Optics, Electricity & Magnetism
Lecture 3

Today's Concepts:

A) The Electric Field

B) Continuous Charge Distributions

1. Course website: https://www.sfu.ca/phys/121/1264/

2. Tutorial, Achieve Prelecture and Bridge, iClicker, their grades are not counted in the first week

3. Online Achieve homework will be counted, but the deadline will be postponed to May 16 (Sat) 1159pm

Electric Field

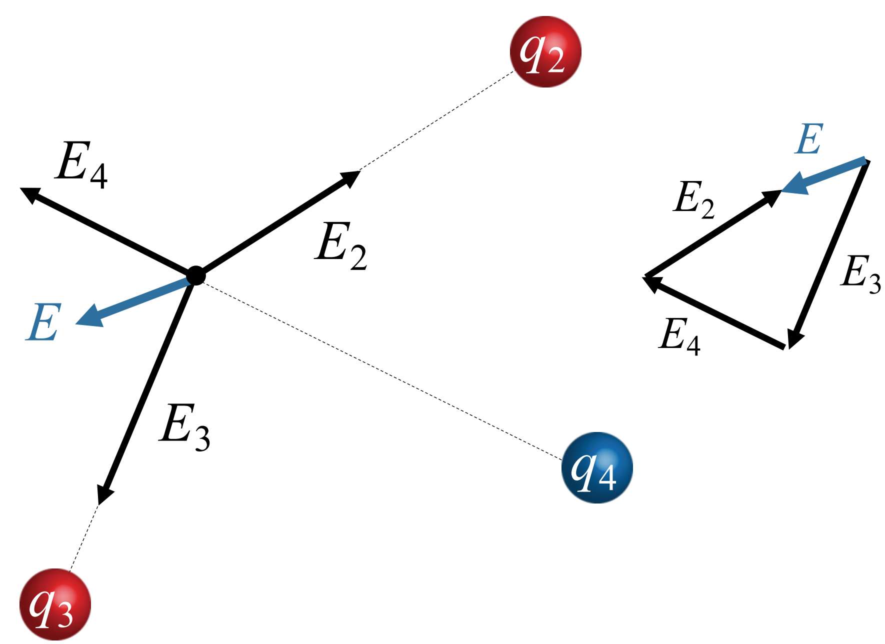
In electrostatics, the electric field E at a point in space is simply the force per unit charge at that point.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle *Q*

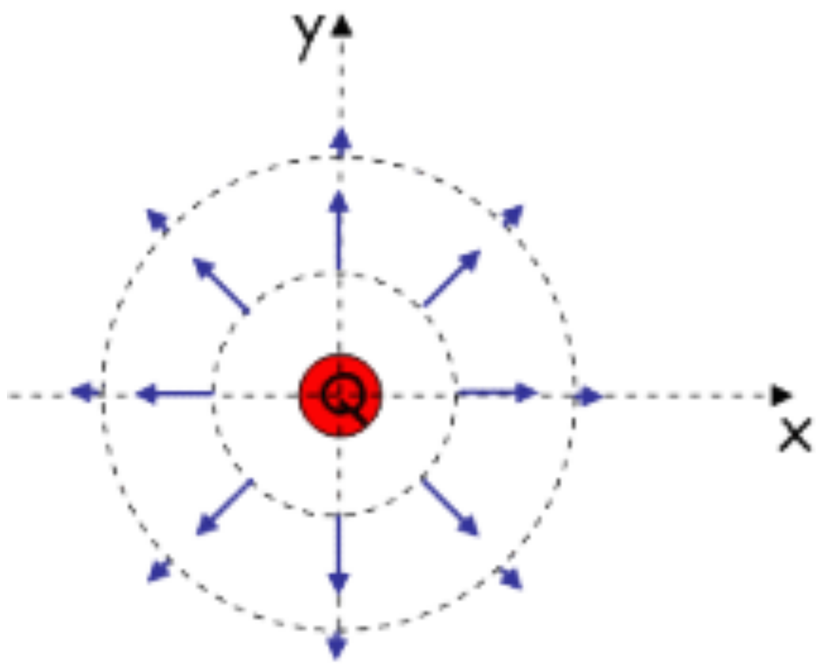
$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

Superposition $\vec{E} = \sum_i \vec{E}_i = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$

