Optics, Electricity & Magnetism
Lecture 4

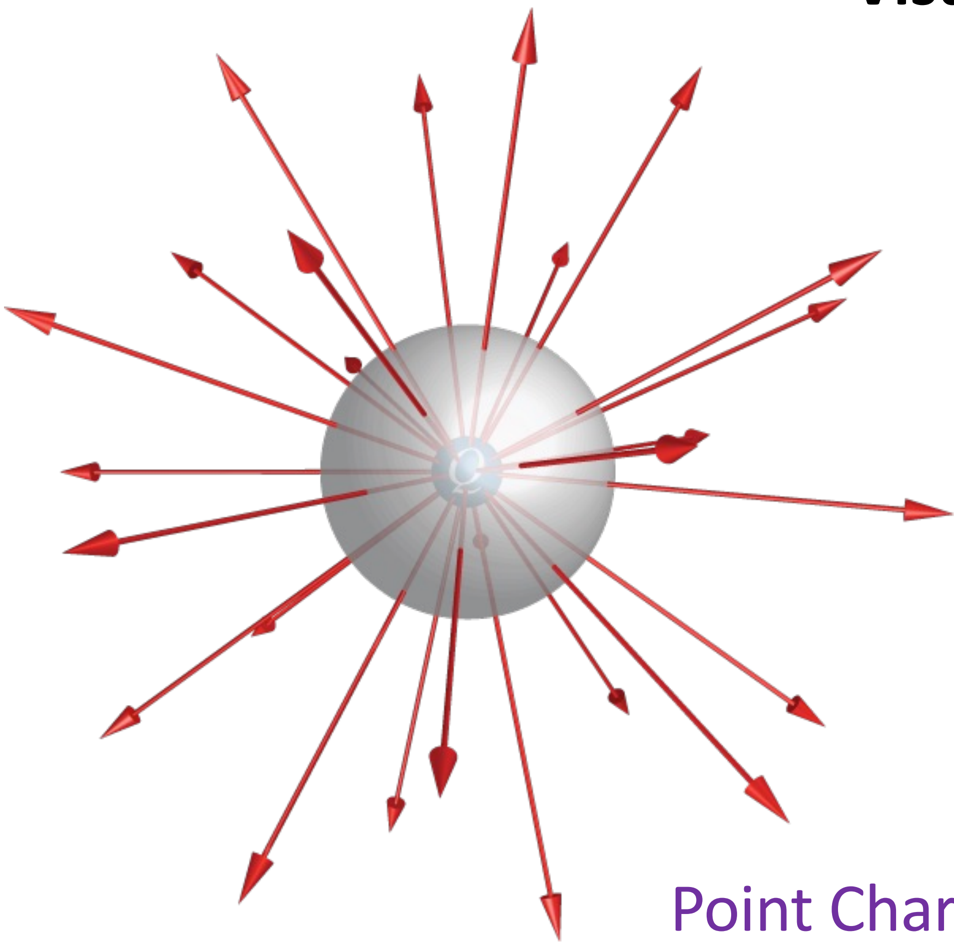
Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

Electric Field Lines



Visual representation of electric field:

Direction of Lines: Direction of E

- Start from +ve charge
- End in –ve charge
- No cross lines and no broken lines in free space

Density of Lines: Magnitude of E

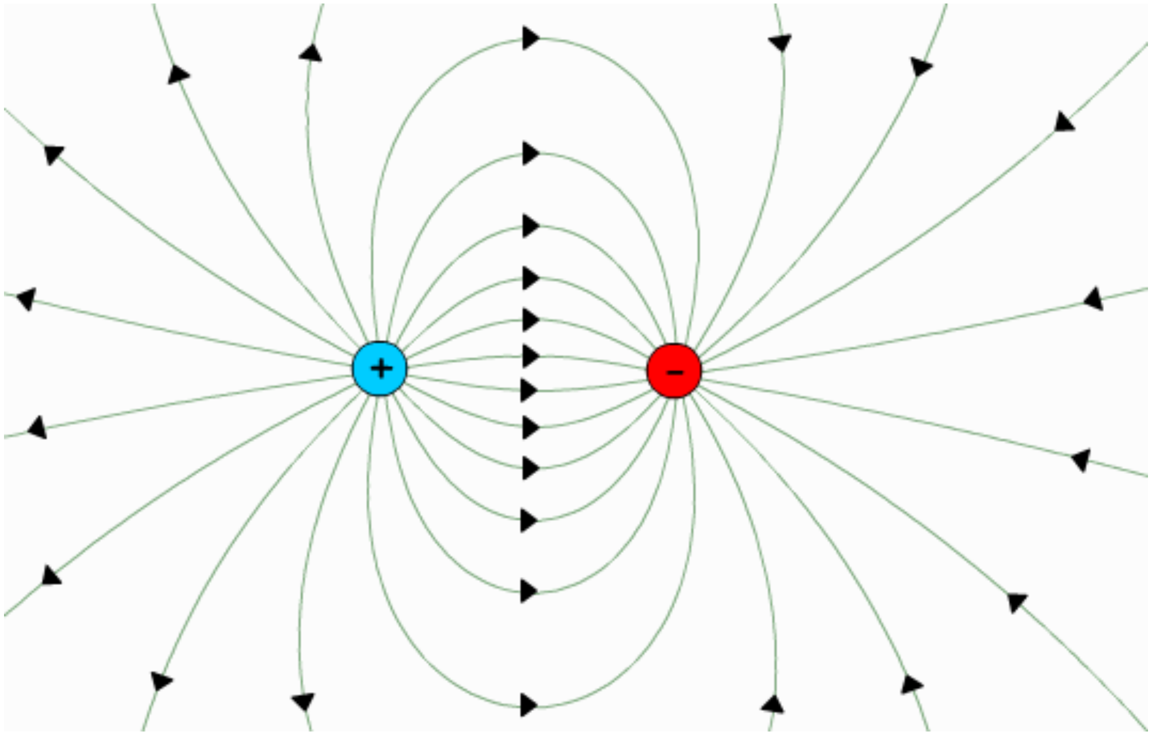
- More charge, more lines

Point Charge:

Direction is radial

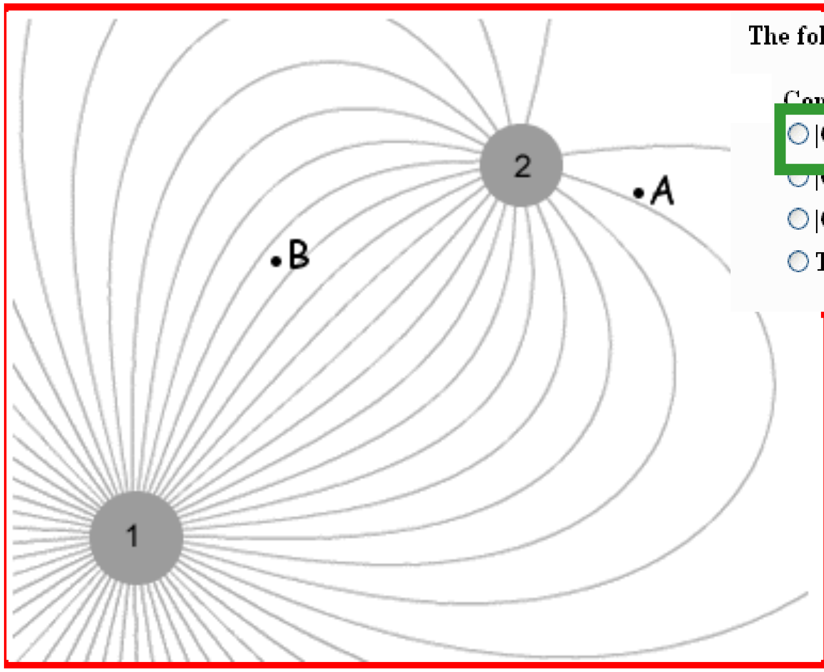
Density $\propto 1/R^2$

Electric Field Lines



Dipole Charge Distribution:
Direction & Density
much more interesting.

