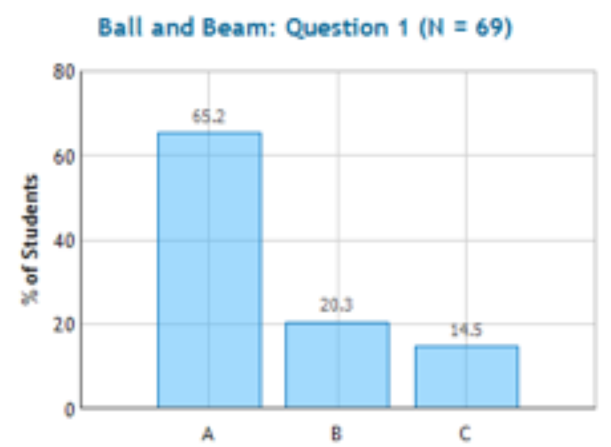


A) Mass M balanced at $L/4$ on left; Mass M balanced $L/4$ on right. So system balanced.

B) The mass on the left is greater than the mass on the right so the object will fall to the left.

C) The center of mass is to the right of the $L/4$ dividing mark, so it will fall to the right.

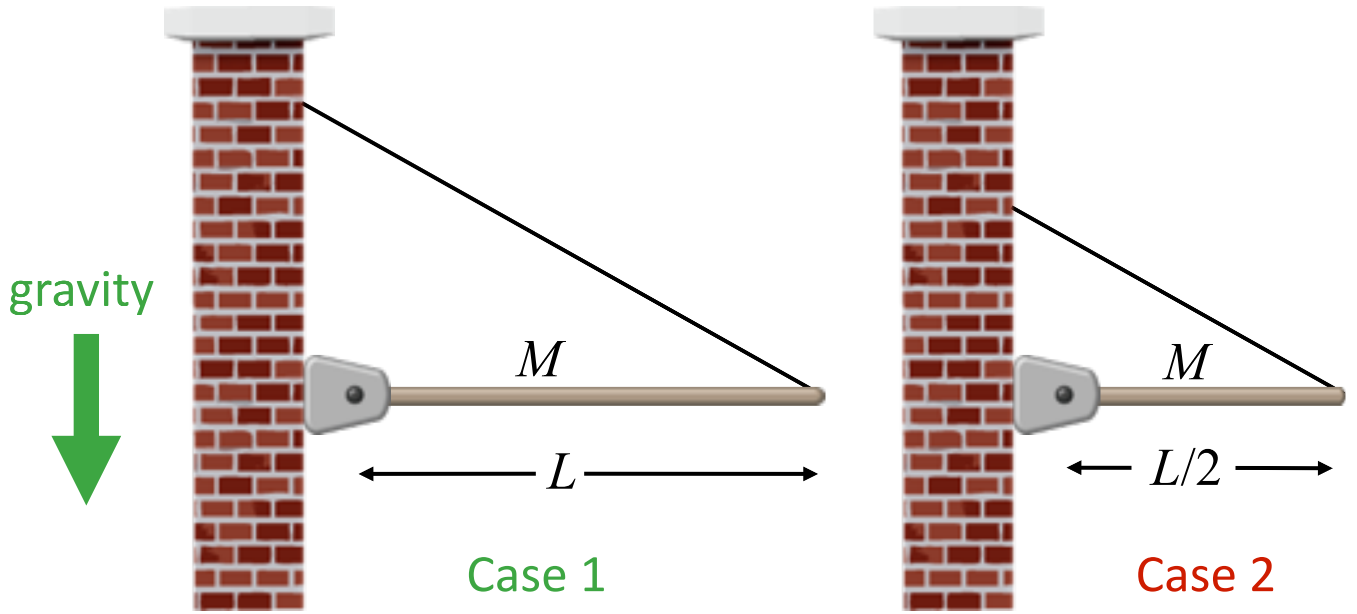


CheckPoint

In Case 1, one end of a horizontal plank of mass M and length L is attached to a wall by a hinge and the other end is held up by a wire attached to the wall. In Case 2 the plank is half the length but has the same mass as in Case 1, and the wire makes the same angle with the plank.

