

# *Electricity & Magnetism*

## *Lecture 5*

### Today's Concepts:

A) Using Gauss' Law

B) Conductors

(Next class: More examples of using Gauss' Law.)

# Gauss' Law

Closed surface

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

Gaussian surface

ALWAYS TRUE!

Generally, integral to calculate flux is difficult... and not useful!

To use **Gauss' Law** to calculate  $E$ , we need to choose surface carefully!

1) Want  $E$  to be constant on surface  $A$  of interest

OR

2) Want  $E \cdot A = 0$ , so does not add to integral

These properties allow us to pull  $E$  outside and get  $E = \frac{Q_{enc}}{A\epsilon_0}$

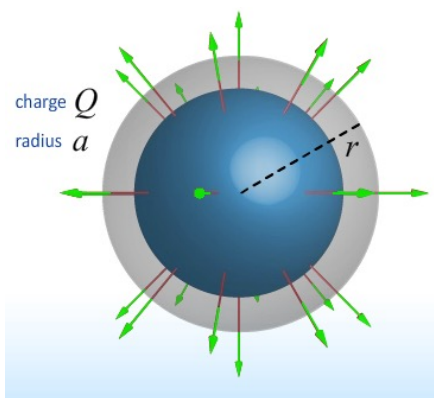
# Using Gauss' Law to calculate E-fields

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE ... but not always useful.

In cases with symmetry, can pull  $E$  outside and get  $E = \frac{Q_{\text{enc}}}{A\epsilon_0}$