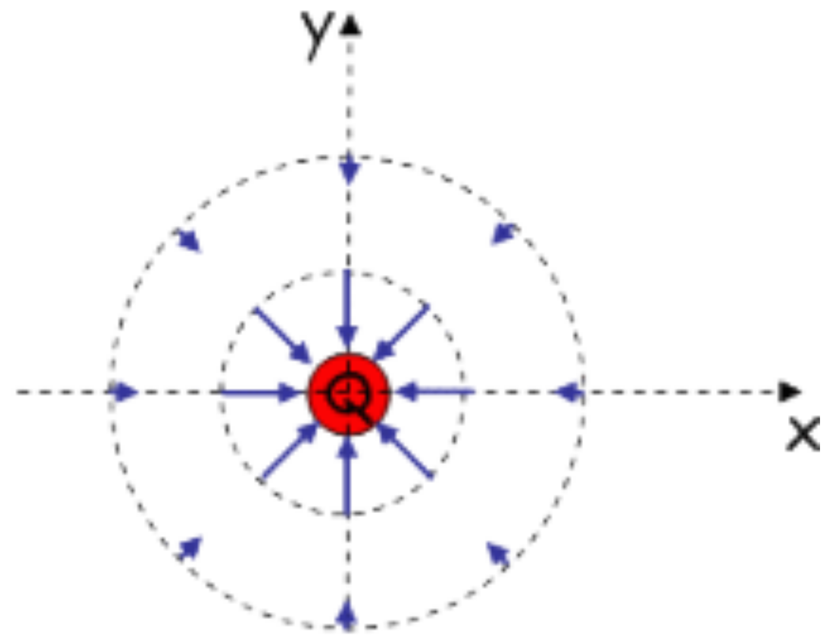


Field points toward negative and away from positive source charges. (field direction – imagine a positive test charge)

Example: electric field of a point charge

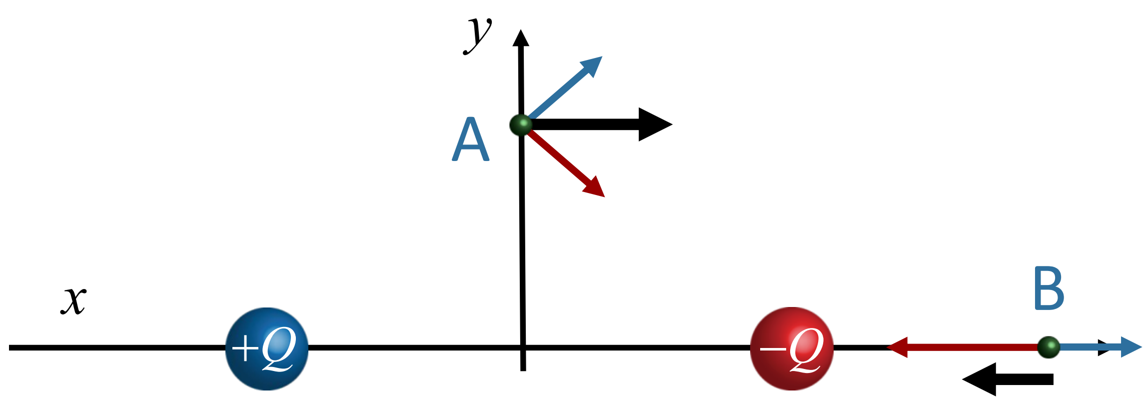


Postive Q



Negative Q

Bridge 1



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

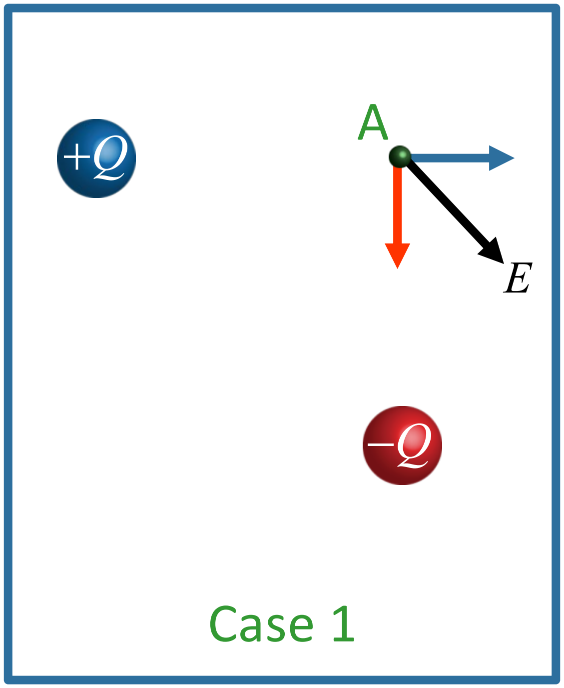
2) What is the direction of the electric field at point A?

○ Up ○ Down ○ Left ○ Right ○ Zero

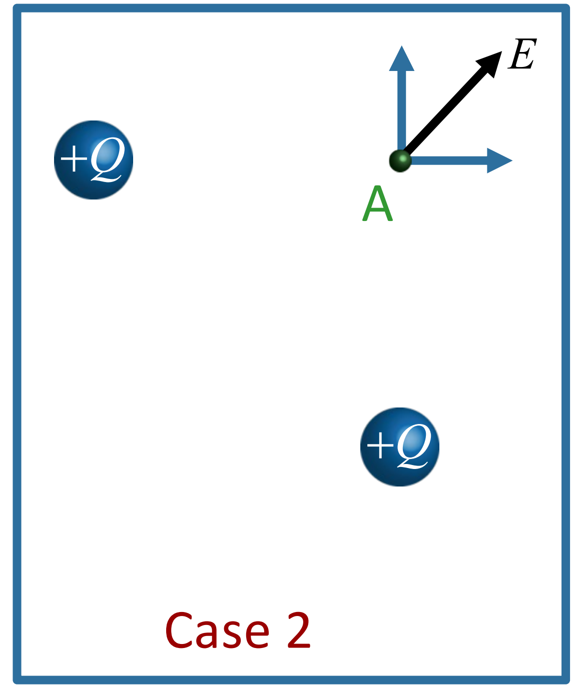
What is the direction of the electric field at point B?