In which case is the tension in the wire biggest?

A) Case 1 B) Case 2 C) Both are the same

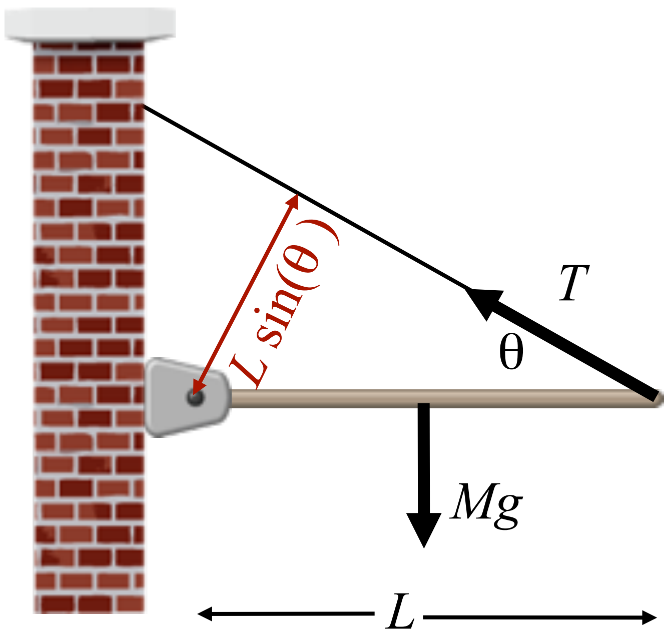


Which equation correctly expresses the fact that the total torque about the hinge is zero?

A) $TL\sin\theta - Mg\dfrac{L}{2} = 0$

B) $TL - Mg\dfrac{L}{2} = 0$

C) $TL - MgL\sin\theta = 0$



The L cancels out: $T\sin\theta = \dfrac{Mg}{2}$

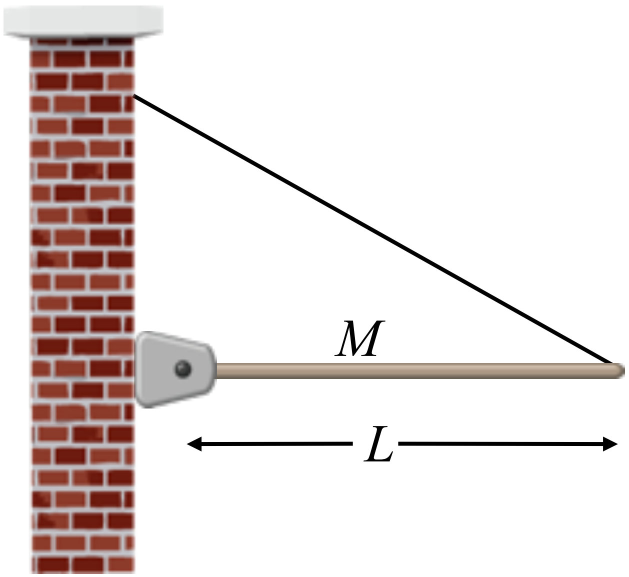
Clicker Question

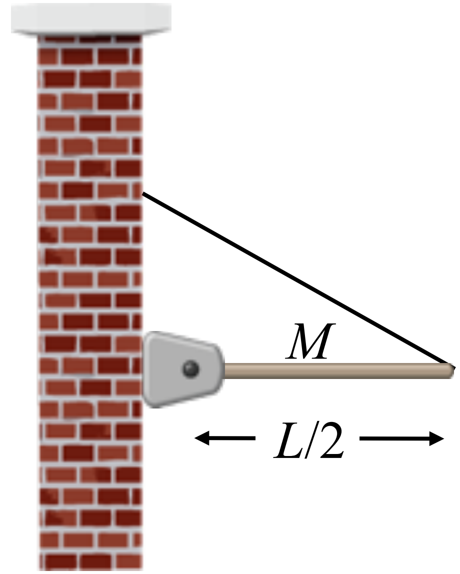
In which case is the tension in the wire biggest?