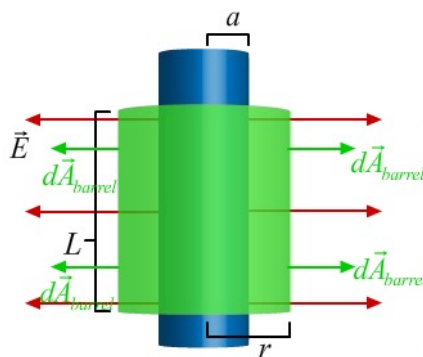
## Spherical



$$A = 4\pi r^2$$

$$E = \frac{q_{\text{enc}}}{4\pi r^2 \epsilon_0}$$

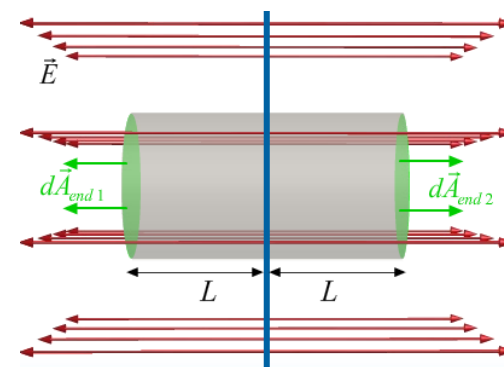
## Cylindrical



$$A = 2\pi rL$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

## Planar



Any  $A$

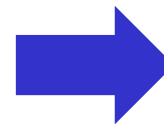
$$E = \frac{\sigma}{2\epsilon_0}$$

# Gauss' Law - Point Charge

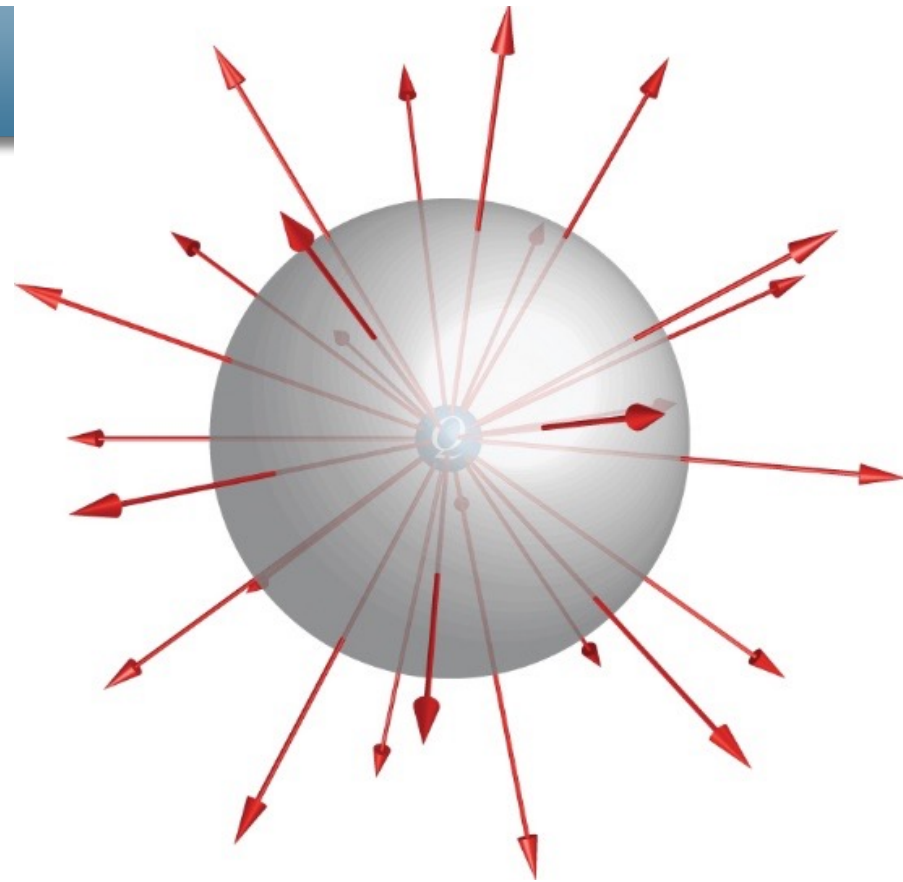
- Spherical symmetry:  $\vec{E} = E(r) \hat{r}$
- Use Gauss' Law: spherical surface centered on charge  $+Q$ .
- $\vec{E}$  is perpendicular to surface everywhere by symmetry, and has constant magnitude on surface.

$$\oint_S \vec{E} \cdot d\vec{A} = \oint_S E dA = E \oint_S dA = \frac{Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

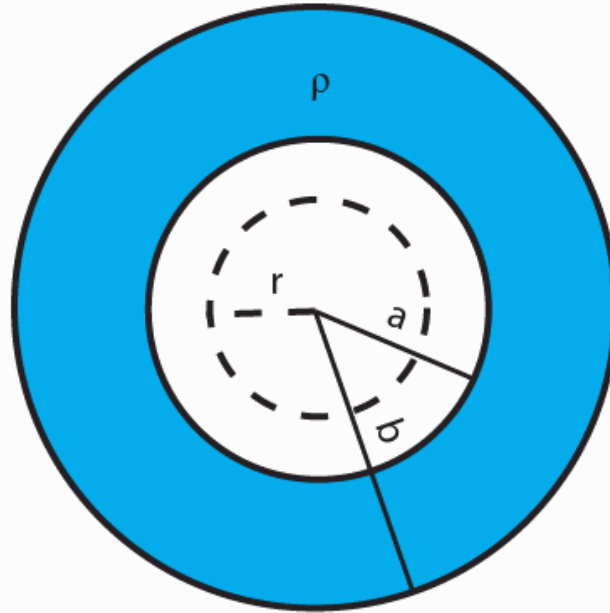


$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$



# Bridge 2

4) A charged spherical insulator shell has inner radius  $a$  and outer radius  $b$ . The charge density on the shell is  $\rho$ .



What is the magnitude of the E-field at a distance  $r$  away from the center of the shell where  $r < a$ ?

- A)  $\rho/\epsilon_0$
- B) zero
- C)  $\rho(b^3 - a^3)/(3\epsilon_0 r^2)$
- D) none of the above

1. Spherical surface centered at origin with radius  $r$
2. Check how much charge is inside the surface
3. Because there is no charge, there is no E field on the surface

# Calculation

Find the electric field everywhere for a **non-conducting** and uniformly charged solid sphere with radius  $a$  and total charge  $+Q$ .