Bridge 3.1



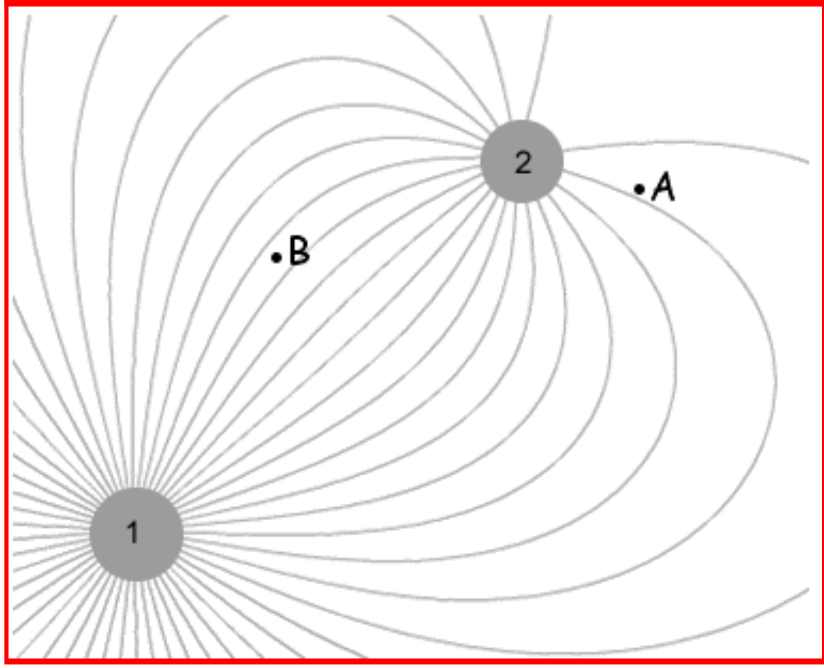
The following three questions pertain to the electric field lines due to two charges are shown above.

Compare the magnitude of the two charges

- ○ $|Q_1| > |Q_2|$
- ○ $|Q_1| = |Q_2|$
- ○ $|Q_1| < |Q_2|$
- ○ There isn't enough information to determine the relative magnitude of the charges.

More field lines are connecting to Q1 than Q2

Bridge 3.2

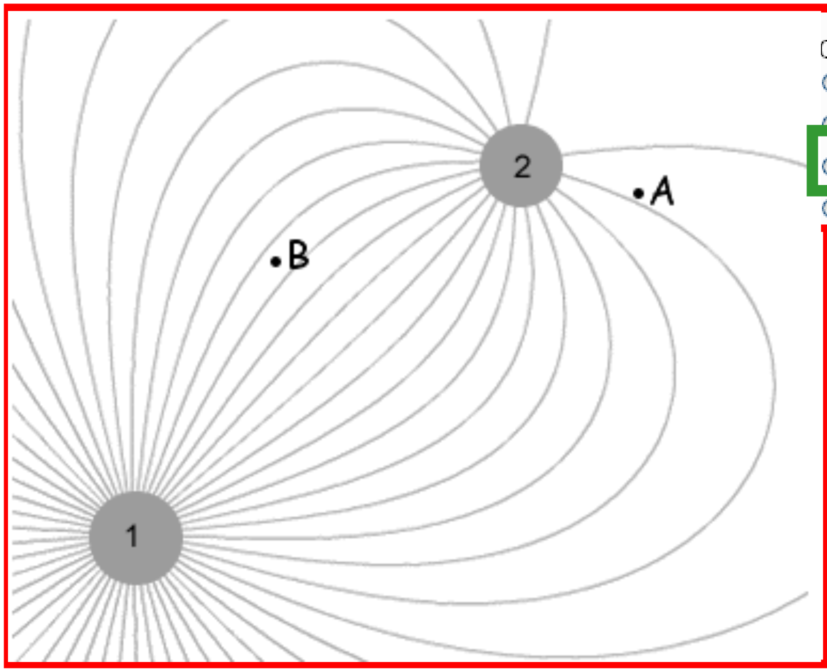


3)
What do we know about the signs of the charges from looking at the picture?

- Q_1 and Q_2 have the same sign
- Q_1 and Q_2 have opposite signs
- There is not enough information in the picture to determine the relative signs of the charges

Field lines start from +ve and end at –ve.
They do not connect charges with the same sign.

Bridge 3.3

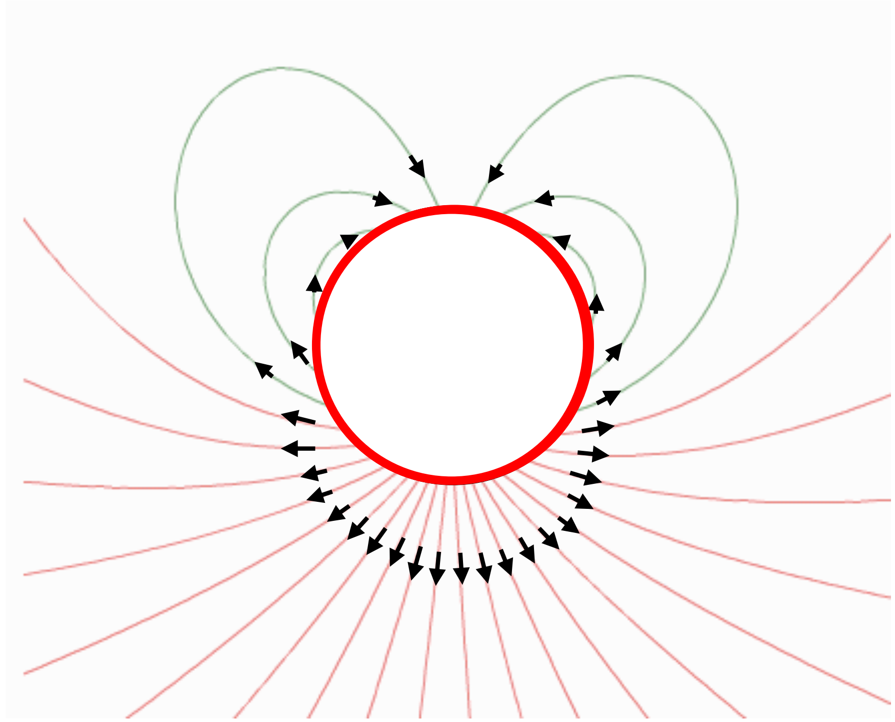


Compare the magnitude of the electric field at points A and B.

