Classical Mechanics
Lecture 19

Today's Concept:

Angular Momentum

So does L=r x p or L= Iω?

Maybe its not about today's lecture ,,,, but can you make more explanations about those vector notations , the "i hat" "j hat" thing ,,, we are confused about how to use them , we saw them on mid terms but we didn't know what to do :)

The conecept of an object having angular momentum when it's moving in a straight line will no doubt be a source of confusion, so we might want to focus on that.

It was pretty understandable. However, if I throw a playdo at a rotating object and friction allows my clay to stick to it and go at the same angular speed, is there loss of momentum? I know energy will be lost

I would like more practise problems for unit 12 and 13 please

I don't understand how the velocity or momentum is up or down from the angle of rotation. I understand how to use the right-hand rule, but what does it MEAN when the velocity or momentum is up? What's going up?

The merry-go-round derivation was very difficult to follow... BUT, did you know that the merry-go-rounds are now banned from school playgrounds because young children have been severely hurt from increasing their omegas too much and then flying off? A kid almost died doing this at my elementary school, then a huge machine/crane thingy uprooted the merry-go-around.

Angular Momentum

We have shown that for a system of particles

$$\vec{F}_{EXT} = \frac{d\vec{p}}{dt}$$

→

Momentum is conserved if
$$\vec{F}_{EXT} = 0$$

What is the rotational version of this?

The rotational analogue of force F is torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Define the rotational analogue of momentum p to be

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$

For a symmetric solid object $\vec{L} = I\vec{\omega}$

$$\vec{\tau}_{EXT} = \frac{d\vec{L}}{dt}$$

where

$$\vec{L} = \vec{r} \times \vec{p}$$
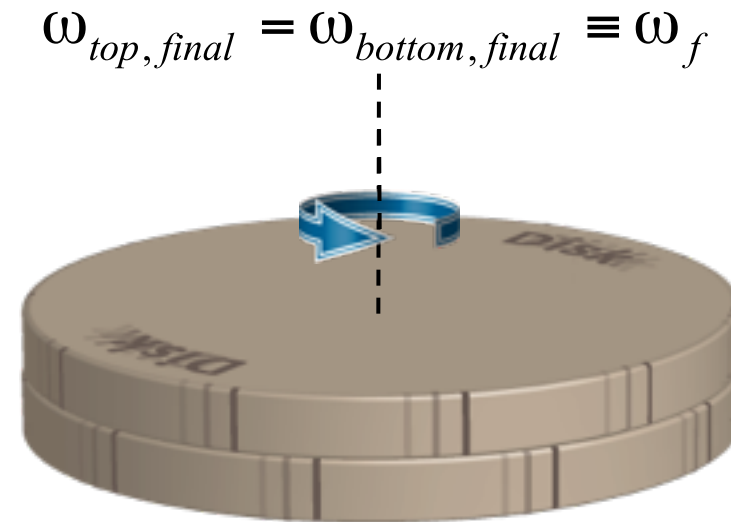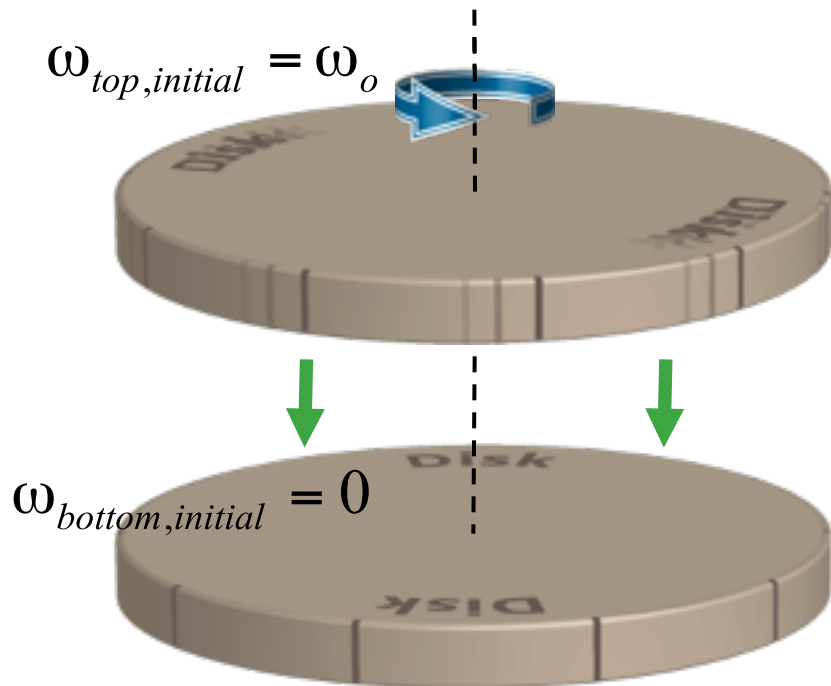$$\vec{L} = I\vec{\omega}$$