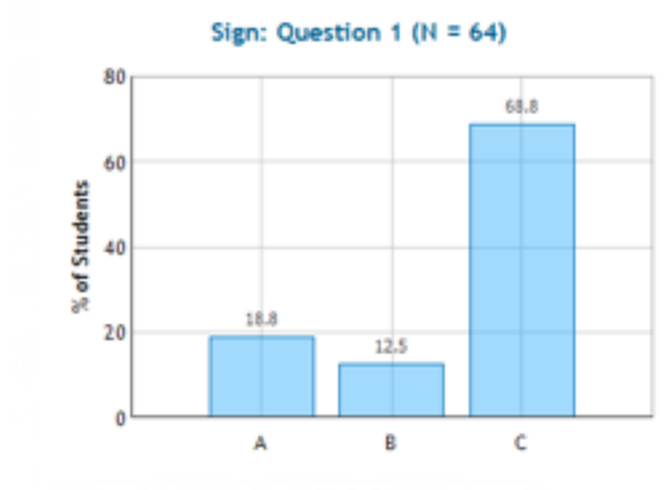
A) Case 1

B) Case 2

C) Same







A) The center of mass is situated further from the hinge and therefore the mass applies a greater force.

B) shorter is not better take it from someone short i know whats up haha

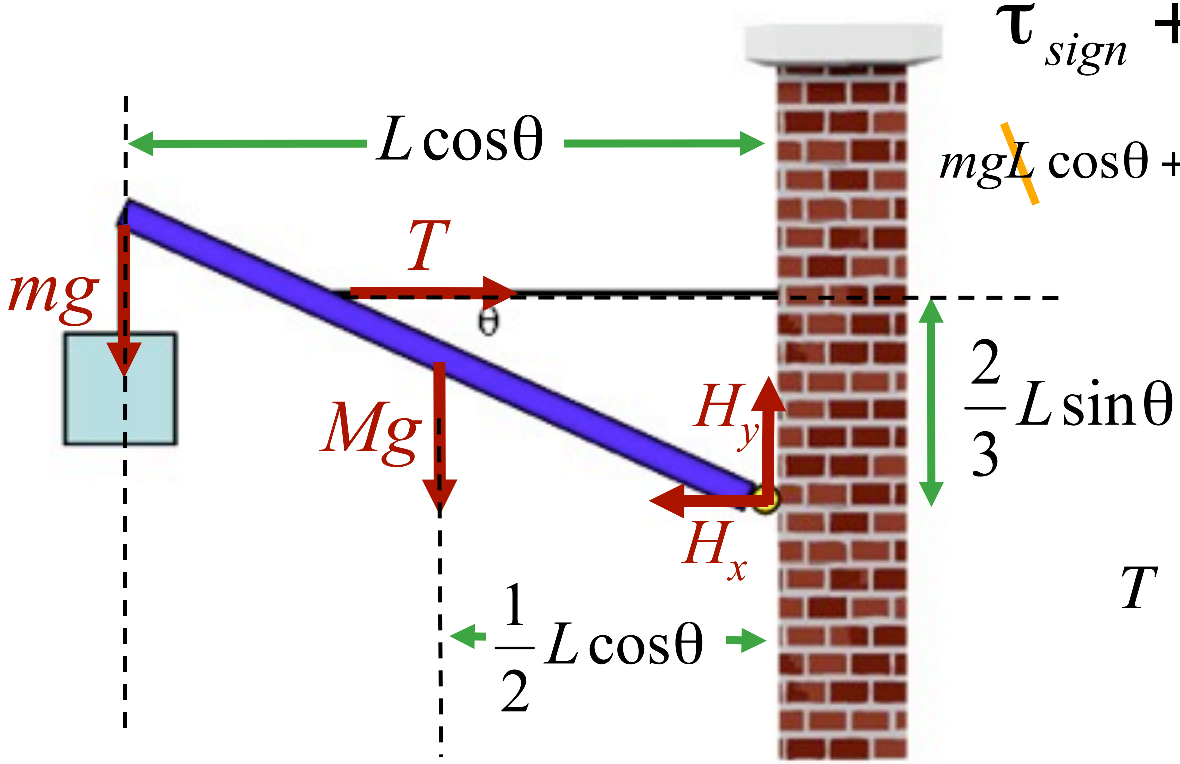
C) Because tension is not dependent on the length of the object. It is dependent on the mass and the angle.