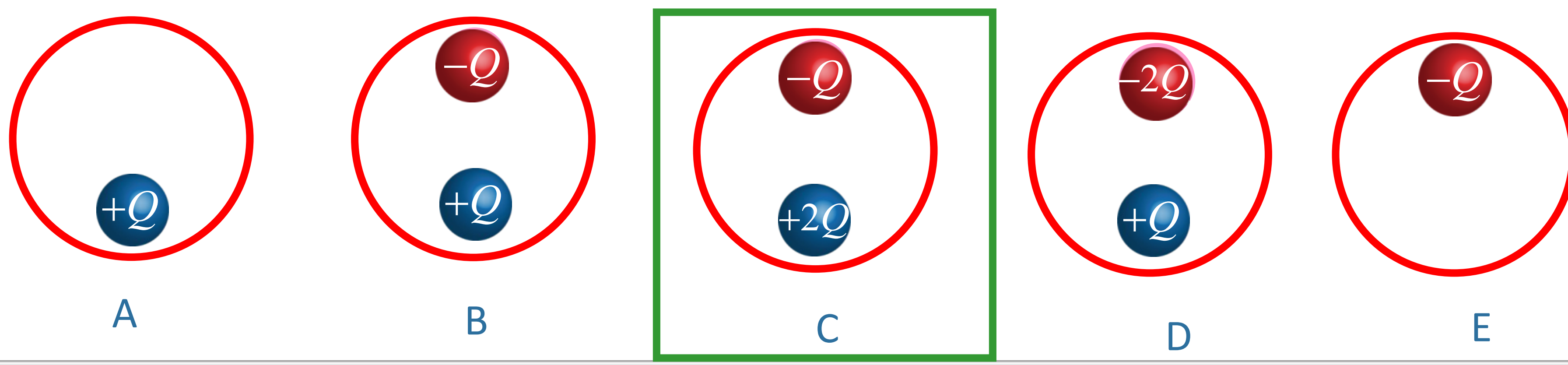
- ○ $|E_A| > |E_B|$
- ○ $|E_A| = |E_B|$
- ○ $|E_A| < |E_B|$
- ○ There isn't enough information to determine the relative magnitude of the electric field at points A and B..

Field lines are denser at point B than point A

Deducing charge from field lines

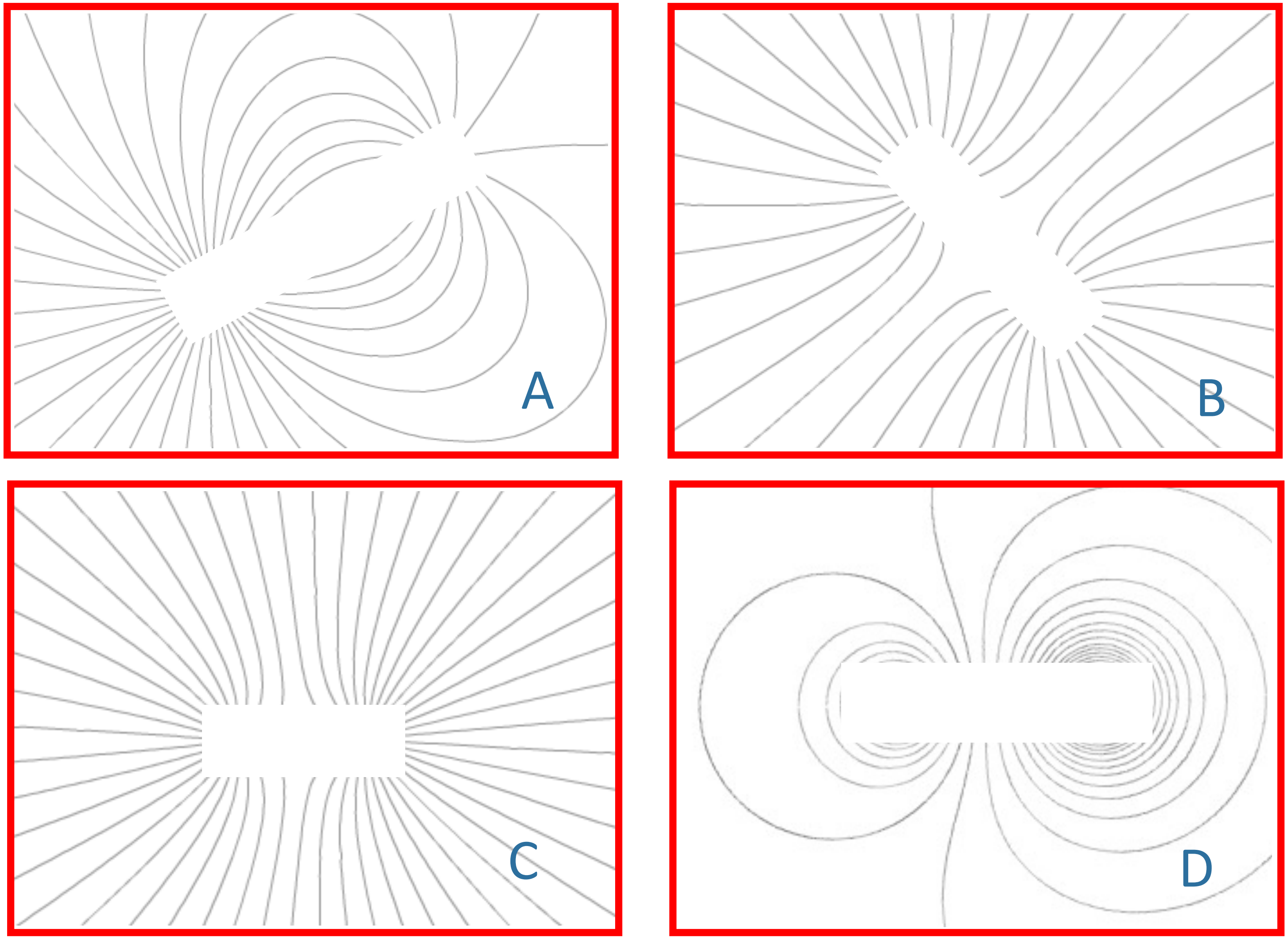


What charges are inside the red circle?



Deducing charge from field lines 2

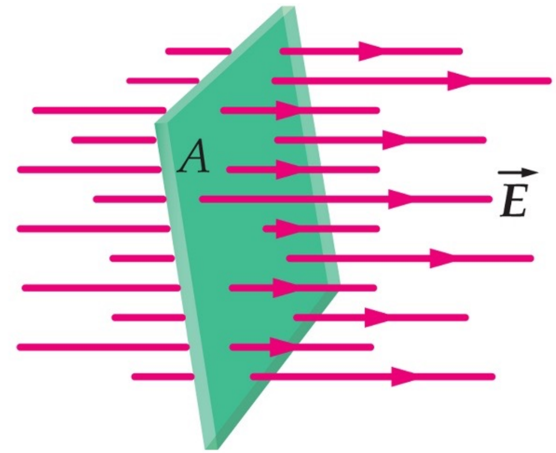
Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?



