Classical Mechanics
Lecture 20

Today's Concepts:

A) Angular Momentum

B) Precession

Recommended Video:

Mechanical Universe 20: Torques and Gyroscopes

Your Comments

I did not completely understand how the torque causes an object to move and have a precession frequency. [http://www.learner.org/vod/login.html?pid=569]

S**t just got real.

Can we spend the whole class spinning the tires again?

How do we determine the direction of precession?

This is all REALLY confusing!!! Please help!

I am doing so so on angular momentum, but quantitative problems, or some sort of practise worksheet would help lots.

OMG this is so hard I spent my whole evening to study physics, I have no entertainment :(

Can you upload lecture 14 on webct? [http://www.sfu.ca/phys/140/1117/Lectures/Lect14.pdf]

Is there a case analogous to perfectly elastic collisions in rotational kinematics?

Your Comments

Precession demos will be almost guaranteed to BLOW THEIR MINDS! But we're definitely going to need to go over precession a few times with physical examples.

Angular momentum isn't that bad, but I'm still having issues with parallel axis theorem that are screwing me up in the later homework

i don't understand how we can include rotation direction in calculating the precession direction.

lets go over this one more time, I get it all, just would be great to have some demos

This video seems to help picture the Gyroscope much better than what was depicted in the slides.http://www.youtube.com/watch?v=ty9QSiVC2g0 But it seems like physics might just be getting interesting again :)

Can we go over how to find the direction of precession?

I need precession help

Gyration and precession is still unclear.

physics yo! das doowwwwwn

This is getting stupidly hard...

Could you please explain the right hand rule for precession?

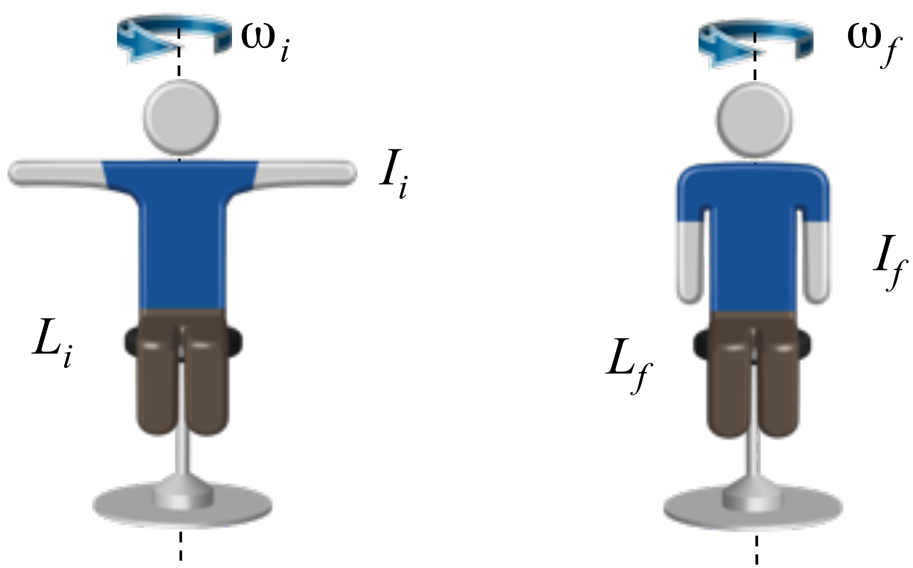
Student on Stool

There are no external torques acting on the student-stool system, so angular momentum will be conserved.

Initially: $L_i = I_i \omega_i$

Finally: $L_f = I_f \omega_f$

$$\boxed{\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}}$$

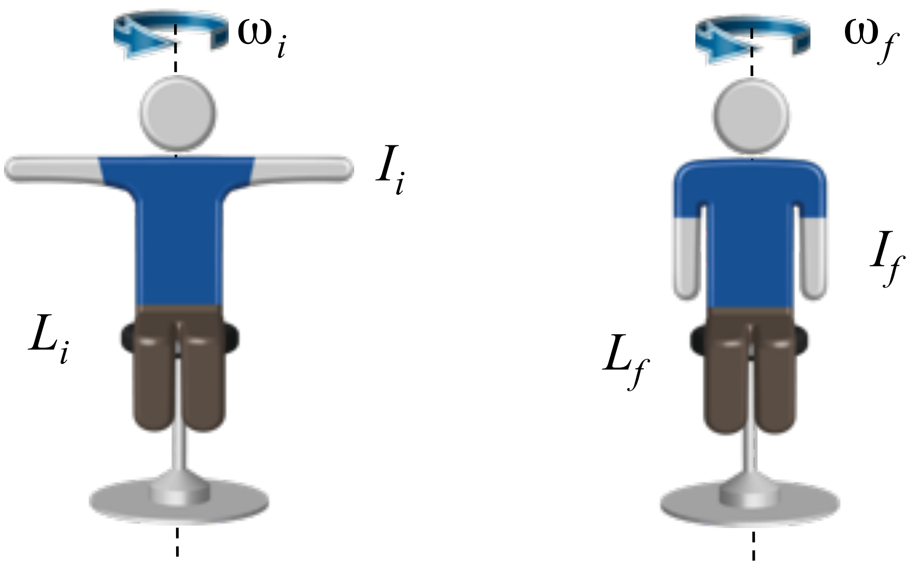


Clicker Question

A student sits on a freely turning stool and rotates with constant angular velocity ω_1. She pulls her arms in and her angular velocity increases to ω_2.

In doing this her kinetic energy:

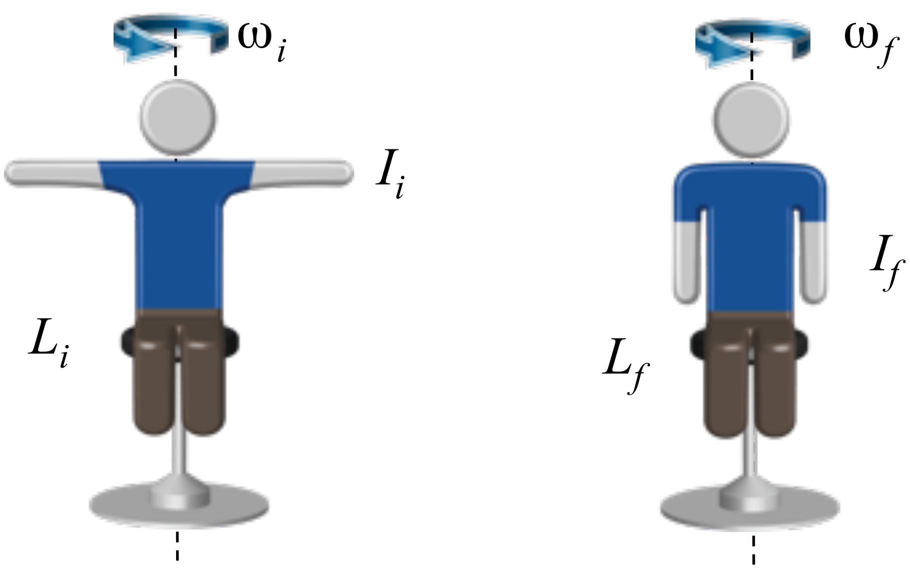
A) Increases B) Decreases C) Stays the same



$$K = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$$ (using $L = I\omega$)