○ Up ○ Down ○ Left ○ Right ○ Zero

Bridge 2



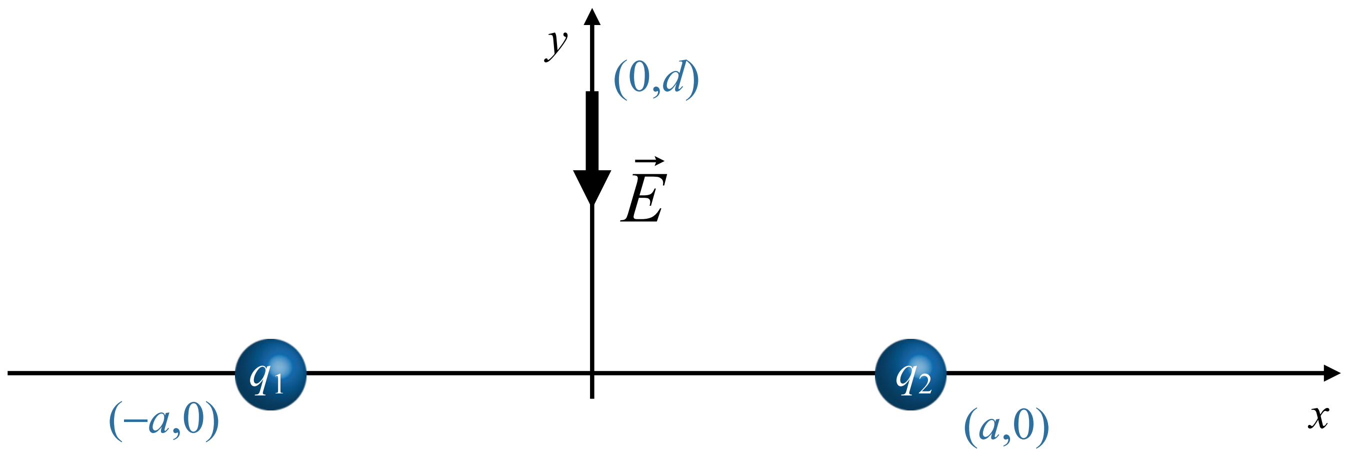
In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?



- x components are the same
- y components are the same in magnitude, but opposite in direction
- By Pythagorean theorem, the total fields are **same in magnitude**

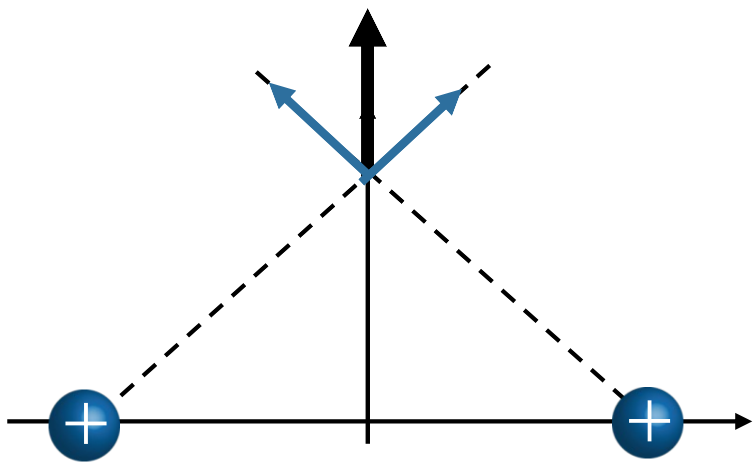
Two Charges

Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.

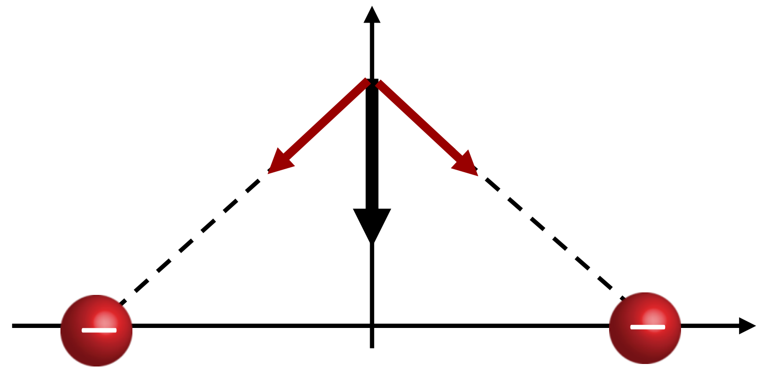


Which of the following statements is true:

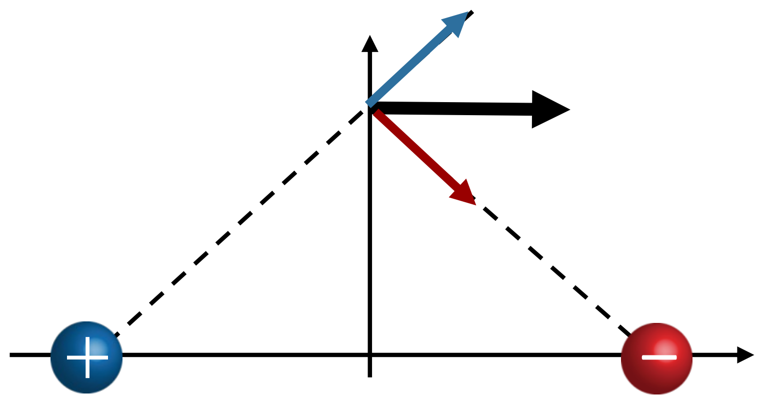
A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related



Both positive:
y-component always points up



Both negative:
y-component always points down
We just need x-component to vanish



Opposite charge:
x-component does not vanish

Bridge 3

A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q.

1) How will the test charge move immediately after being released?

A) to the left B) to the right C) stay still D) other

$$F_Q \propto \frac{Q}{r^2}$$

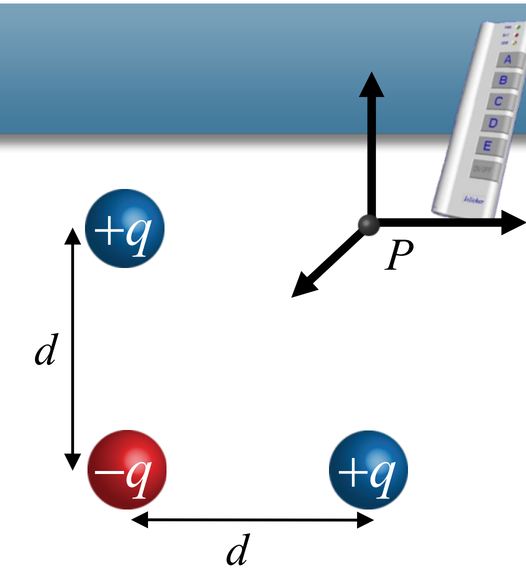
$$F_{2Q} \propto \frac{2Q}{(2r)^2} = \frac{Q}{2r^2} < F_Q$$

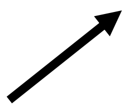
all charges positive, repel

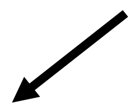
∴ q will move to the right.

Example

What is the direction of the electric field at point P, the unoccupied corner of the square?



A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{(\sqrt{2}d)^2}\cos\frac{\pi}{4}\right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{(\sqrt{2}d)^2}\sin\frac{\pi}{4}\right)$$

Continuous Charge Distributions

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

WHAT DOES THIS MEAN ?

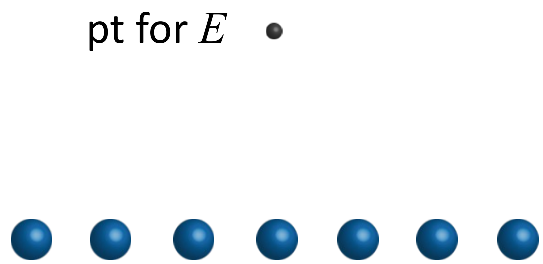
Integrating (summing) over all charges (dq)

$\hat{r}$ is vector from dq to the point at which $\vec{E}$ is defined

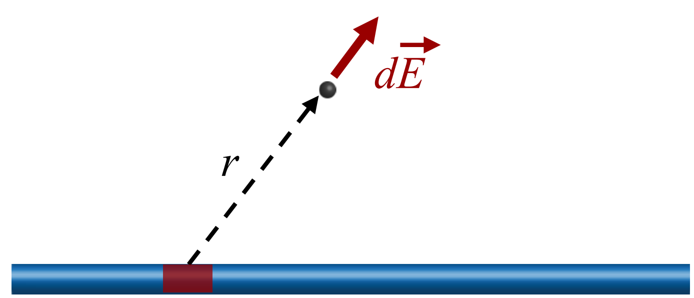
Line charge Example:

$$\lambda = Q/L$$

pt for E



charges



$dq = \lambda\, dx$

Charge Density

Linear $(\lambda = Q/L)$ Coulomb/metre

Surface $(\sigma = Q/A)$ Coulomb/metre^2

Volume $(\rho = Q/V)$ Coulomb/metre^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?