Electric Flux "Counts Field Lines"

Electric flux through surface S perpendicular to $\vec{E}$:

$$\Phi_S = EA$$

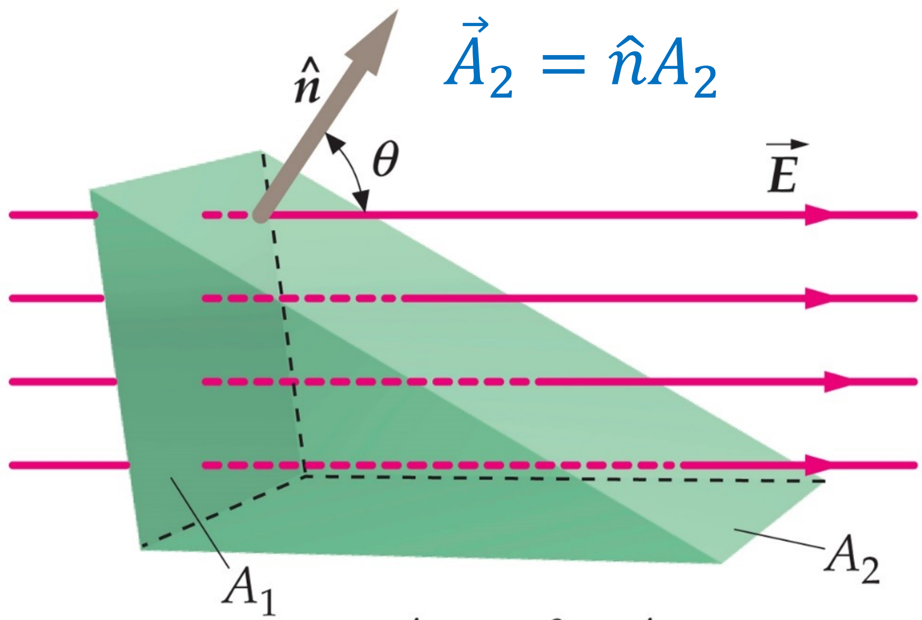


If the surface is tilted with respect to $\vec{E}$, we need to generalize to:

$$\Phi_S = \vec{E}\cdot\vec{A} = \vec{E}\cdot\hat{n}A$$

$$= EA\cos\theta$$

$\vec{A}_2 = \hat{n}A_2$



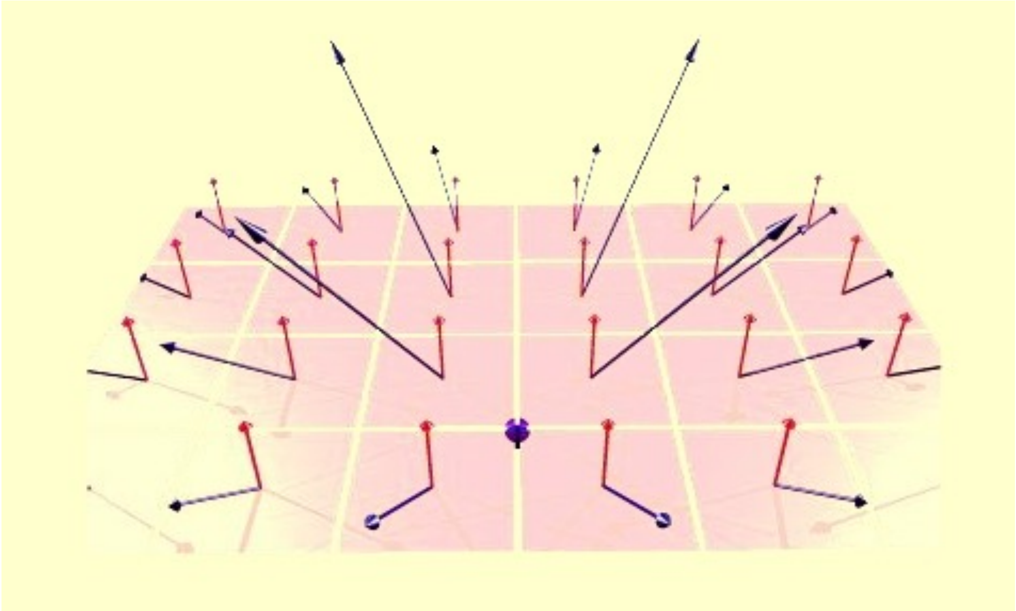
Electric Flux "Counts Field Lines"

For arbitrary E field and surface, we must break the surface up into infinitesimal elements and sum over them.

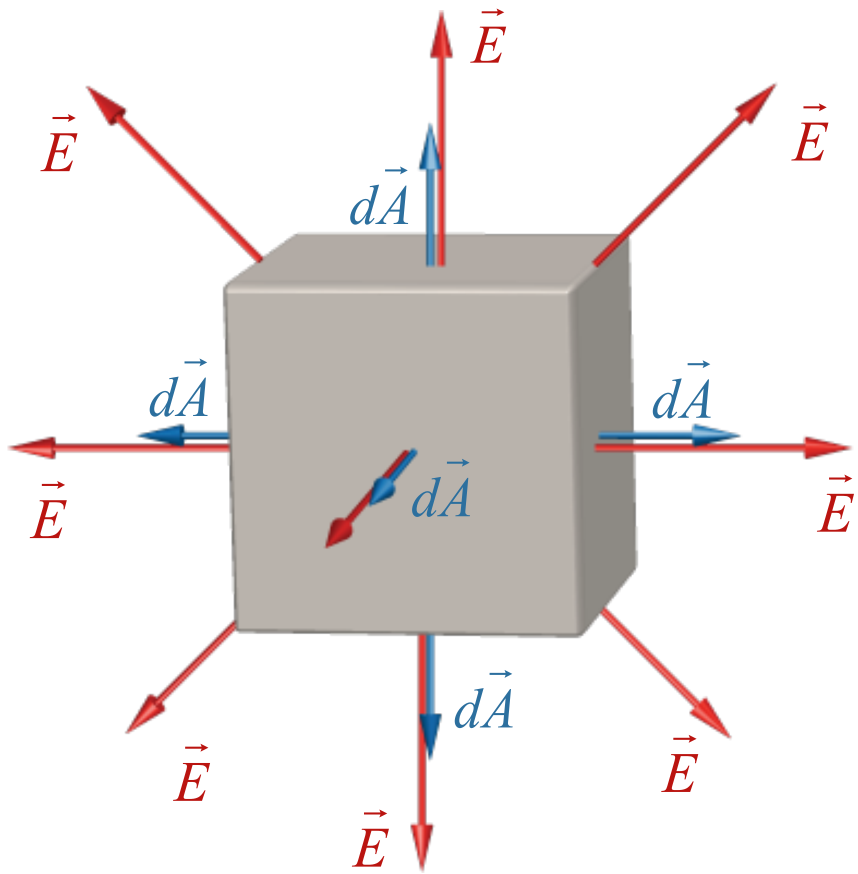
$$\Phi_S = \int_S \vec{E} \cdot d\vec{A}$$