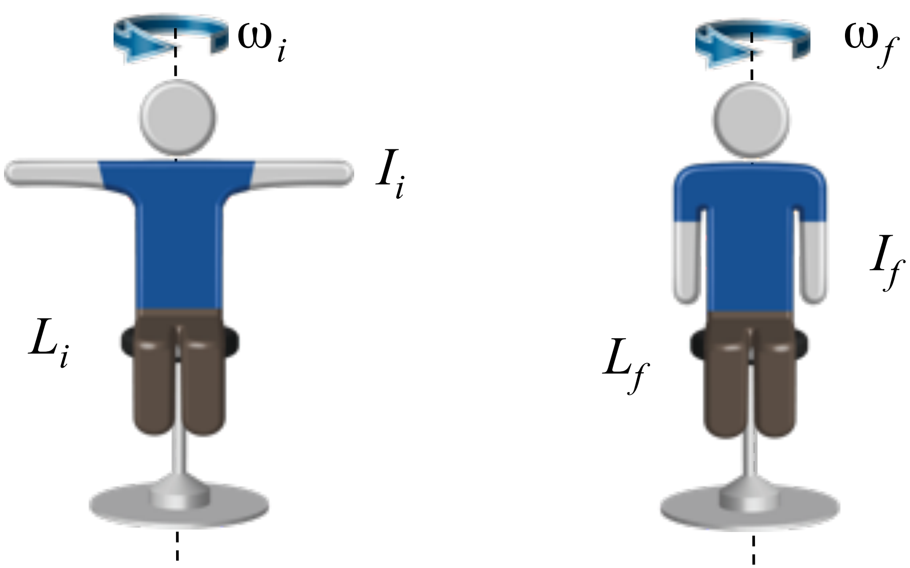
L is conserved:

$I_i < I_f$ → $K_f > K_i$ K increases!

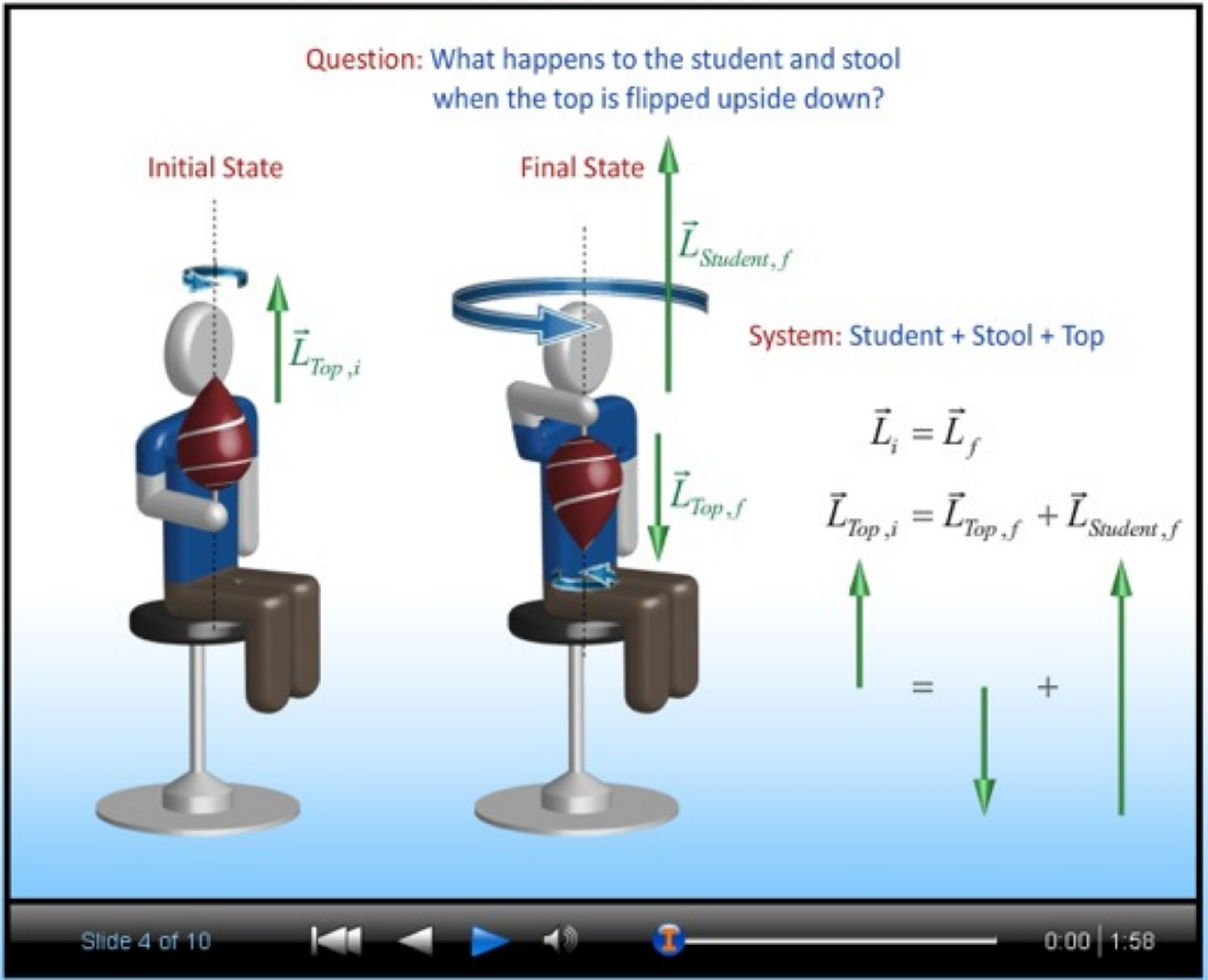


Since the student has to force her arms to move toward her body, she must be doing positive work!

The work/kinetic energy theorem states that this will increase the kinetic energy of the system!