A) A sphere w/ radius R= 2 meters and volume charge density ρ = 2 C/m^3

B) $\boxed{\text{A sphere w/ radius R= 2 meters and surface charge density } \sigma = 2\ \text{C/m}^2}$

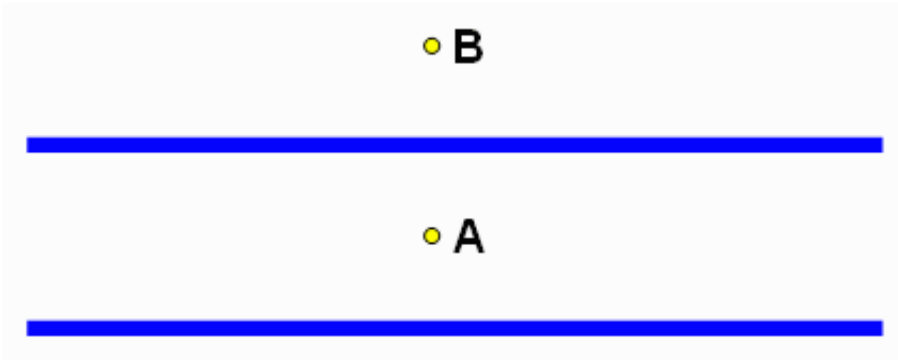
C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3 \mathbf{=64/3\ \pi\ C}$$

$$Q_B = \sigma A = \sigma 4\pi R^2 \mathbf{=32\ \pi\ C}$$

Bridge 4

Two infinite lines of charge are shown below.



Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown. How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- $E_A < E_B$
- $E_A = E_B$
- $E_A > E_B$

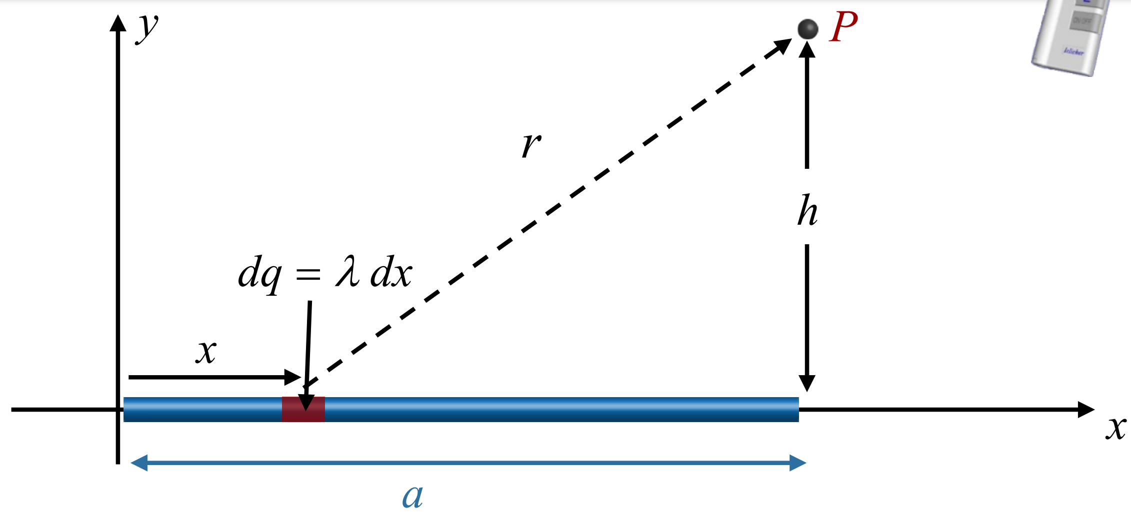
Point A: in the middle, E field from both lines cancel each other

Point B: E field from both lines are pointing up

E field from uniform, infinitely long line charge: $E = \dfrac{2k\lambda}{r}$

Example

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



*See also Tipler Sec. 22-1

We know:

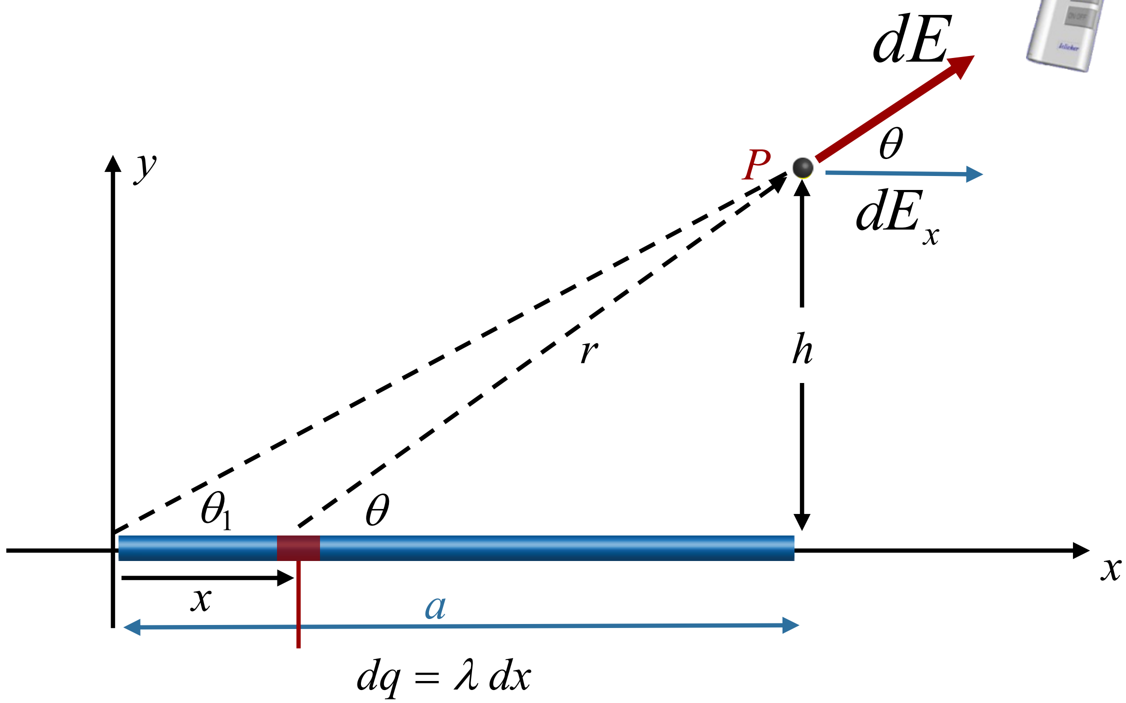
$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$?

A) $\dfrac{dx}{x^2}$ B) $\dfrac{dx}{a^2 + h^2}$ C) $\dfrac{\lambda dx}{a^2 + h^2}$ D) $\boxed{\dfrac{\lambda dx}{(a - x)^2 + h^2}}$ E) $\dfrac{\lambda dx}{x^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int \underbrace{k\frac{dq}{r^2}\hat{r}}_{d\vec{E}}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x$$

What is dE_x?

A) $dE\cos\theta_1$

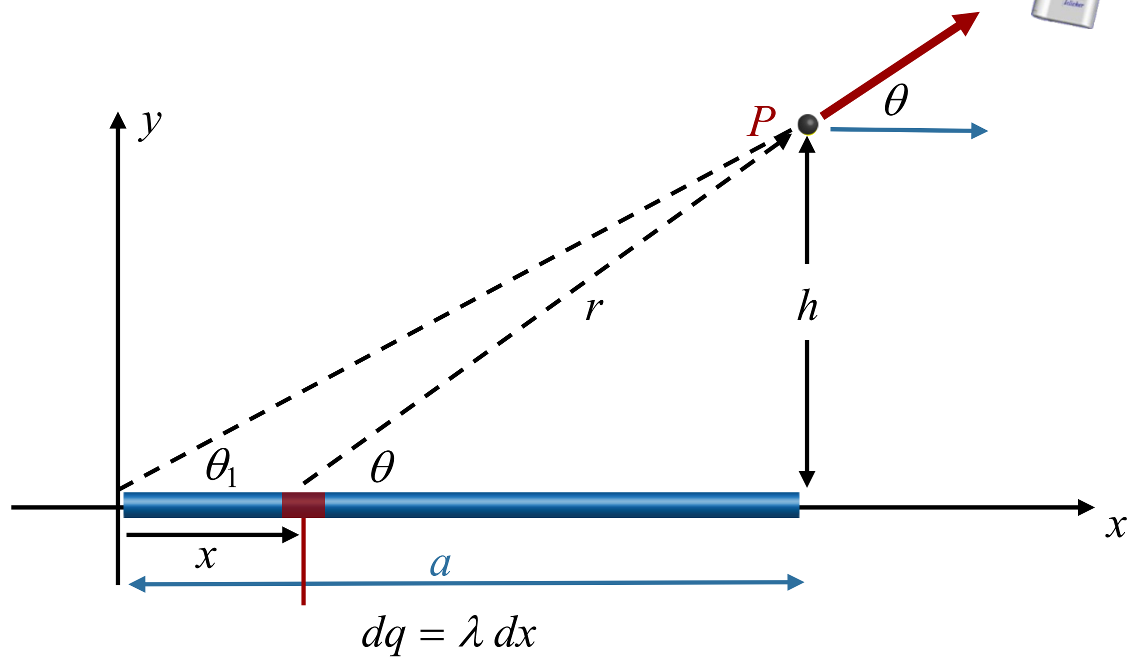
B) $\boxed{dE\cos\theta}$

C) $dE\sin\theta_1$

D) $dE\sin\theta$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos\theta$$

What is E_x ?