and

$$\vec{\tau}_{ext} = \vec{r} \times \vec{F}_{EXT}$$

In the absence of external torques

$$\vec{\tau}_{EXT} = \frac{d\vec{L}}{dt} = 0$$

Total angular momentum is conserved

$$\omega_{top,initial} = \omega_o$$

$$\omega_{bottom,initial} = 0$$

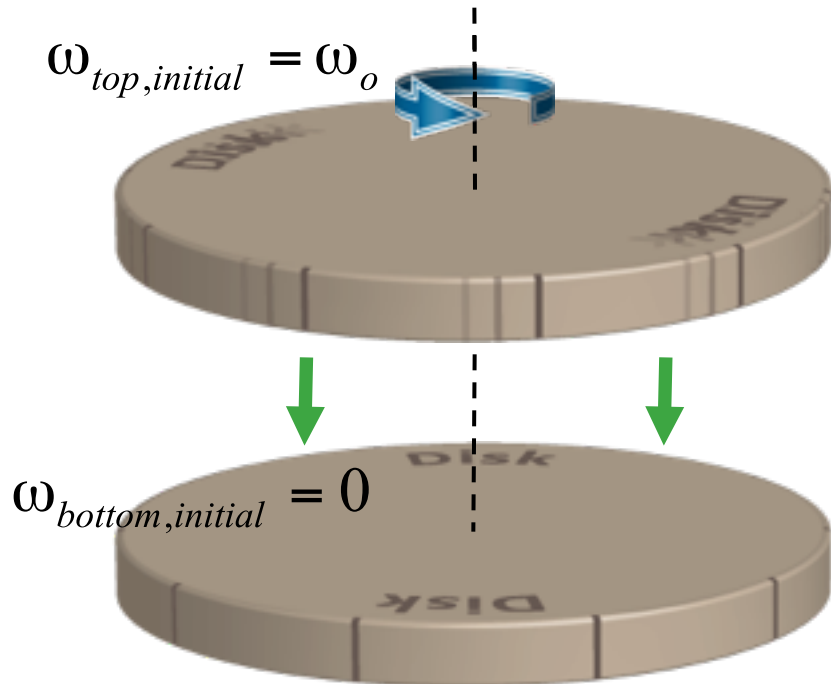$$\omega_{top,final} = \omega_{bottom,final} \equiv \omega_f$$

$$L_{initial} = \frac{1}{2}MR^2\omega_o + 0 \qquad = \qquad L_{final} = \frac{1}{2}MR^2\omega_f + \frac{1}{2}MR^2\omega_f$$
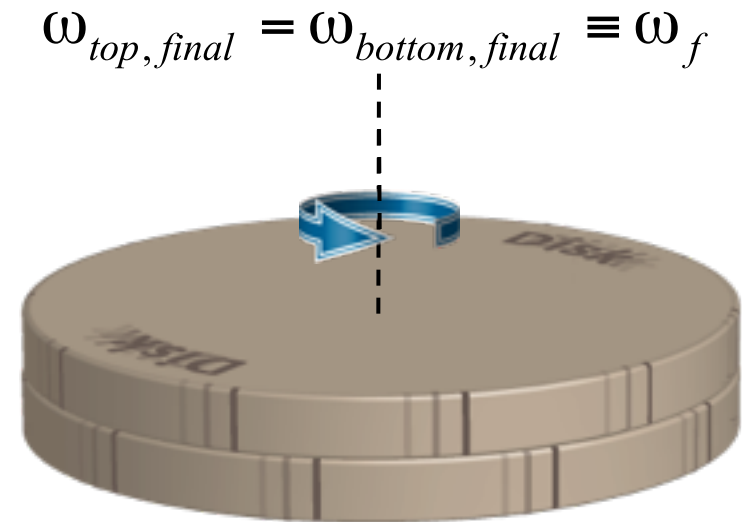
$$\frac{1}{2}MR^2\omega_o = MR^2\omega_f \qquad \longrightarrow \qquad \boxed{\omega_f = \frac{1}{2}\omega_0}$$

What about Kinetic Energy?