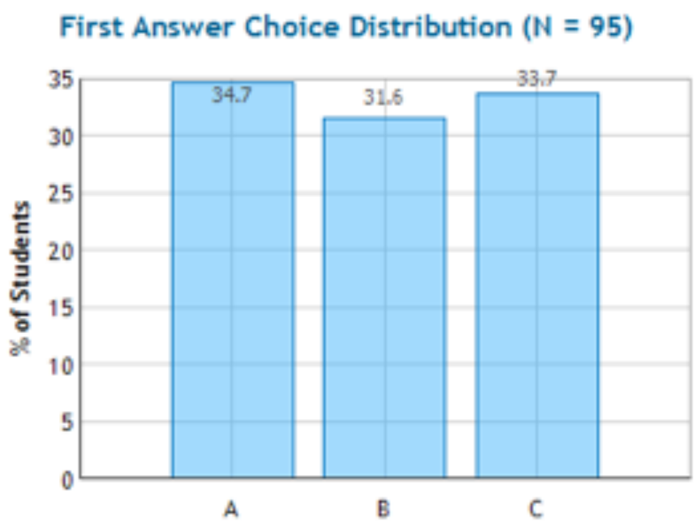
Vector Nature of Angular Momentum

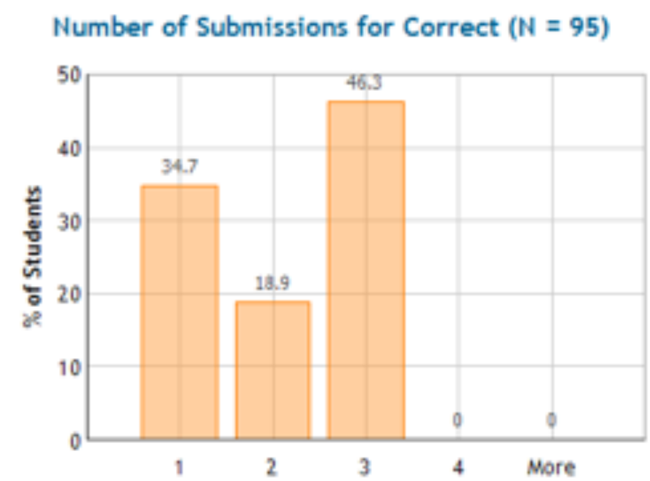


I would like to see the student sitting in the chair with a top in a real life demo.

A student is initially at rest on a stool that can rotate freely without friction in the horizontal plane. The student is holding a wheel spinning as shown in (1) . He turns the wheel over and as a result he and the stool start to rotate (2). If he keeps turning the wheel over in the same direction until it ends up in its original orientation (3), his rotation will...

- stop.
- double.
- stay the same.

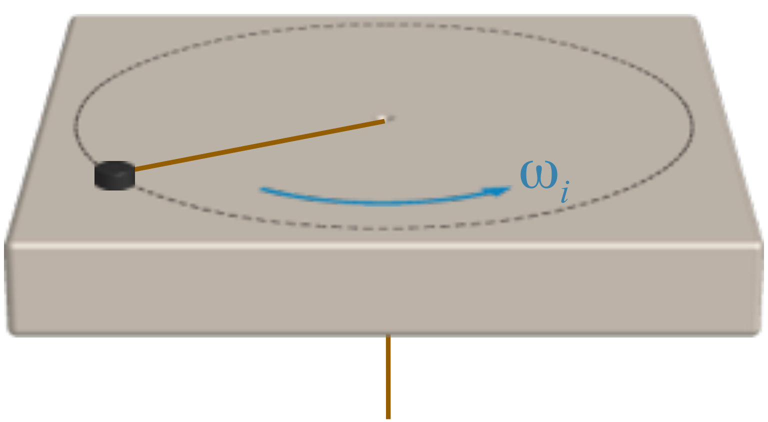


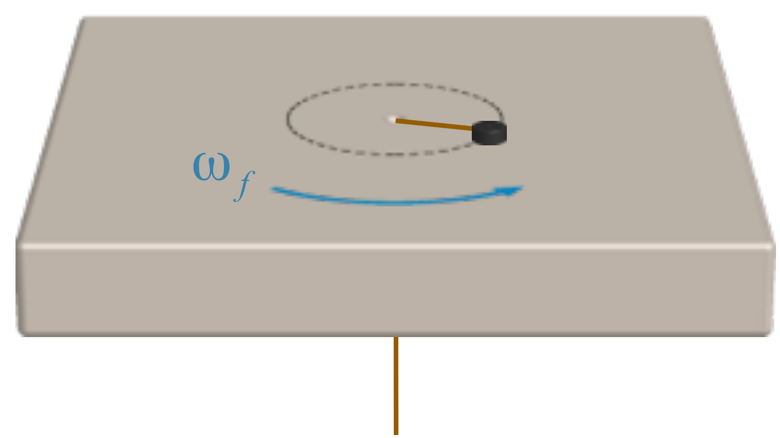


Clicker Question (like CheckPoint)

A puck slides in a circular path on a horizontal frictionless table. It is held at a constant radius by a string threaded through a frictionless hole at the center of the table. If you pull on the string such that the radius decreases by a factor of 2, by what factor does the angular velocity of the puck increase?

A) 2 B) 4 C) 8



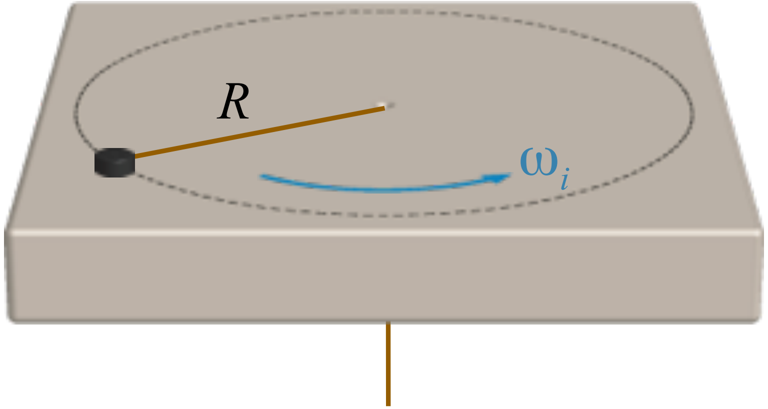


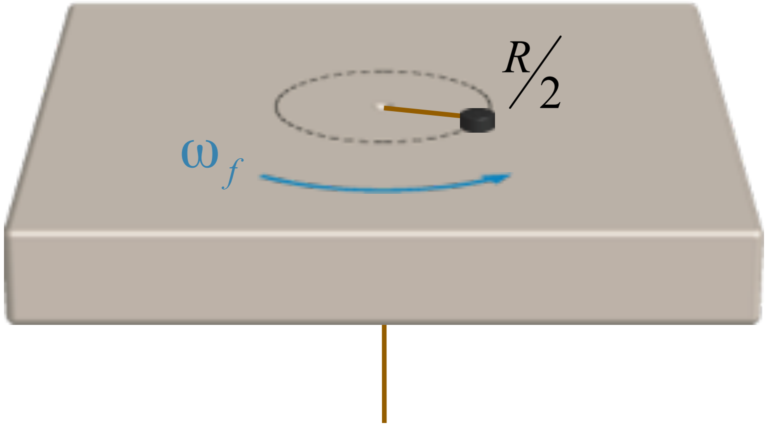
Clicker Question

$$L_1 = I_1\omega_1 = mR^2\omega_1 = L_2 = I_2\omega_2 = m\left(\frac{R}{2}\right)^2\omega_2$$