Charge density:  $\rho = \frac{Q}{4/3\pi a^3}$

Outside sphere ( $r > a$ ):

Charge enclosed by  $S_1 = Q$

Gauss's Law:  $E(4\pi r^2) = Q/\epsilon_0$

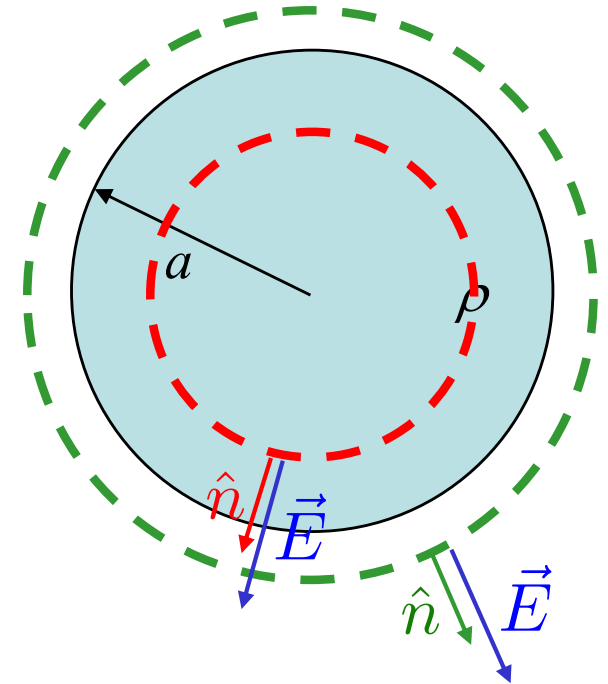
$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Inside sphere ( $r < a$ ):

Charge enclosed by  $S_2 = q = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{r^3}{a^3}Q$

Gauss's Law:  $E(4\pi r^2) = q/\epsilon_0$

$$E = \frac{Qr}{4\pi\epsilon_0 a^3}$$



# Clicker Question



A rod of infinite length has a charge per unit length of  $\lambda$ . Gauss' Law makes it easy to determine that the electric field strength at a perpendicular distance  $r$  from the rod is, in terms of  $k = (4\pi\epsilon_0)^{-1}$ ,