$$\omega_{top,initial} = \omega_o$$

$$\omega_{bottom,initial} = 0$$

$$\omega_{top,final} = \omega_{bottom,final} \equiv \omega_f$$

$$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

$$K_{before} = \frac{L^2}{2I_{before}}$$

same
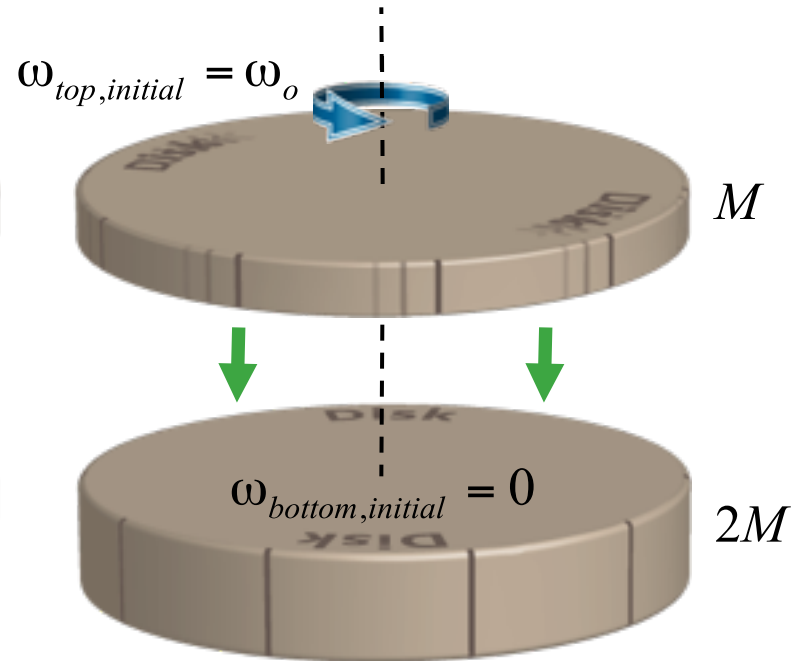
$$K_{after} = \frac{L^2}{2I_{after}}$$

x2

smartPhysics

CheckPoint

In both cases shown below a solid disk of mass M, radius R, and initial angular velocity ω_o is dropped onto an initially stationary second disk having the same radius. In Case 2 the mass of the bottom disk is twice as big as in Case 1. If there are no external torques acting on either system, in which case is the final kinetic energy of the system biggest?
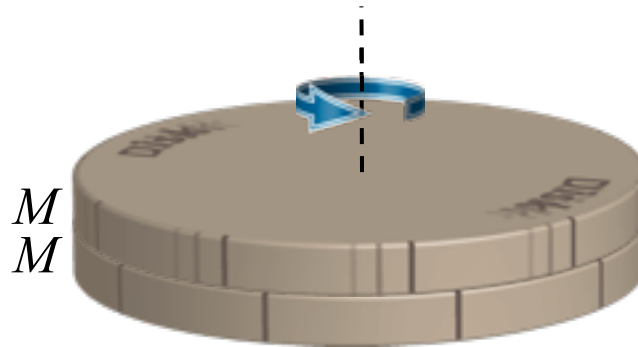
A) Case 1

B) Case 2

C) Same

$\omega_{top,initial} = \omega_o$

$\omega_{top,initial} = \omega_o$

M

M

$\omega_{bottom,initial} = 0$

$\omega_{bottom,initial} = 0$

M

$2M$

Same initial L

Case 1

Case 2

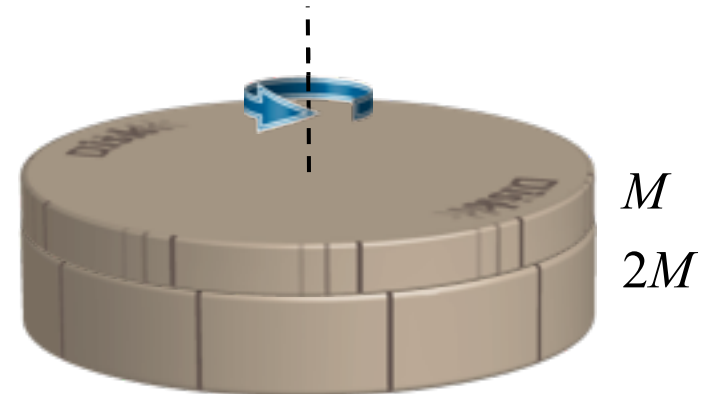smartPhysics
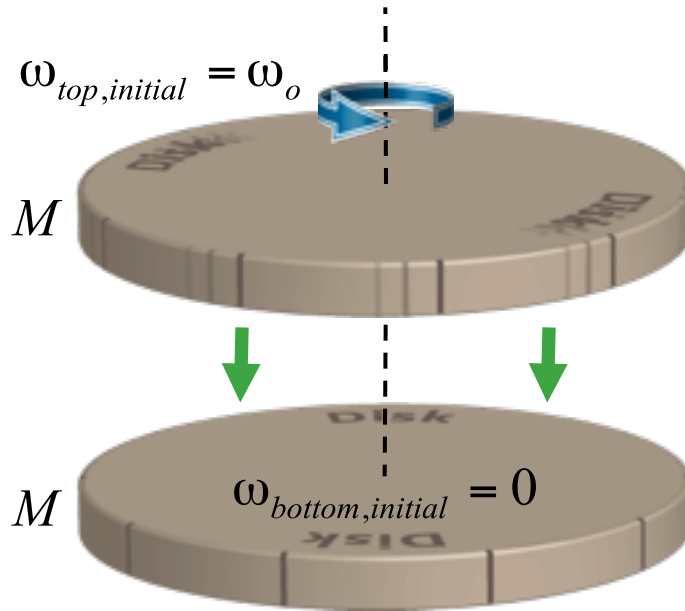
CheckPoint

In which case is the final kinetic energy of the system biggest?