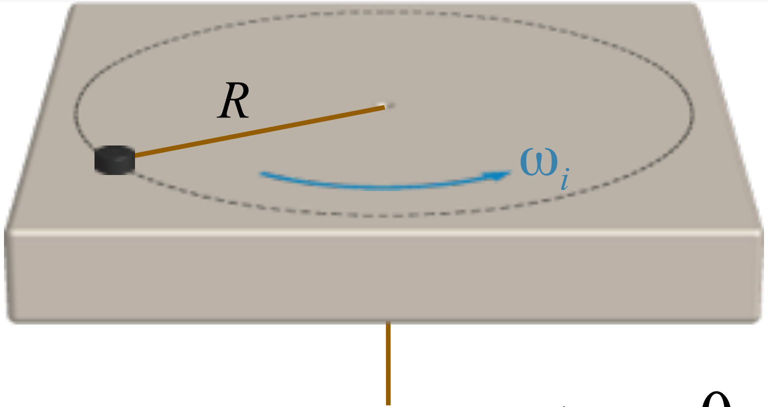
$$mR^2\omega_1 = m\frac{1}{4}R^2\omega_2$$

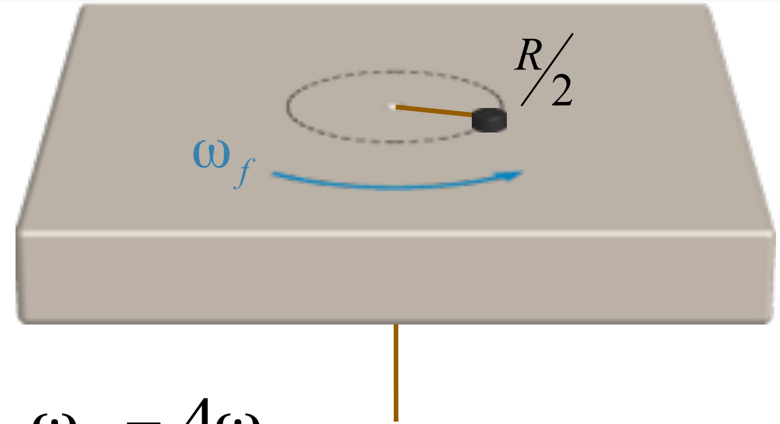
$$\omega_1 = \frac{1}{4}\omega_2 \rightarrow \omega_2 = 4\omega_1$$





Food for thought (not on any test)





$$\tau_{ext} = 0$$

$$\omega_2 = 4\omega_1$$

$$\rightarrow \alpha \neq 0$$

But $\tau = I\alpha$ So how do we get an α without a τ ?

$$\tau = \frac{dL}{dt} = \frac{d(I\omega)}{dt} = \underbrace{\frac{dI}{dt}}_{\text{usually 0, but not now}}\omega + I\underbrace{\frac{d\omega}{dt}}_{I\alpha}$$

Food for thought (not on any test)

$$\boxed{\tau_{EXT} = I\alpha + \omega \frac{dI}{dt}}$$

Now suppose $\tau_{EXT} = 0$:

$$I\alpha + \omega \frac{dI}{dt} = 0 \qquad \alpha = -\frac{\omega}{I}\frac{dI}{dt}$$

So in this case we can have an α without an external torque

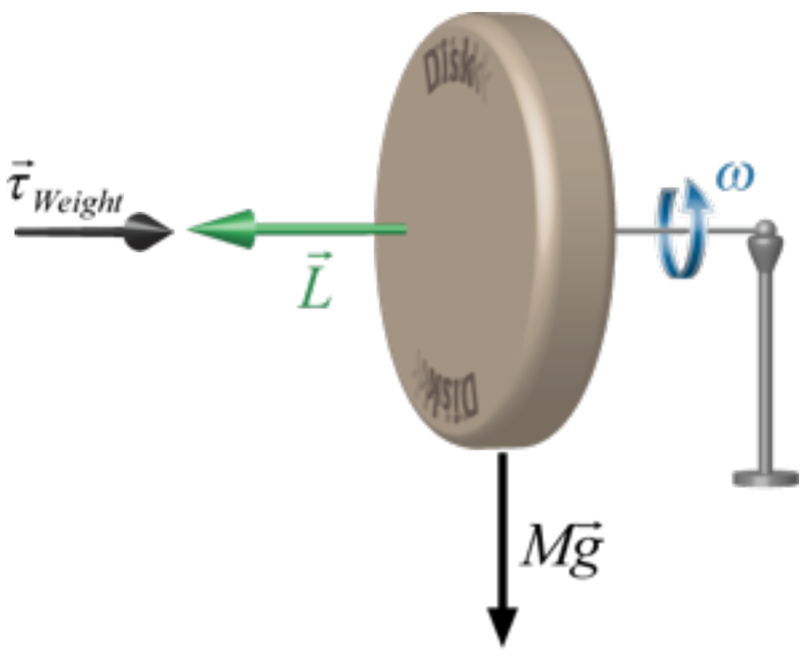
Precession

The magnitude of the torque about the pivot is $\tau = Mgd$.

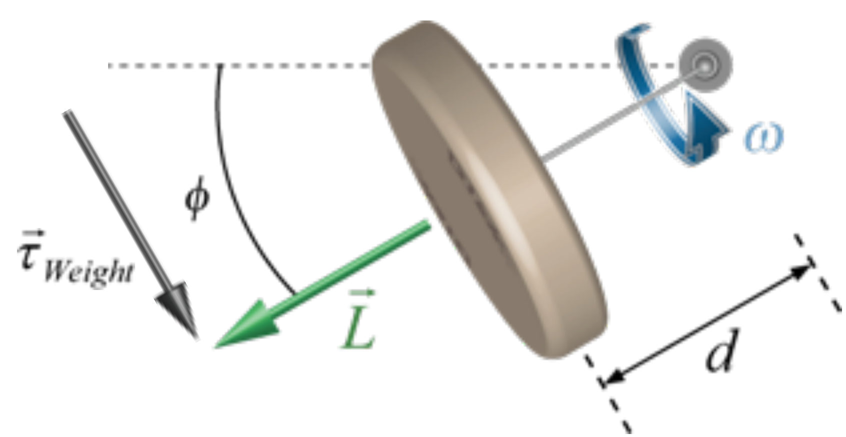
The direction of this torque at the instant shown is out of the page (using the right hand rule).

$$\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$$

The change in angular momentum at the instant shown must also be out of the page!