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S



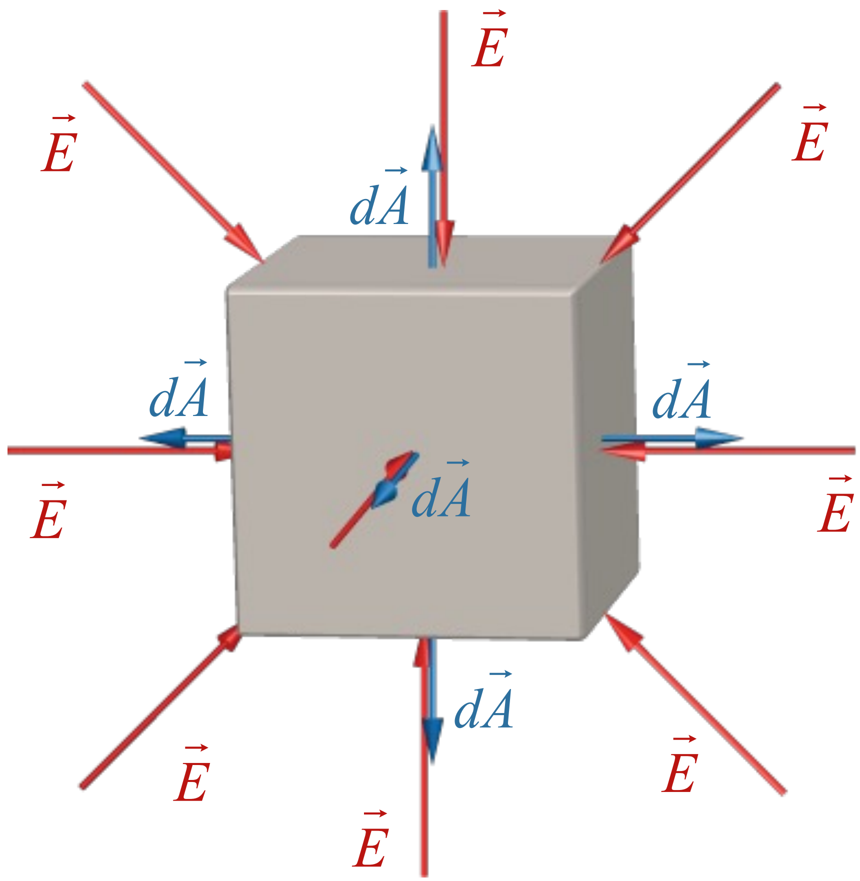
Direction Matters:



For a closed surface S, $d\vec{A}$ points outward by convention.

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

Direction Matters:



For a closed surface S, $d\vec{A}$ points outward by convention.

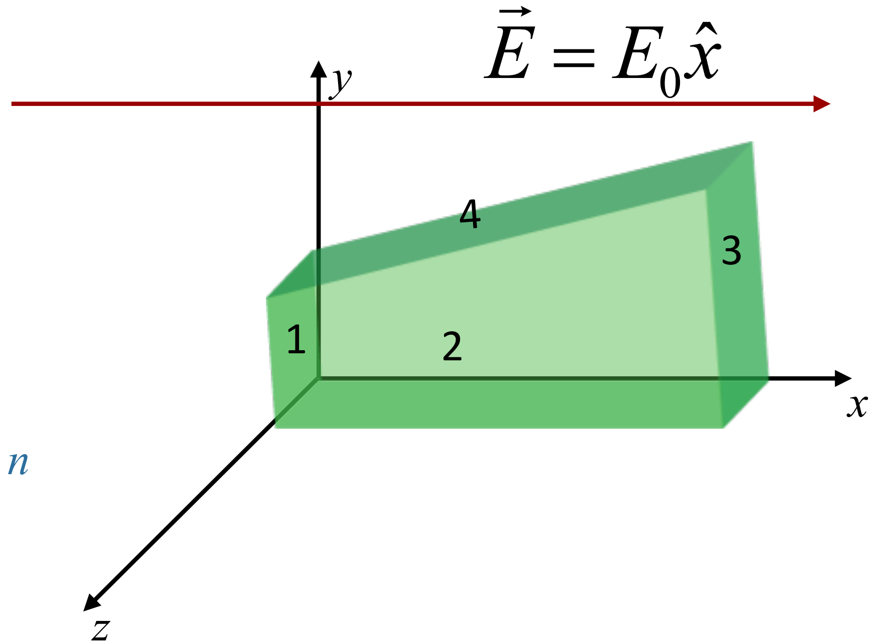
$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

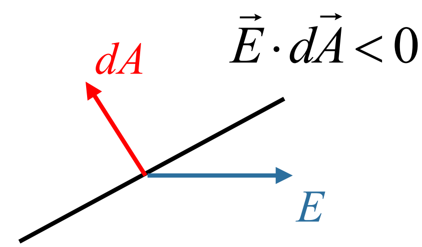
Trapezoid in Constant Field

Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define Φ_n = Flux through Face n





A) $\Phi_1 < 0$
B) $\Phi_1 = 0$
C) $\Phi_1 > 0$

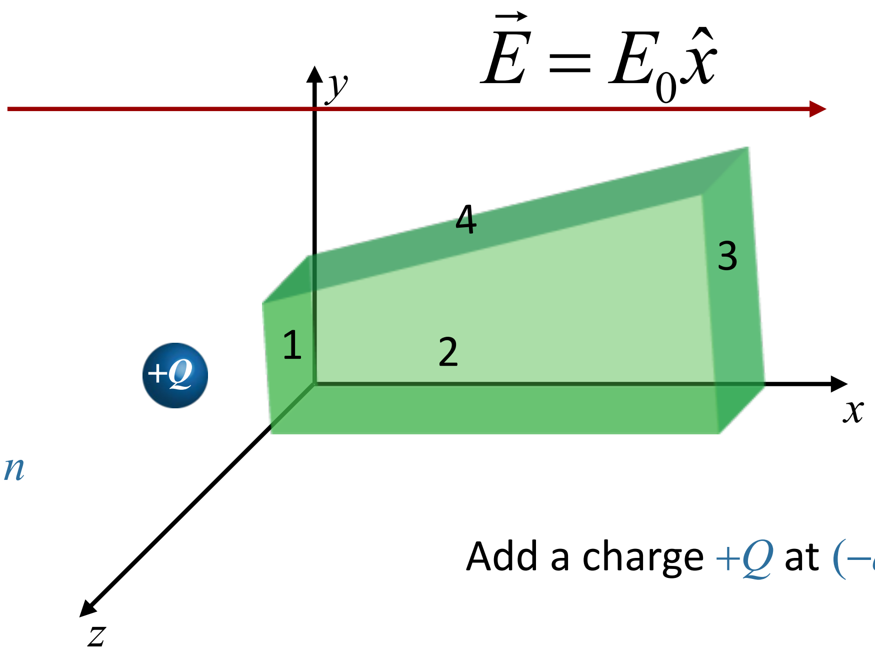
A) $\Phi_2 < 0$
B) $\Phi_2 = 0$
C) $\Phi_2 > 0$

A) $\Phi_3 < 0$
B) $\Phi_3 = 0$
C) $\Phi_3 > 0$

A) $\Phi_4 < 0$
B) $\Phi_4 = 0$
C) $\Phi_4 > 0$

Trapezoid in Constant Field + Q

Label faces:
1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted



Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change?