A) Case 1

B) Case 2

C) Same

M
M

M
$2M$

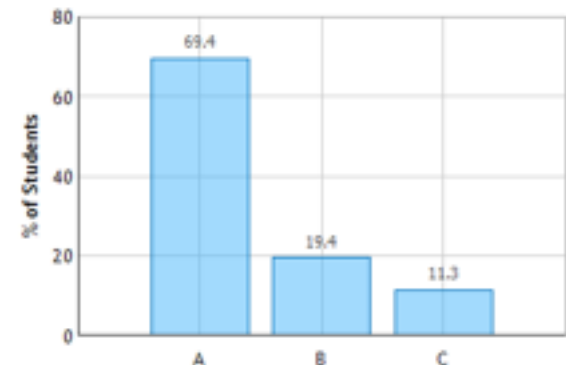Case 1   Case 2

$$K = \frac{L^2}{2I}$$

A) L is equal but bigger mass in case 2

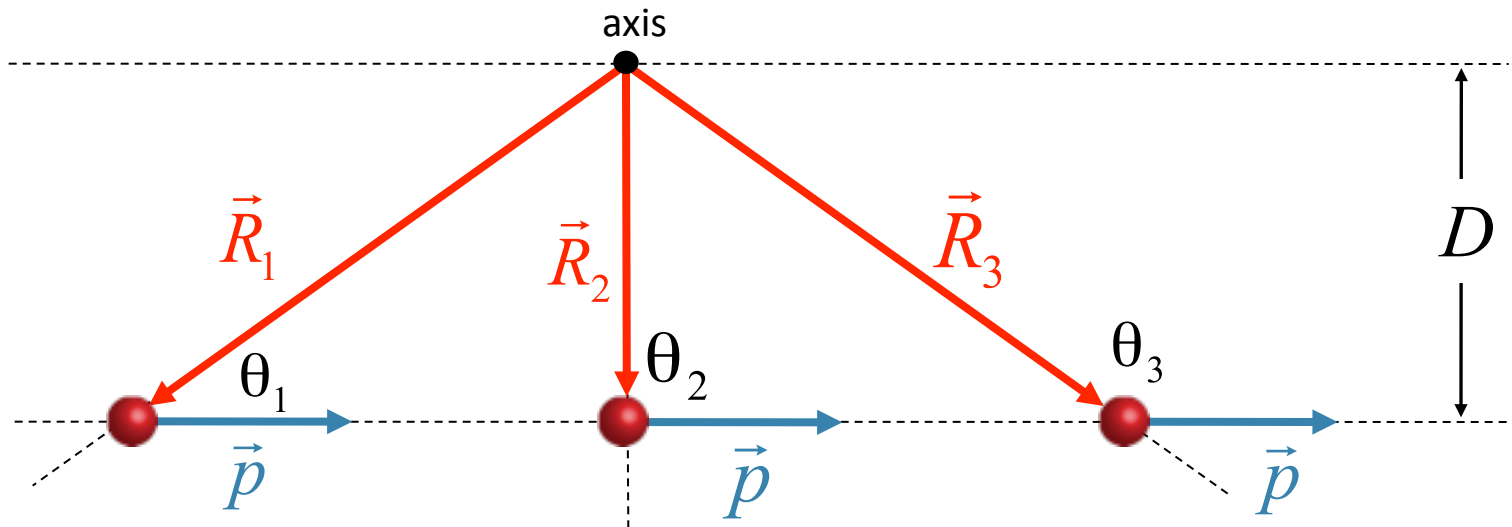B) Case 2 has greater mass

C) Conservation of angular momentum

Disk Collisions: Question 1 (N = 62)

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$$\vec{L} = \vec{R} \times \vec{p}$$

axis

$\vec{R}_1$ $\vec{R}_2$ $\vec{R}_3$

D

θ_1 θ_2 θ_3

$\vec{p}$ $\vec{p}$ $\vec{p}$

$$\vec{L}_1 = \vec{R}_1 \times \vec{p}$$
$$= R_1 \, p \sin(\theta_1)$$
$$= pD$$

$$\vec{L}_2 = \vec{R}_2 \times \vec{p}$$
$$= R_2 \, p \sin(\theta_2)$$
$$= pD$$

$$\vec{L}_3 = \vec{R}_3 \times \vec{p}$$
$$= R_3 \, p \sin(\theta_3)$$
$$= pD$$

$$\vec{L} = \vec{R} \times \vec{p} \quad \longrightarrow \quad L = mvD = pD$$

axis

D

$\vec{p}$

Direction given by right hand rule
(out of the page in this case)

Playground Example

Before

After

Top
View

M

R

ω

v

m

$$L_{Before} = mvR$$

$$L_{After} = I\omega = (\frac{1}{2}MR^2 + mR^2)\omega$$

$$L_{Before} = L_{After} \quad \longrightarrow \quad \omega = \frac{v}{R}\frac{1}{\left(1 + M/2m\right)}$$