A) $\dfrac{\lambda \cos\theta}{4\pi\varepsilon_o} \displaystyle\int_{-\infty}^{\infty} \frac{dx}{(a-x)^2 + h^2}$

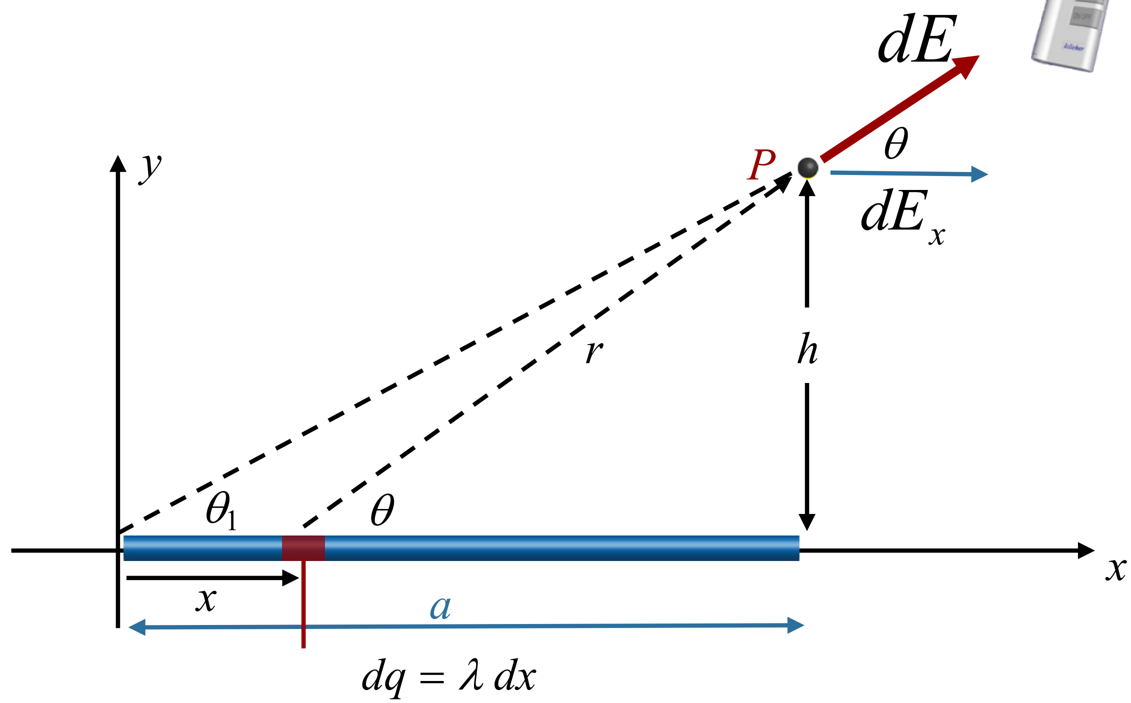
B) $\dfrac{\lambda \cos\theta}{4\pi\varepsilon_o} \displaystyle\int_{0}^{a} \frac{dx}{(a-x)^2 + h^2}$

$\boxed{\text{C) neither}}$

$\cos\theta$ DEPENDS ON x!

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos\theta$$

What is $\cos\theta$?

A) $\dfrac{x}{\sqrt{a^2 + h^2}}$

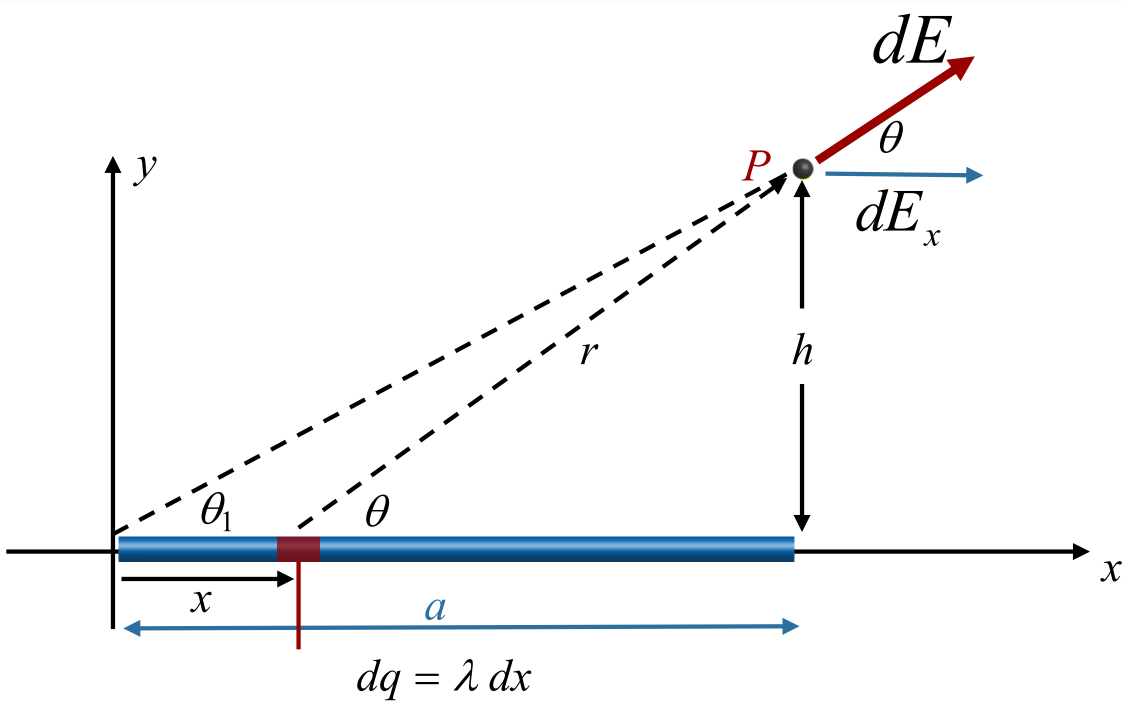
B) $\boxed{\dfrac{a-x}{\sqrt{(a-x)^2 + h^2}}}$