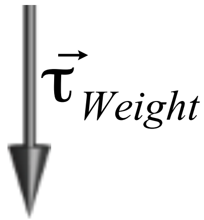


Side View with Perspective
($\vec{g}$ points down the page)

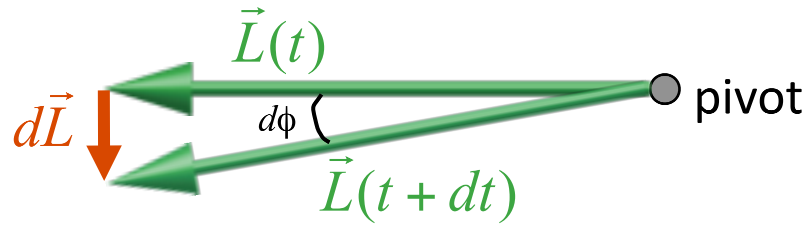


Aerial View
($\vec{g}$ points into the page)

Precession



Aerial View



$$dL = L_{top}\, d\phi$$

$$\underbrace{\frac{dL}{dt}}_{\tau_{EXT}} = L_{top} \underbrace{\frac{d\phi}{dt}}_{\Omega} = L_{top}\,\Omega \quad \rightarrow \quad \Omega = \frac{\tau_{ext}}{L_{top}}$$

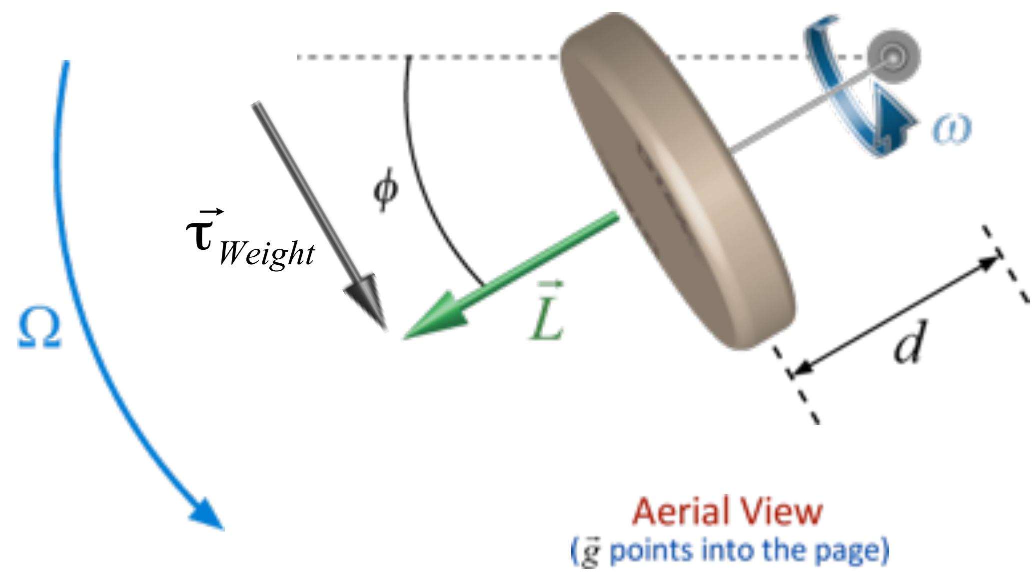
Precession

$$\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}} \rightarrow \boxed{\Omega = \frac{\tau_{ext}}{L_{top}}}$$

In this example:

$$\tau_{ext} = Mgd$$
$$L_{top} = I\omega$$
$$\rightarrow \quad \Omega = \frac{Mgd}{I\omega}$$

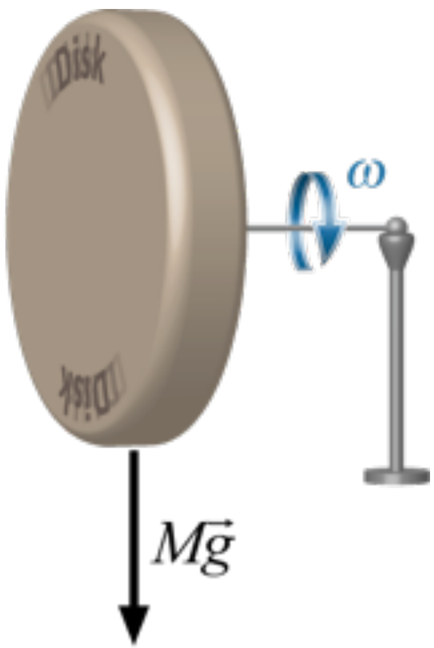
Direction:
The tip of L moves
in the direction of τ.



CheckPoint

A disk is spinning with angular velocity ω on a pivoted horizontal axle as shown. Gravity acts down. In which direction does precession cause the disk to move?

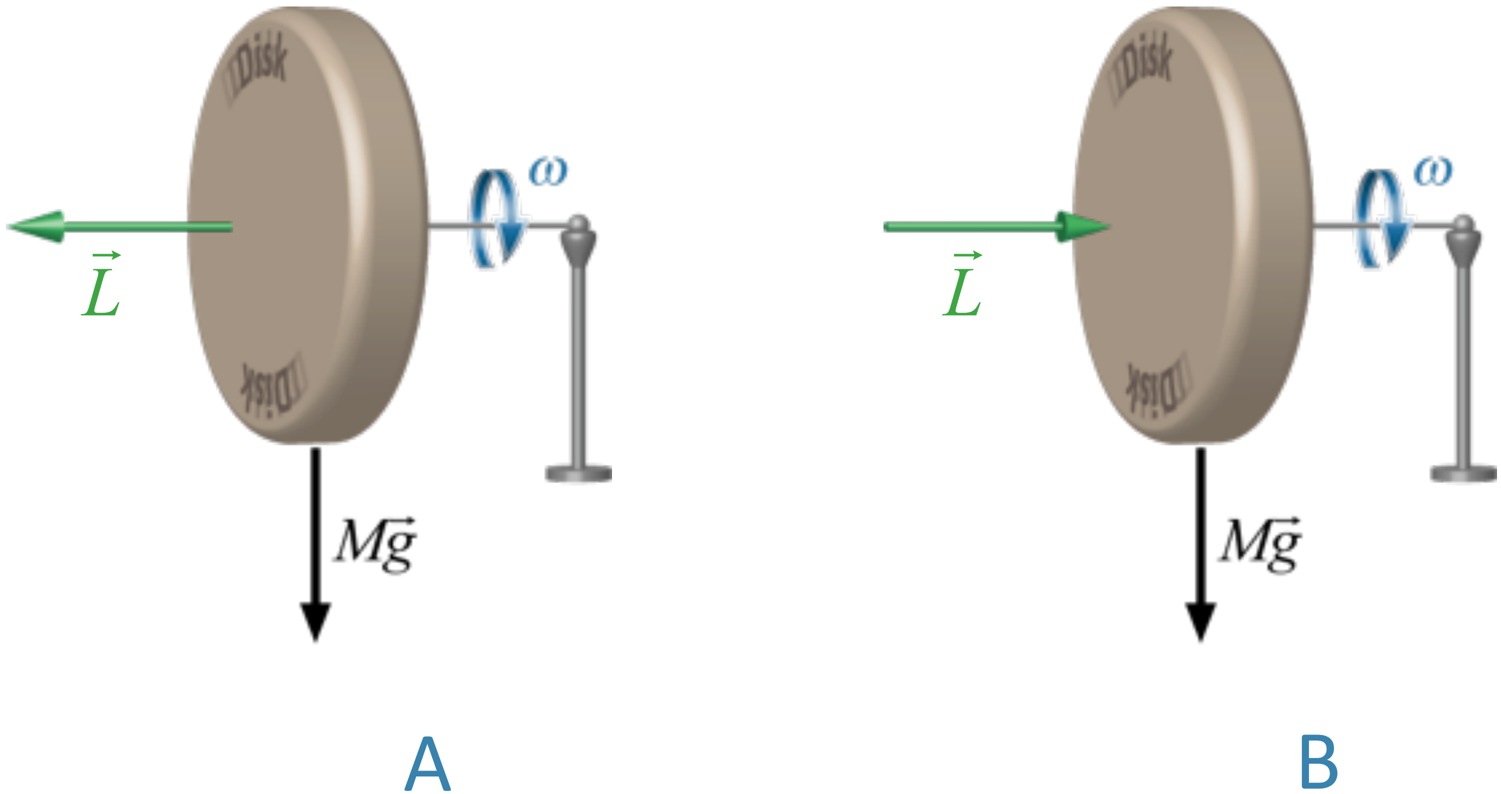
A) Out of the page B) Into the page C) Up D) Down