Hanging Beam

Use hinge as axis

$$\tau_{sign} + \tau_{beam} + \tau_{wire} = 0$$

$$mgL\cos\theta + Mg\frac{1}{2}L\cos\theta - T\frac{2}{3}L\sin\theta = 0$$



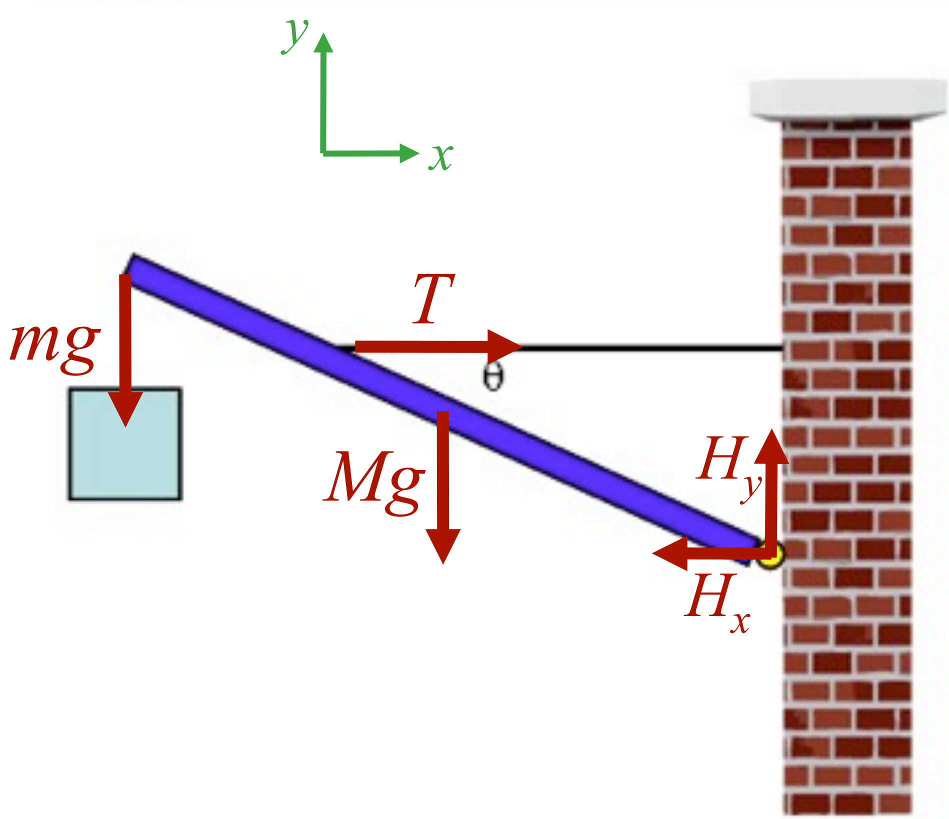
$$T = \left(\frac{3m}{2} + \frac{3M}{4}\right) g\cot\theta$$

A purple beam is hinged to a wall to hold up a blue sign. The beam has a mass of m_b = 6 kg and the sign has a mass of m_s = 17 kg. The length of the beam is L = 2.81 m. The sign is attached at the very end of the beam, but the horizontal wire holding up the beam is attached 2/3 of the way to the end of the beam. The angle the wire makes with the beam is θ = 34.1°.