A) Φ_1 increases
B) Φ_1 decreases
C) Φ_1 remains same

A) Φ_3 increases
B) Φ_3 decreases
C) Φ_3 remains same

A) Φ increases
B) Φ decreases
C) Φ remains same

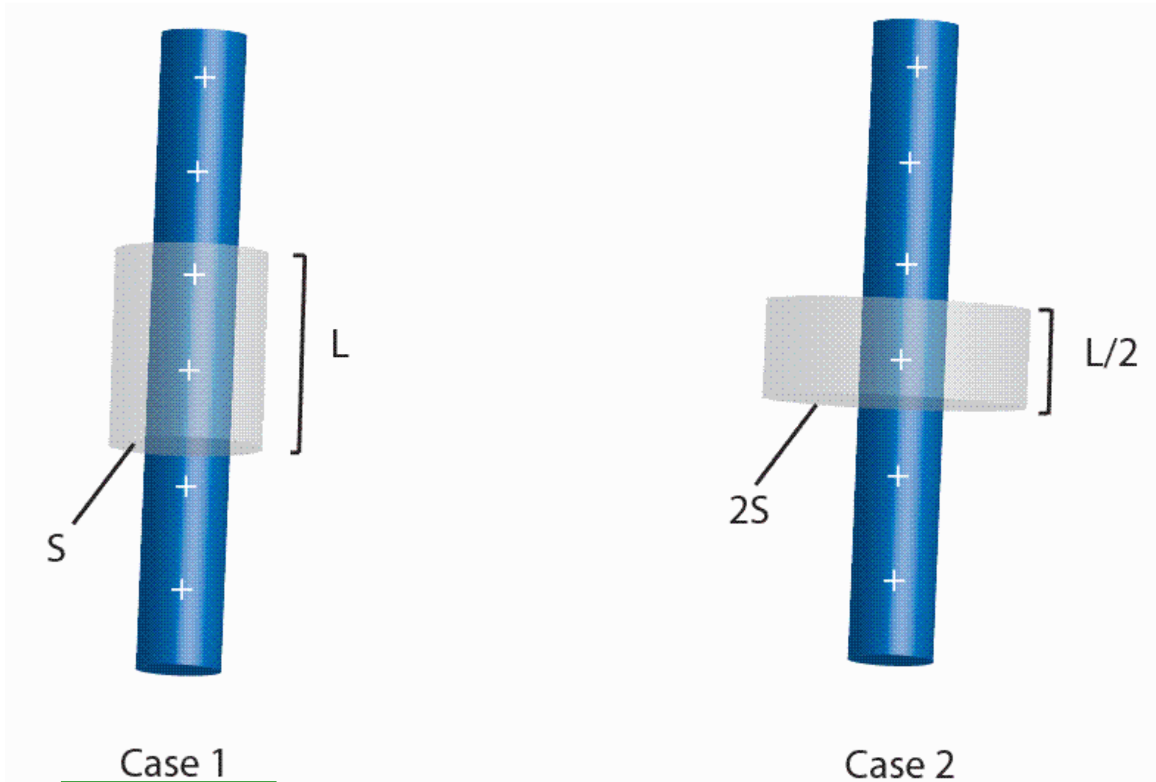
Bridge 1

More charge, more flux:

- Case 1: charge = λ L
- Case 2: charge = λ L/2

Note: Enclosed volume is the larger for case 2, but most region does not have charge

An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

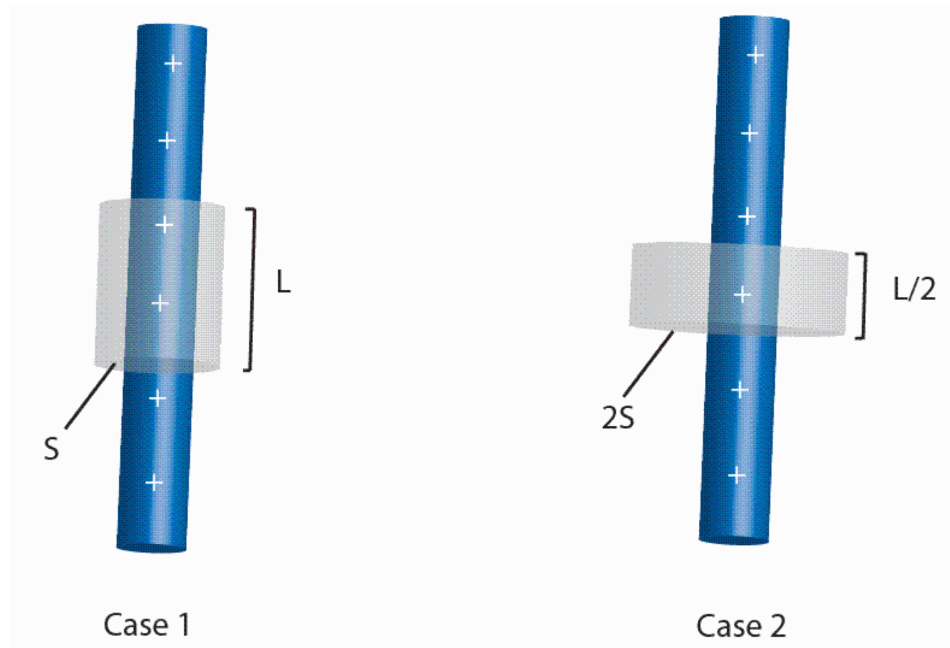


(A)	(B)	(C)	(D)
$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none

Bridge 1

An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

Calculate the flux for the two cases:



$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none
(A)	(B)	(C)	(D)

Definition of Flux:

$$\Phi \equiv \int_{surface} \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi = E \int_{barrel} d\vec{A} = EA_{barrel}$$

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 s}$$

$$A_1 = 2\pi s L$$

$$\Phi_1 = \frac{\lambda L}{\varepsilon_0}$$

Case 2

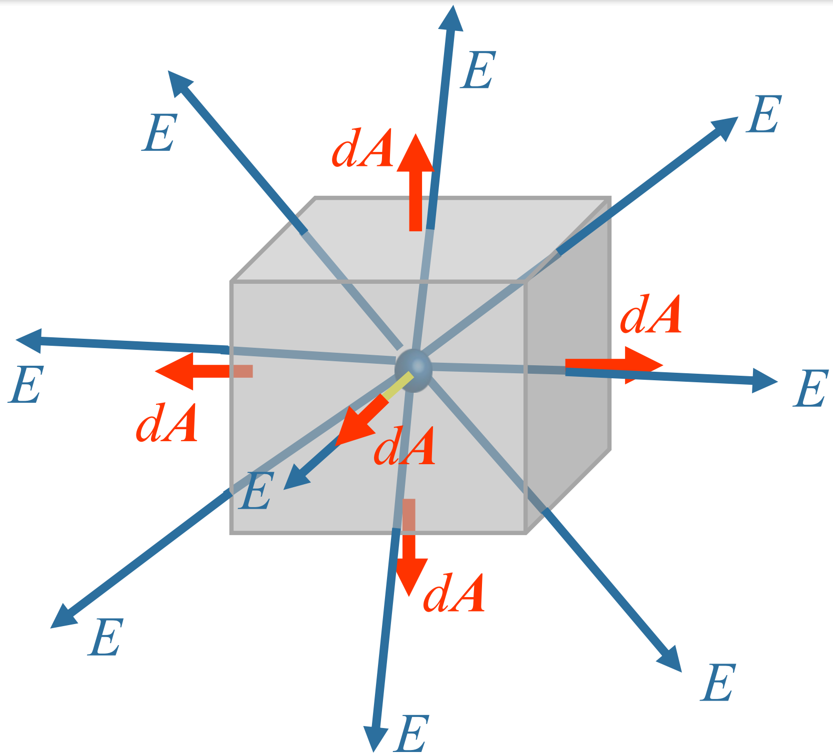
$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 (2s)}$$