Torque is out of the page

CheckPoint

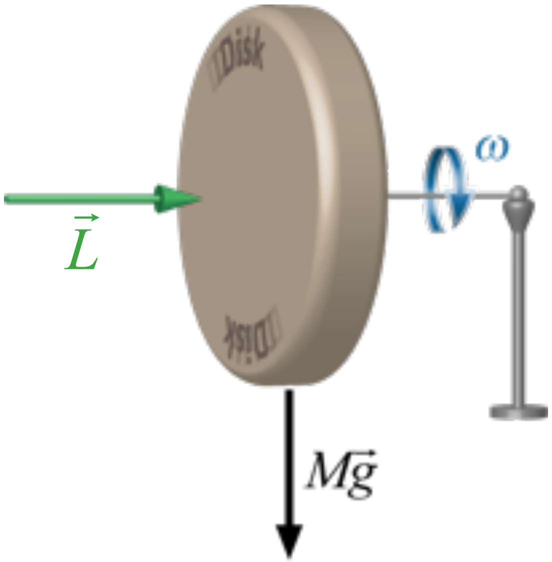
In which direction does $\vec{L}$ point?



CheckPoint

In which direction does precession cause the disk to move?

A) Into the page
B) Out of the page
C) Up
D) Down



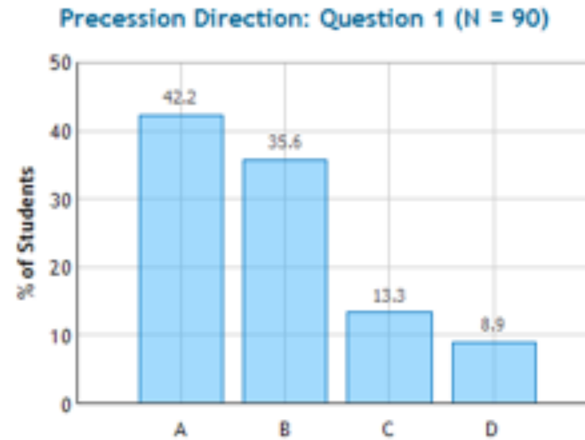
$$\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$$

Torque is out of the page

A) The torque provided by gravity is going out of the page. Therefore, L must change out of the page. However, since L is in the opposite direction of the axel when drawn from the pivot, precession must make the disk go into the page.

B) Gravity exerts a net torque out of the page, so the disk precesses in a direction out of the page.

D) Torque is down so precession is down

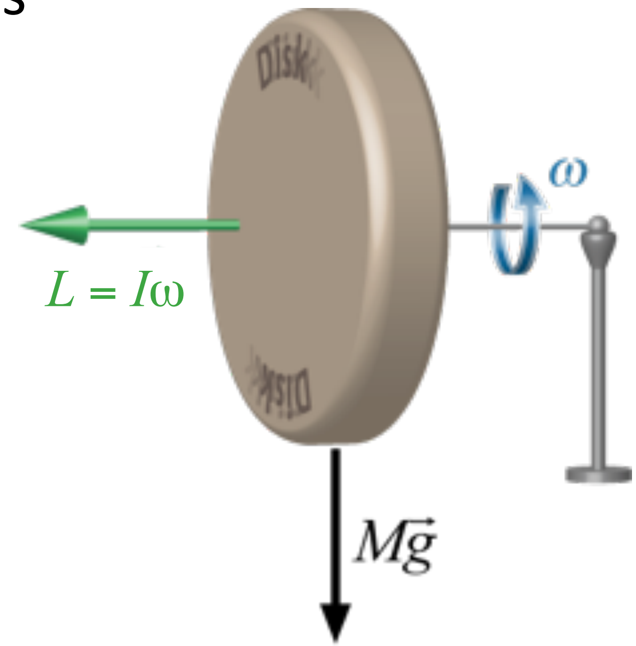


http://www.smartphysics.com/smartphysics/course/instructor/charts/stdhw_choice_histogram.aspx?qid=593054

Clicker Question

A disk is spinning with angular velocity ω on a pivoted horizontal axle as shown. If the mass of the disk were doubled but its radius and angular velocity were kept the same:

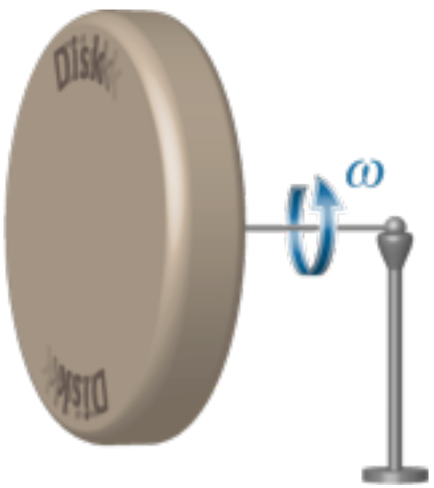
A) The angular momentum of the disk doubles
B) The torque about the pivot doubles
C) Both A and B