smartPhysics
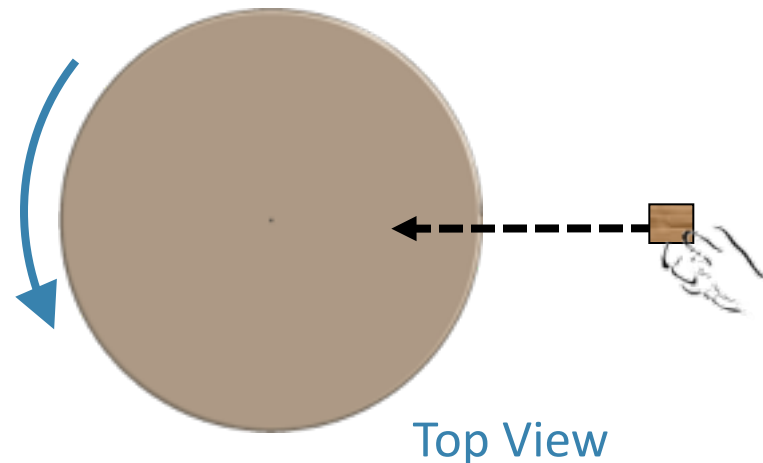
The magnitude of the angular momentum of a freely rotating disk around its center is L. You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. What is the magnitude of the final angular momentum of the disk-block system:

A) $> L$

B) $= L$

C) $< L$

Top View

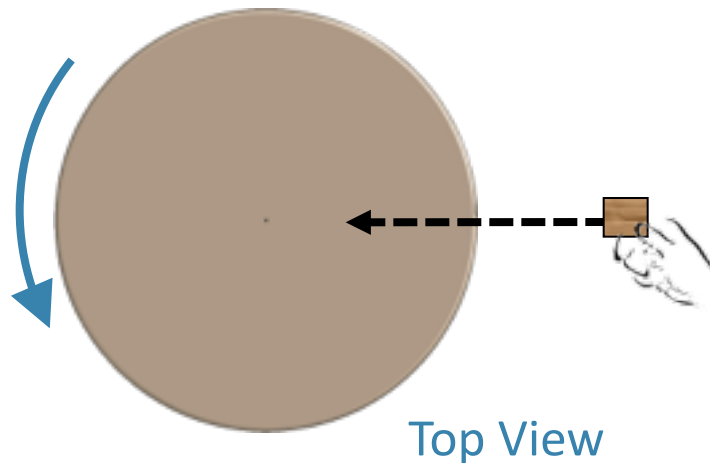Instead of a block, imagine it's a kid hopping onto a merry go round

What is the magnitude of the final angular momentum of the disk-block system:
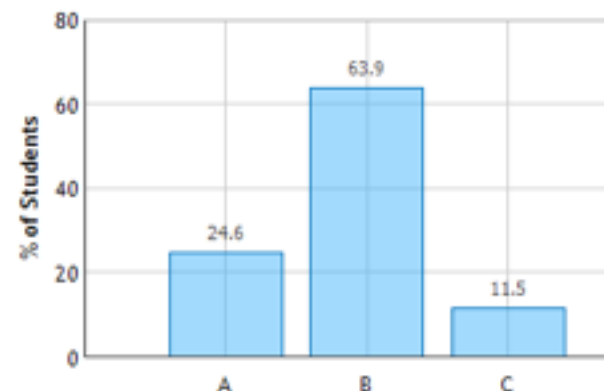
A) $> L$

B) $= L$

C) $< L$

Top View

A) With the added weight the momentum must be greater.

B) Initial total momentum is L (L + 0*m*v) Because there is no external force, this momentum is conserved.

C) Friction decrease the speed of disk

Block on Spinning Disk 1a: Question 1 (N = 6

% of Students

80

63.9

60

40

24.6

20

11.5

0

A   B   C

The magnitude of the angular momentum of a freely rotating disk around its center is L. You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. What is the magnitude of the final angular momentum of the disk-block system:

A) $> L$

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C) $< L$

Top View

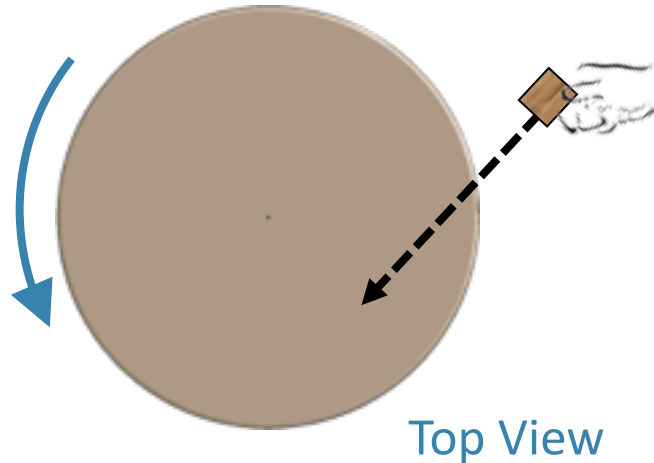Instead of a block, imagine it's a kid hopping onto a merry go round