$$A_2 = (2\pi(2s))L/2 = 2\pi s L$$

$$\Phi_2 = \frac{\lambda(L/2)}{\varepsilon_0}$$

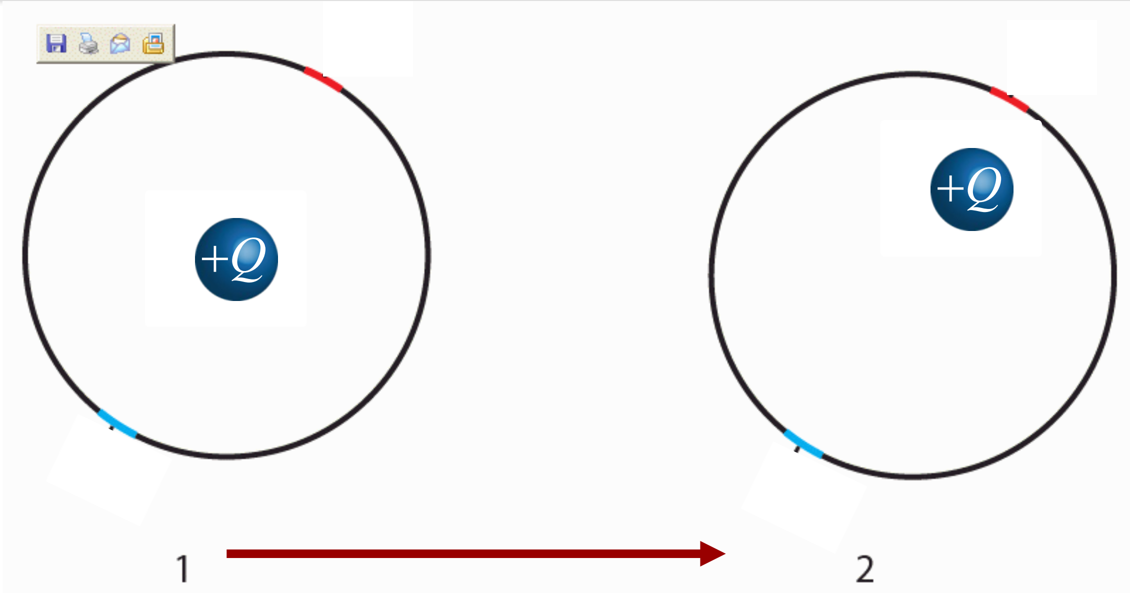
Gauss' Law

$$\Phi_S = \int_{\substack{closed \\ surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



- Net flux >0 if positive charge enclosed (a "source").
- Net flux < 0 if negative charge enclosed (a "sink").