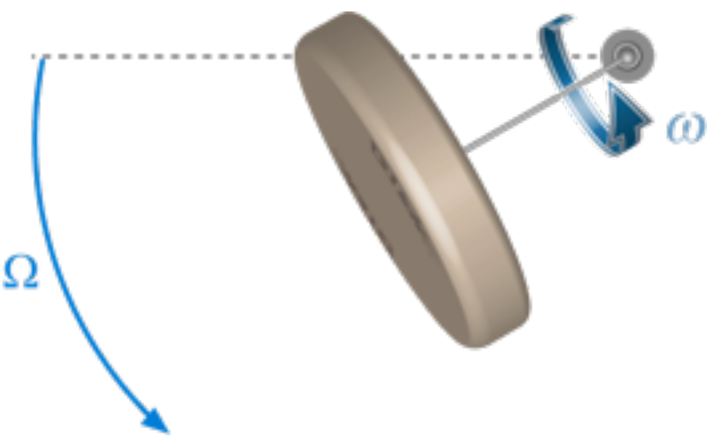
CheckPoint

A disk is spinning with angular velocity ω on a pivoted horizontal axle as shown. Gravity acts down and the disk has a precession frequency Ω. If the mass of the disk were doubled but its radius and angular velocity were kept the same, the precession frequency would:

A) Increase B) Decrease C) Stay the same



Side View with Perspective
($\vec{g}$ points down the page)



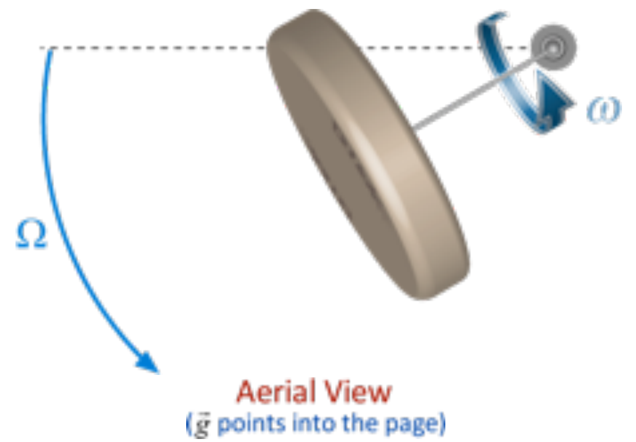
Aerial View
($\vec{g}$ points into the page)

CheckPoint

If the mass of the disk were doubled but its radius and angular velocity were kept the same, the precession frequency would

A) Increase
B) Decrease
C) Stay the same

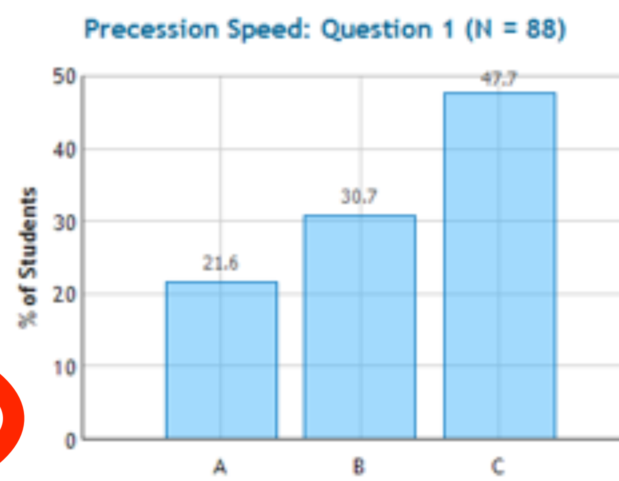
$$\Omega = \frac{\tau_{ext}}{L_{top}}$$



A) torque due to weight increases in the numerator.

B) When mass is doubled, angular momentum doubles, so precession frequency decreases.

C) Increasing the mass increases both the angular momentum and the torque equally, so the precession frequency stays the same



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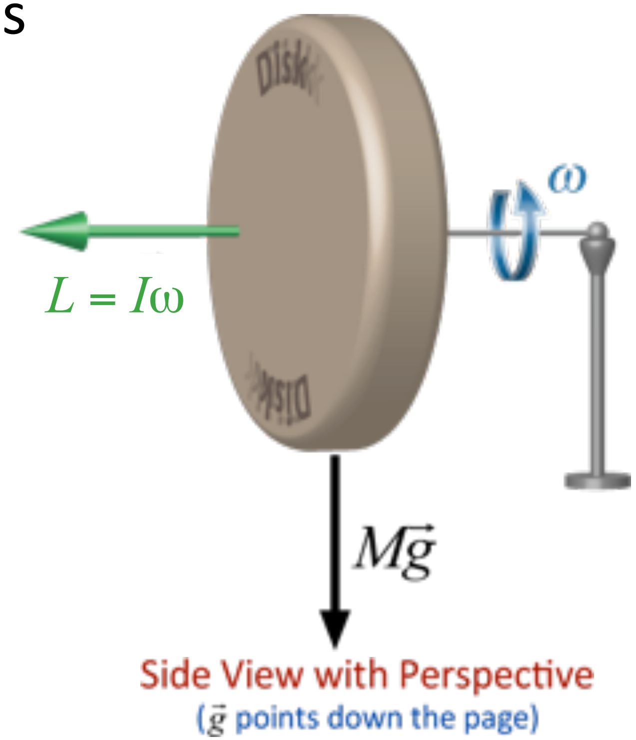

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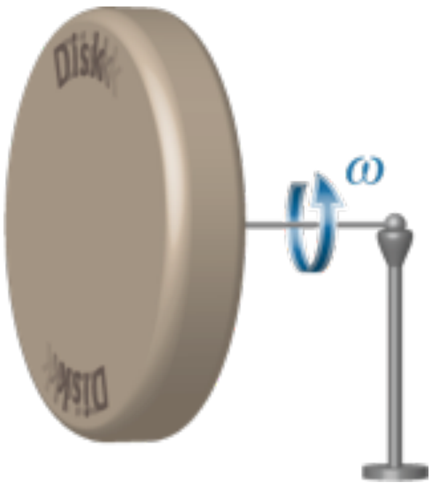
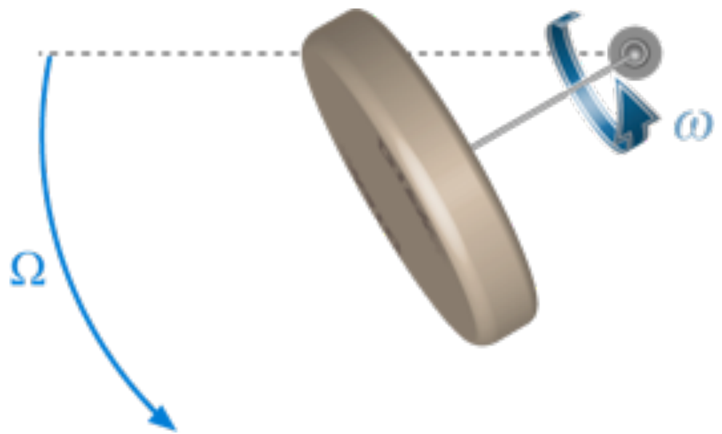
A) Increase B) Decrease C) Stay the same



Side View with Perspective
($\vec{g}$ points down the page)