What is the magnitude of the final angular momentum of the disk-block system:
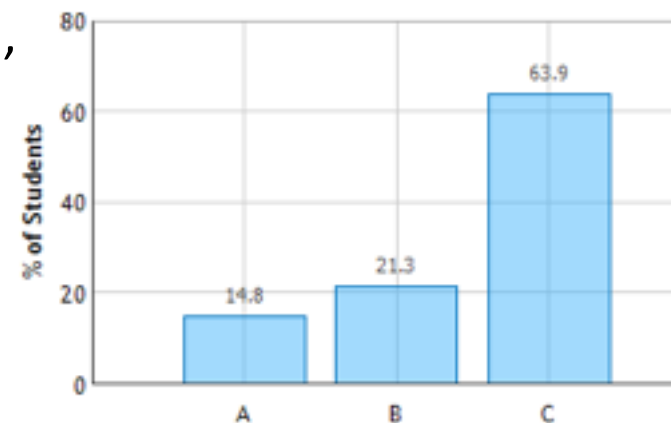
A) $> L$

B) $= L$

C) $< L$

Top View

Block on Spinning Disk 2a: Question 1 (N = 61
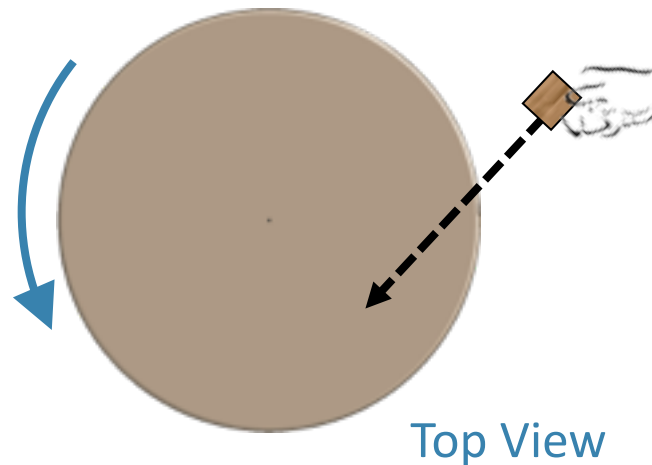
A) Since the block has angular momentum to begin with, that is added into L.

B) L is conserved so $L_{initial}=L_{final}$

C) The block adds momentum going in the opposite direction, so the total momentum is smaller than the momentum of the disk.

% of Students

80

63.9

60

40

21.3

20

14.8

0

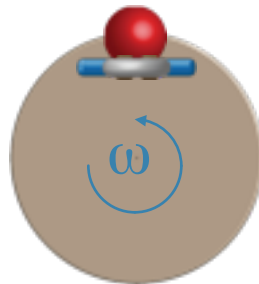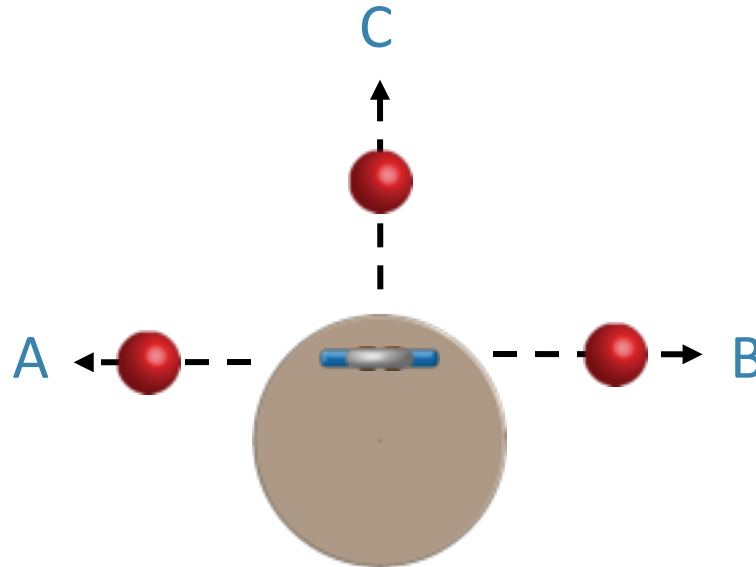A   B   C

A student holding a heavy ball sits on the outer edge a merry go round which is initially rotating counterclockwise. Which way should she throw the ball so that she stops the rotation?