1) What is the tension in the wire?

[ ] N Submit

Hanging Beam



$$\sum F_x = 0$$

$$T - H_x = 0$$

$$H_x = T$$

$$\sum F_y = 0$$

$$H_y - mg - Mg = 0$$

$$H_y = (m + M)g$$

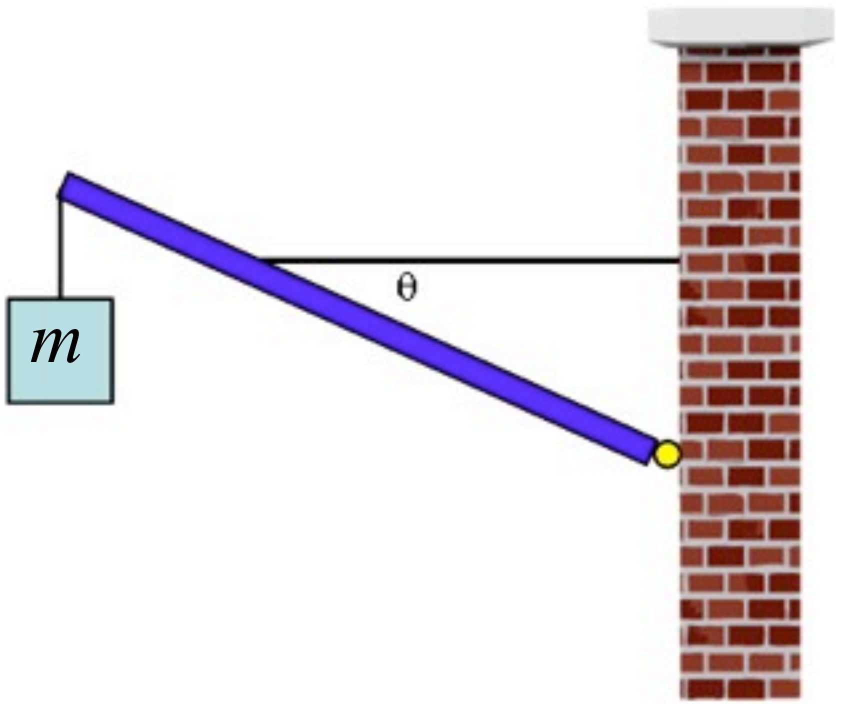
A purple beam is hinged to a wall to hold up a blue sign. The beam has a mass of m_b = 6 kg and the sign has a mass of m_s = 17 kg. The length of the beam is L = 2.81 m. The sign is attached at the very end of the beam, but the horizontal wire holding up the beam is attached 2/3 of the way to the end of the beam. The angle the wire makes with the beam is θ = 34.1°.

2) What is the net force the hinge exerts on the beam?

N Submit

$$\left|\vec{H}\right| = \sqrt{H_x^2 + H_y^2}$$

Hanging Beam



$$T = \left(\frac{3m}{2} + \frac{3M}{4}\right) g \cot\theta$$

A purple beam is hinged to a wall to hold up a blue sign. The beam has a mass of m_b = 6 kg and the sign has a mass of m_s = 17 kg. The length of the beam is L = 2.81 m. The sign is attached at the very end of the beam, but the horizontal wire holding up the beam is attached 2/3 of the way to the end of the beam. The angle the wire makes with the beam is θ = 34.1°.

3) The maximum tension the wire can have without breaking is T = 891 N.

What is the maximum mass sign that can be hung from the beam?