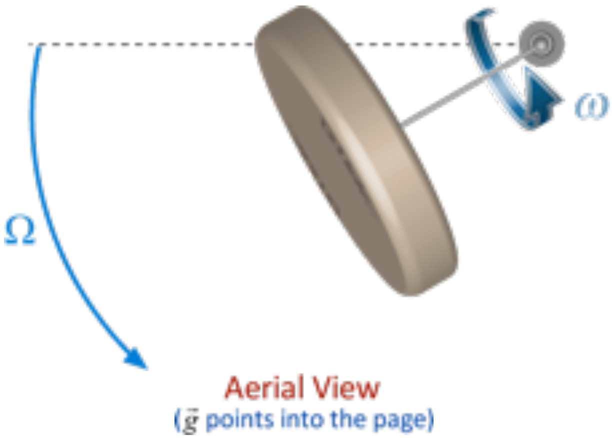
Aerial View
($\vec{g}$ points into the page)

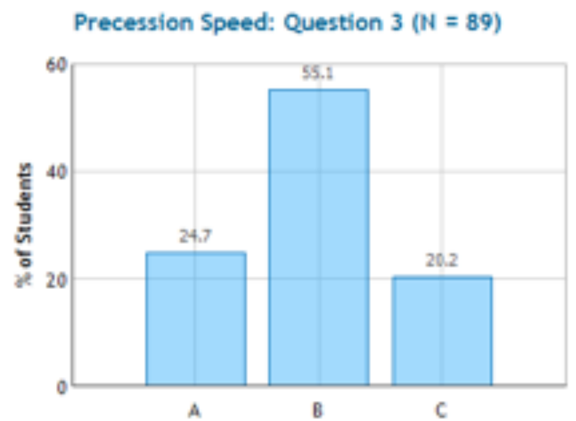
CheckPoint

If the radius of the disk were doubled but its mass and angular velocity were kept the same, the precession frequency would

A) Increase
B) Decrease
C) Stay the same

$$\Omega = \frac{\tau_{ext}}{L_{top}}$$





A) The moment of inertia would increase, so the angular momentum would increase, so omega would also increase

B) If you increase radius, you increase angular momentum, and thus decrease the precession frequency.

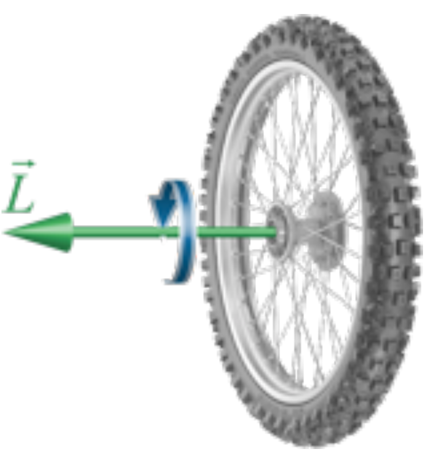
C) the frequency would remain the same because the radius has no impact on the frequency

All of this stuff doesn't really seem practical. If you can't prove me wrong then we should all just get A's for this section of the course.

http://www.youtube.com/watch?v=cquvA_IpEsA (see 2:30)

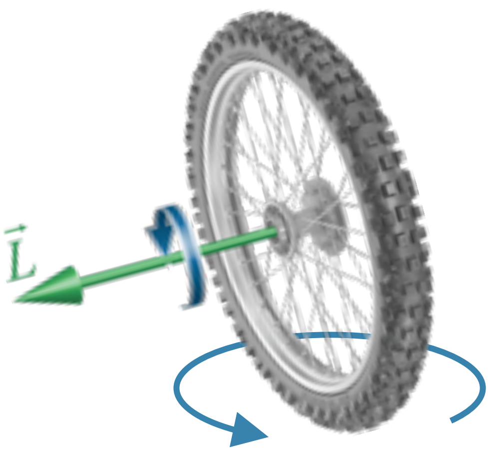
Practical Application: Keeps you from falling off your bike when you ride using no hands!

Riding straight
($\tau = 0$)



Wheel steers straight

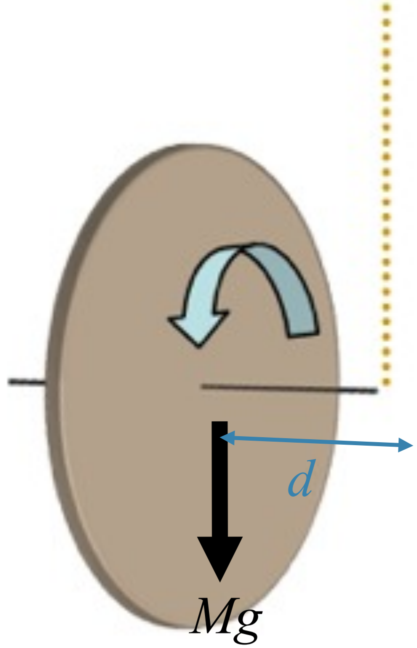
Lean Left
(τ out of page)



Wheel steers left

Lean Right
(τ into page)

Wheel steers right



$$I = \frac{1}{2}MR^2 \qquad \omega = 2\pi f$$

A disk with mass m = 5.2 kg and radius R = 0.43 m hangs from a rope attached to the ceiling. The disk spins on its axis at a distance r = 1.22 m from the rope and at a frequency f = 18.2 rev/s (with a direction shown by the arrow).

1) What is the magnitude of the angular momentum of the spinning disk?

kg-m^2/s Submit

$$L = I\omega$$

2) What is the torque due to gravity on the disk?

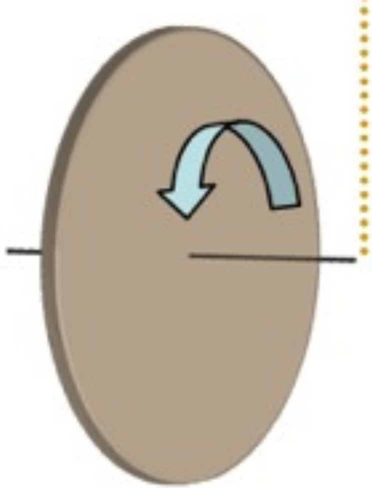
N-m Submit

$$\tau = Mgd$$

3) What is the period of precession for this gyroscope?

s Submit

$$\Omega = \frac{\tau}{L}$$



L (using right hand rule)

A disk with mass m = 5.2 kg and radius R = 0.43 m hangs from a rope attached to the ceiling. The disk spins on its axis at a distance r = 1.22 m from the rope and at a frequency f = 18.2 rev/s (with a direction shown by the arrow).

4) What is the direction of the angular momentum of the spinning disk at the instant shown in the picture?

- up
- down
- left
- right

Submit

5) What is the direction of the precession of the gyroscope?

- it does not precess
- clockwise as seen from above (looking down the rope)
- counterclockwise as seen from above (looking down the rope)