A) To her left

B) To her right

C) Radially outward

top view: initial
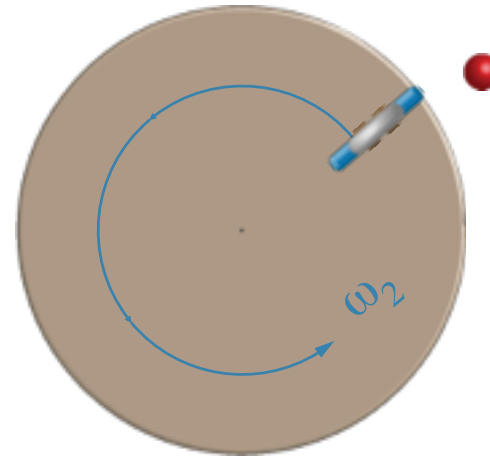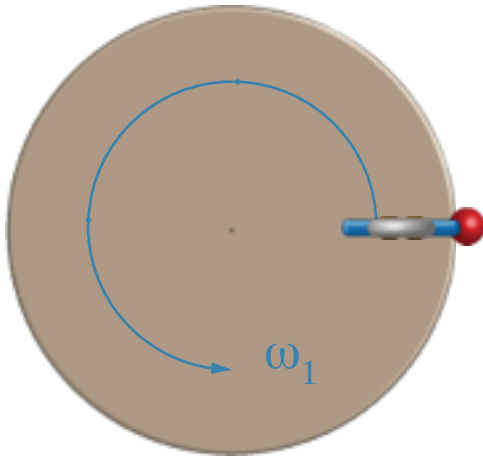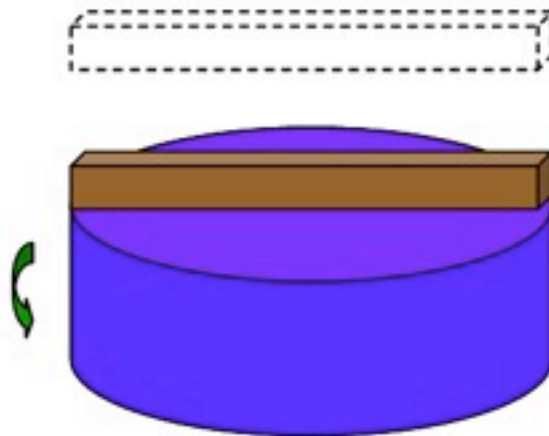
final

A student is riding on the outside edge of a merry-go-round rotating about a frictionless pivot. She holds a heavy ball at rest in her hand.   If she releases the ball, the angular velocity of the merry-go-round will:

A) Increase   B) Decrease   C) Stay the same

Rod and Disk

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

1) What is the initial angular momentum of the rod and disk system?

[   ] kg-m²/s [Submit]

2) What is the initial rotational energy of the rod and disk system?

[   ] J [Submit]

3) What is the final angular velocity of the disk?

[   ] rad/s [Submit]

4) ⏱ What is the final angular momentum of the rod and disk system?

[   ] kg-m²/s [Submit]

5) ⏱ What is the final rotational energy of the rod and disk system?
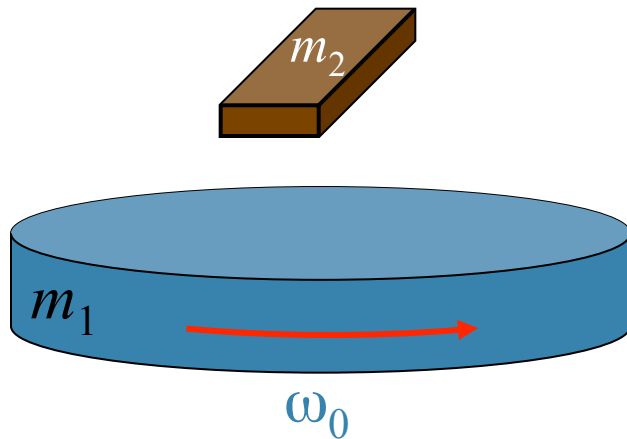
[   ] J [Submit]

6) The rod took t = 7 s to accelerate to its final angular speed with the disk, what average torque was exerted on it by the disk?

[   ] N-m [Submit]

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

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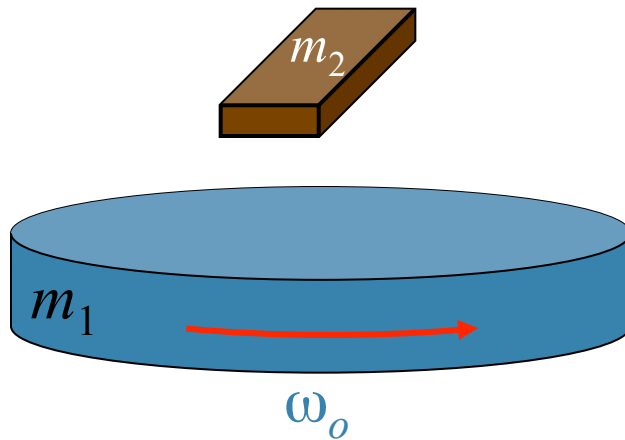
[   ] kg-m²/s [Submit]

m_2

m_1

ω_0

$$I_{disk} = \frac{1}{2} m_1 R^2$$

$$L_{initial} = I_{disk} \omega_o$$

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A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