Bridge 2.2



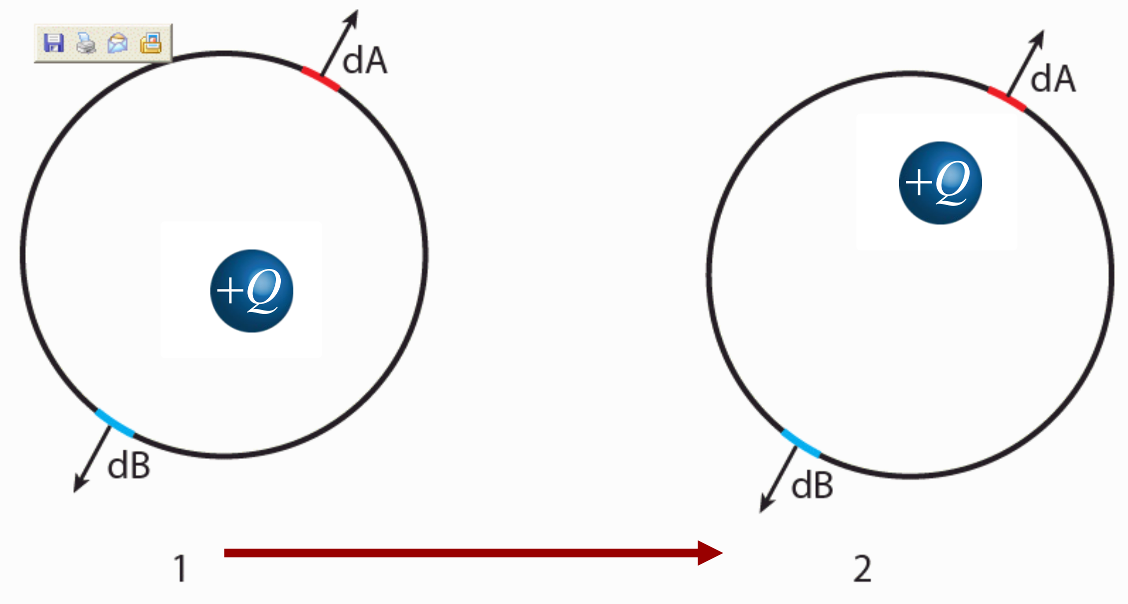
A
Φ_E increases

B
Φ_E decreases

C
Φ_E stays same

Total enclosed charge is the same, so total flux is the same

Bridge 2.1



A

$d\Phi_A$ increases
$d\Phi_B$ decreases

B

$d\Phi_A$ decreases
$d\Phi_B$ increases

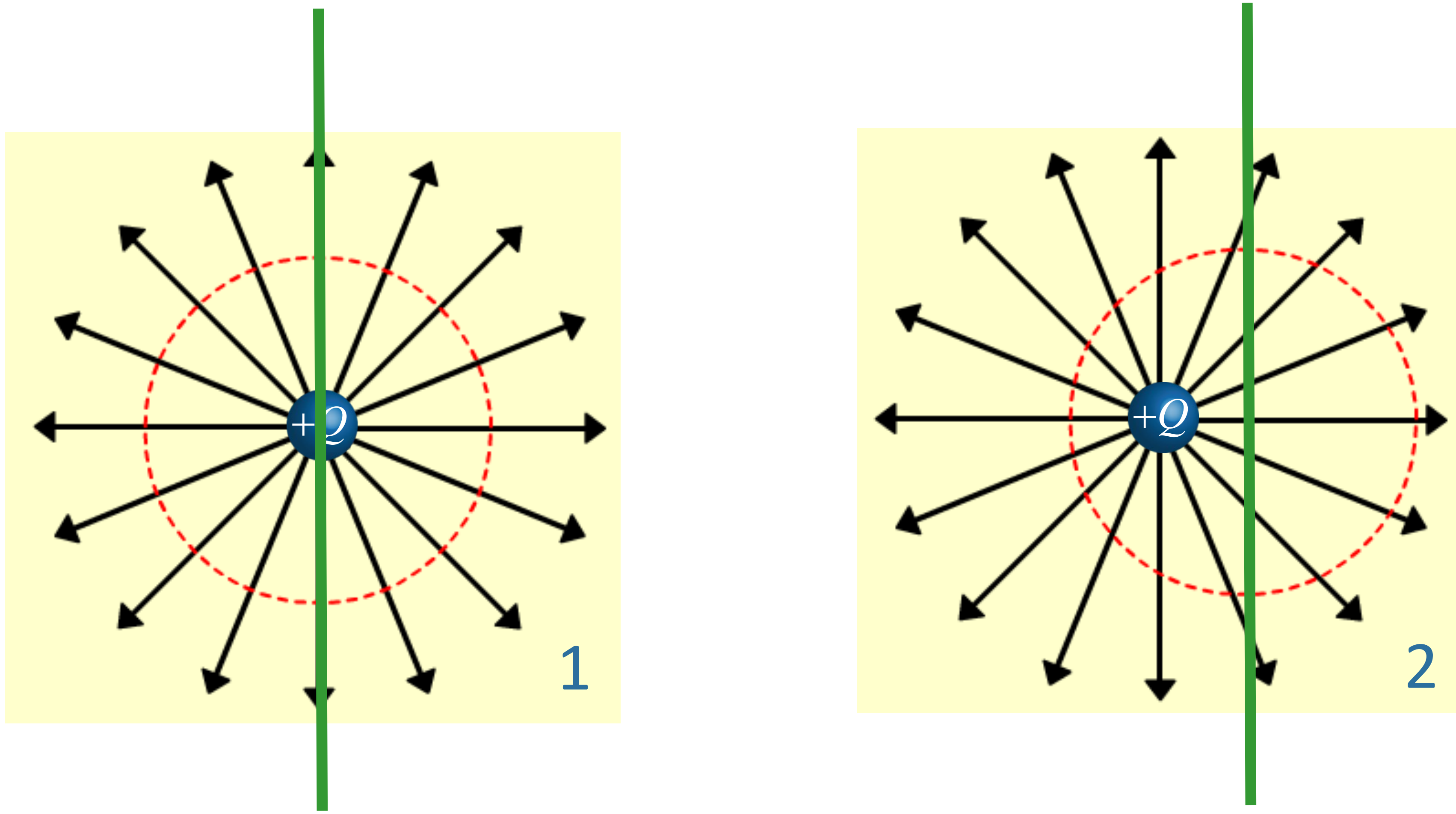
C

$d\Phi_A$ stays same
$d\Phi_B$ stays same

dA: E field becomes stronger when charge moves closer

dB: E field becomes weaker when charge moves away

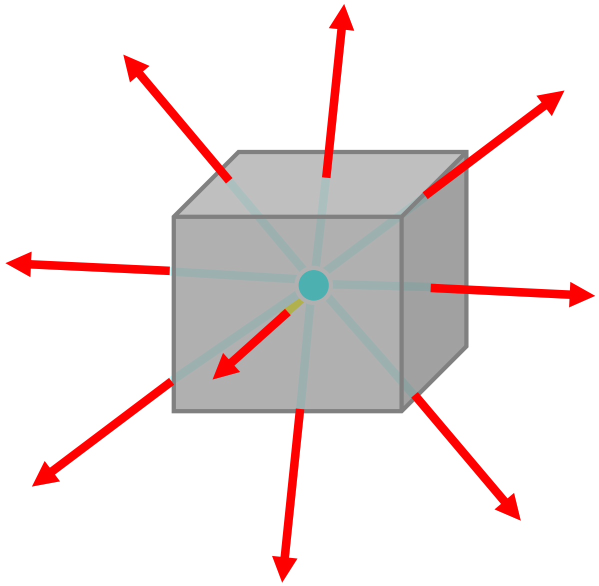
Think of it this way:



The total flux is the same in both cases (just the total number of lines)

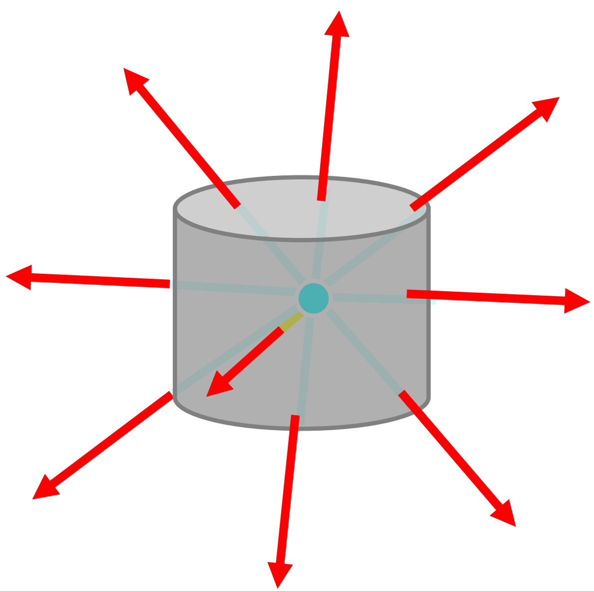
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

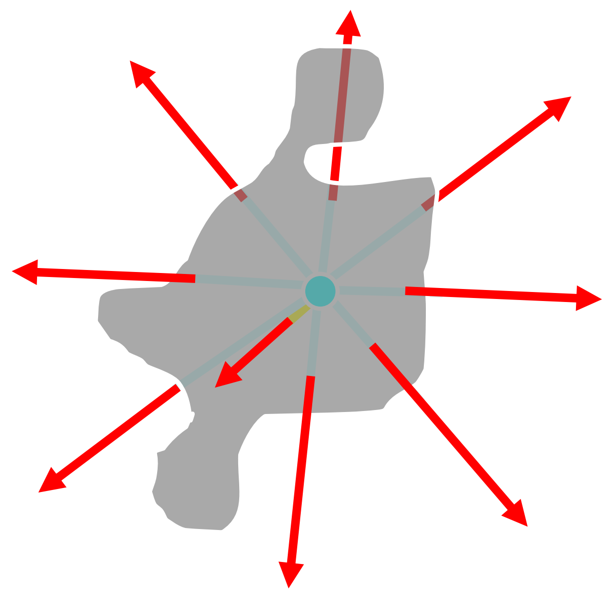
Things to notice about Gauss' Law



$$\Phi_S = \int_{\substack{closed\\surface}} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.





Things to notice about Gauss' Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

In cases of high symmetry it may be possible to bring E outside the integral. In these cases we can solve Gauss Law for E

$$E \int dA = EA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next two lectures.

Final thoughts

1. Tutorial starts this week.
2. Set up Achieve and iClicker ASAP
3. Online homework due this Friday (May 15) 11:59pm.
4. Please complete Ch22: Gauss's Law Prelecture and Bridge before Friday 8am.