C) $\dfrac{a}{\sqrt{a^2 + h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2 + h^2}}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know: $$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

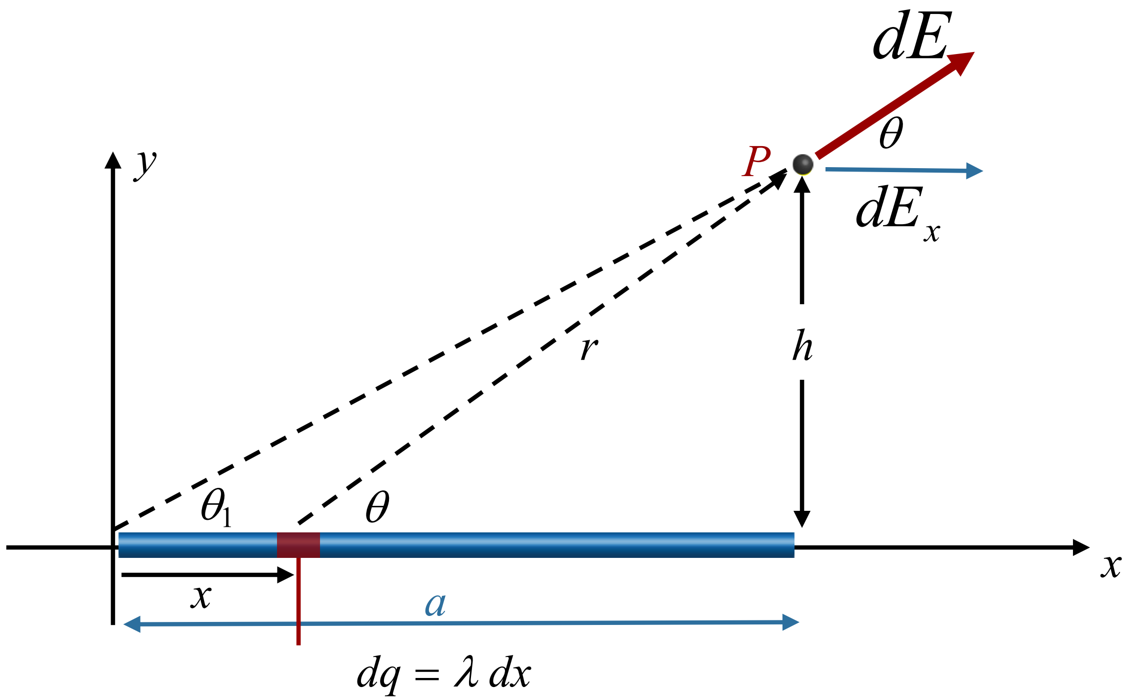
$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE_x = \int dE\cos\theta \qquad \cos\theta = \frac{a-x}{\sqrt{(a-x)^2+h^2}}$$

What is $E_x(P)$?

$$E_x(P) = \frac{\lambda}{4\pi\varepsilon_o}\int_0^a dx \frac{a-x}{\left((a-x)^2+h^2\right)^{3/2}} \quad \longrightarrow \quad E_x(P) = \frac{\lambda}{4\pi\varepsilon_o h}\left(1 - \frac{h}{\sqrt{h^2+a^2}}\right)$$

Observation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



Note that our result can be rewritten more simply in terms of θ_1.

$$E_x(P) = \frac{\lambda}{4\pi\varepsilon_o h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right) \longrightarrow E_x(P) = \frac{\lambda}{4\pi\varepsilon_o h}(1 - \sin\theta_1)$$

Exercise for student:

Change variables: write x in terms of θ

Result: obtain simple integral in θ

$$\longrightarrow E_x(P) = \frac{\lambda}{4\pi\varepsilon_o h}\int_{\theta_1}^{\pi/2} d\theta \cos\theta$$

Final thoughts

1. Tutorial starts this week.

2. Set up Achieve and iClicker ASAP

3. We will go over Ch22: Electric Flux and Field Lines in 1130am class.

4. Online homework due this Sat (May 16) 11:59pm.

5. Please complete Ch22: Gauss's Law prelecture and bridge before Friday 8am.