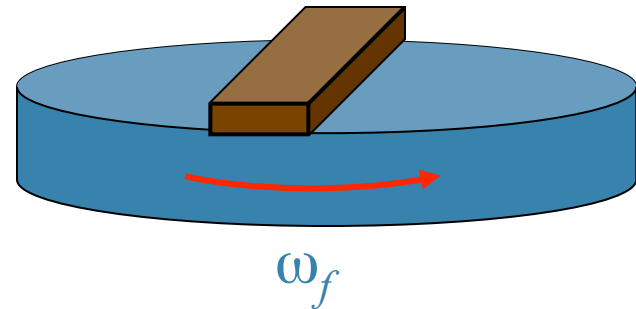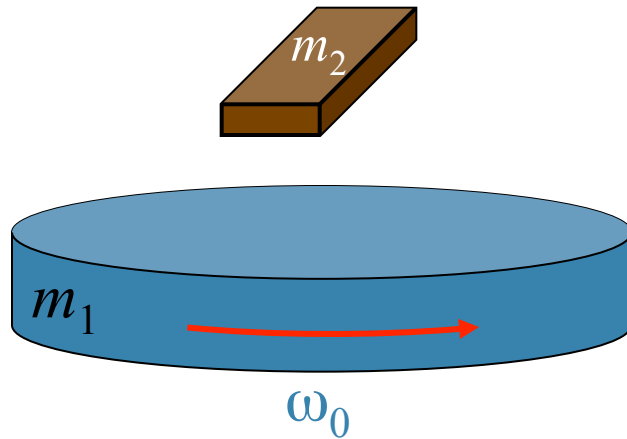2) What is the initial rotational energy of the rod and disk system?

[   ] J   [Submit]

$$I_{disk} = \frac{1}{2} m_1 R^2$$

$$K_{initial} = \frac{1}{2} I_{disk} \omega_o^2$$

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

3) What is the final angular velocity of the disk?

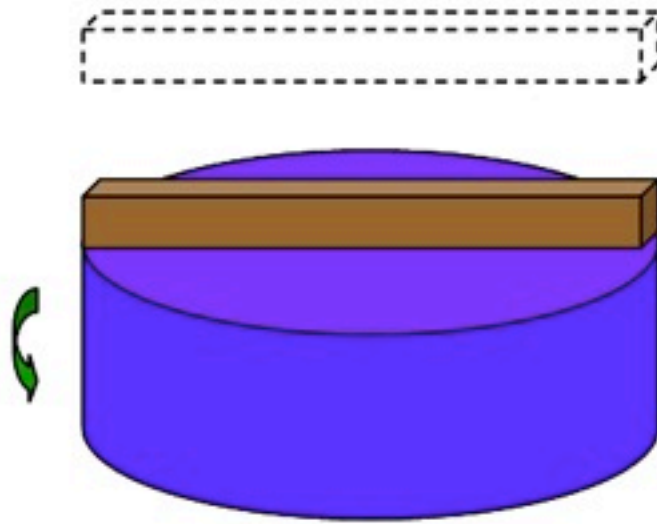[   ] rad/s [Submit]

m_2

m_1

ω_0

ω_f

$$L_{initial} = I_{disk}\omega_0$$

$$L_{final} = \left(I_{disk} + I_{rod}\right)\omega_f$$

$$L_{final} = L_{initial}$$

$$I_{rod} = \frac{1}{12}m_2L^2$$

Solve for ω_f

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

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[   ] kg-m^2/s [Submit]

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[   ] J [Submit]

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[   ] rad/s [Submit]

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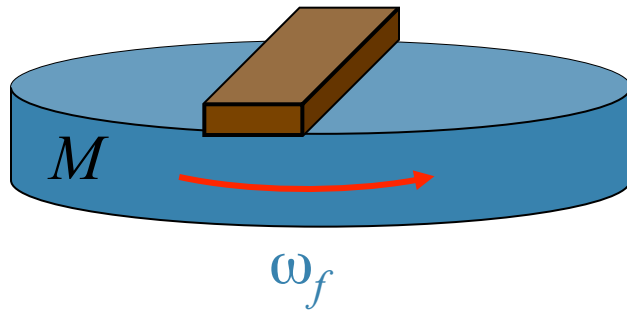
[   ] kg-m^2/s [Submit]

$$L_{final} = L_{initial}$$

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

5) What is the final rotational energy of the rod and disk system?

[_____] J [Submit]

M

ω_f

$$K_{final} = \frac{1}{2}\left(I_{disk} + I_{rod}\right)\omega_f^2$$

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.
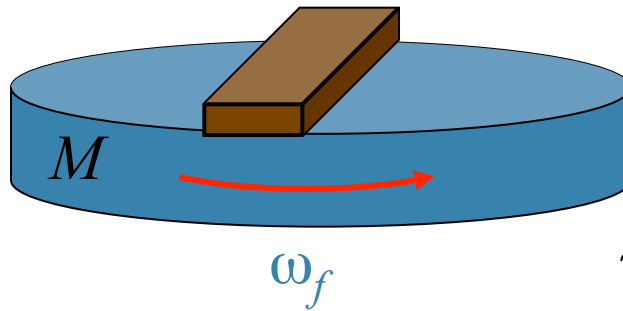
6) The rod took t = 7 s to accelerate to its final angular speed with the disk, what average torque was exerted on it by the disk?

[   ] N-m   Submit

Just like

$$F_{average} = \frac{\Delta P}{\Delta t}$$

for linear momentum

$$\tau_{average} = \frac{\Delta L}{\Delta t}$$ for angular momentum

$$\Rightarrow \Delta L_{rod} = I_{rod}\left(\omega_f - \omega_0\right) \Rightarrow \tau_{average} = \frac{I_{rod}\left(\omega_f - \omega_0\right)}{\Delta t}$$