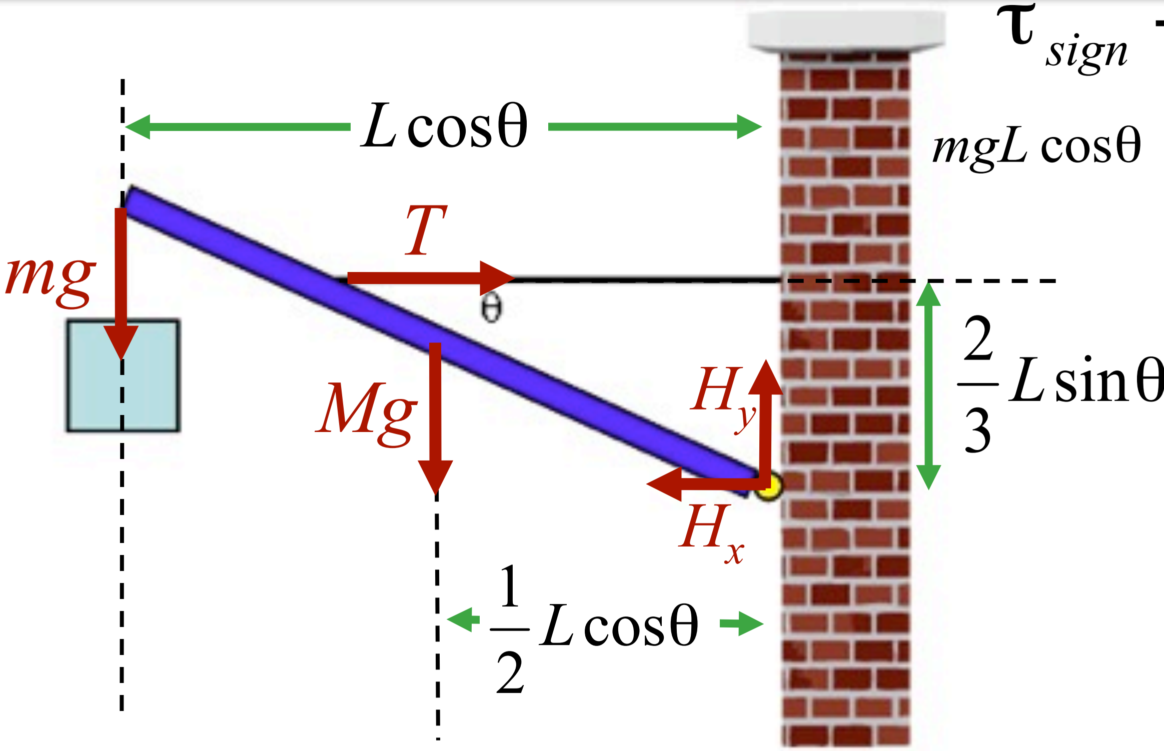
[ ] kg Submit

Solve for m

Use hinge as axis

$$\tau_{sign} + \tau_{beam} + \tau_{wire} = 0$$

$$mgL\cos\theta + Mg\frac{1}{2}L\cos\theta - T\frac{2}{3}L\sin\theta = 0$$



A purple beam is hinged to a wall to hold up a blue sign. The beam has a mass of m_b = 6 kg and the sign has a mass of m_s = 17 kg. The length of the beam is L = 2.81 m. The sign is attached at the very end of the beam, but the horizontal wire holding up the beam is attached 2/3 of the way to the end of the beam. The angle the wire makes with the beam is θ = 34.1°.

4) What else could be done in order to be able to hold a heavier sign?

- ☐ while still keeping it horizontal, attach the wire to the end of the beam
- ☐ keeping the wire attached at the same location on the beam, make the wire perpendicular to the beam
- ☐ attach the sign on the beam closer to the wall
- ☐ shorten the length of the wire attaching the box to the beam

Which of these things make T smaller?