A.  $\frac{k\lambda}{r^2}$

B.  $\frac{k\lambda}{r}$

C.  $\frac{4\pi k\lambda}{r}$

D.  $\frac{2k\lambda}{r}$

E. zero

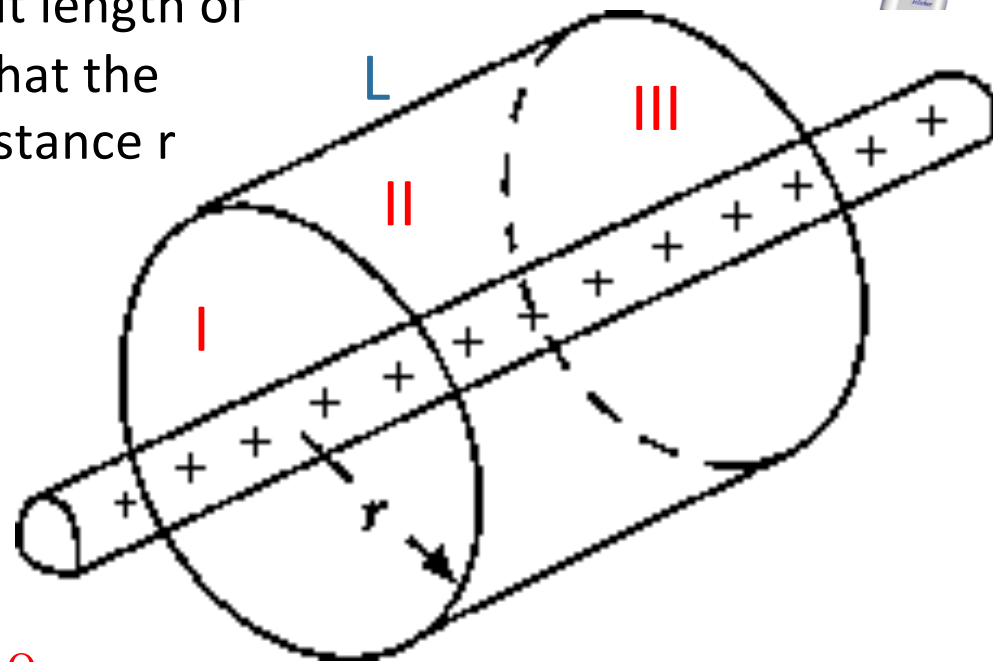
$$Q_{\text{enc}} = \lambda L$$

Surface I:  $\int \vec{E} \cdot d\vec{A} = 0$

Surface III:  $\int \vec{E} \cdot d\vec{A} = 0$

All surface:  $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$

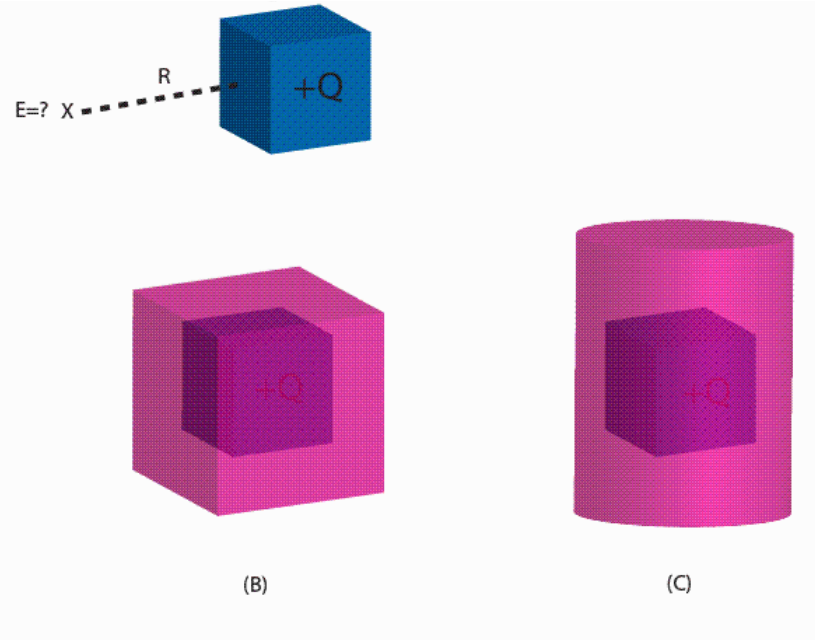
Surface II:  $\int \vec{E} \cdot d\vec{A} = E \int dA = E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$



$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

# Bridge 1

You are told to use Gauss' Law to calculate the electric field at a distance  $R$  away from a charged cube of dimension  $a$ . Which of the following Gaussian surfaces is best suited for this purpose?



D) None of the above

E) The field cannot be calculated using Gauss' Law

**Cube does not have sufficient symmetry**  
E field at these surfaces is **not perpendicular, or parallel, or constant**

# Conductors = Charges Free to Move

Claim:  $E = 0$  inside any conductor **at equilibrium**

- If  $E \neq 0$ , then charge feels force and moves (accelerates)!
- Charges in conductor move to make  $E$  field zero inside. (Induced charge distribution).

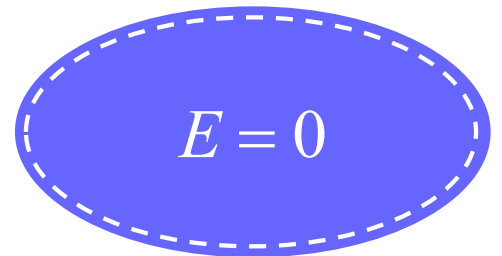
Claim: Excess charge on conductor only on surface at equilibrium

Why? **Apply Gauss' Law**

- Take Gaussian surface to be just inside conductor surface.

- $E = 0$  everywhere inside conductor  $\rightarrow \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = 0$

- Gauss' Law:  $\oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \rightarrow Q_{enc} = 0$



# Charged conducting shell

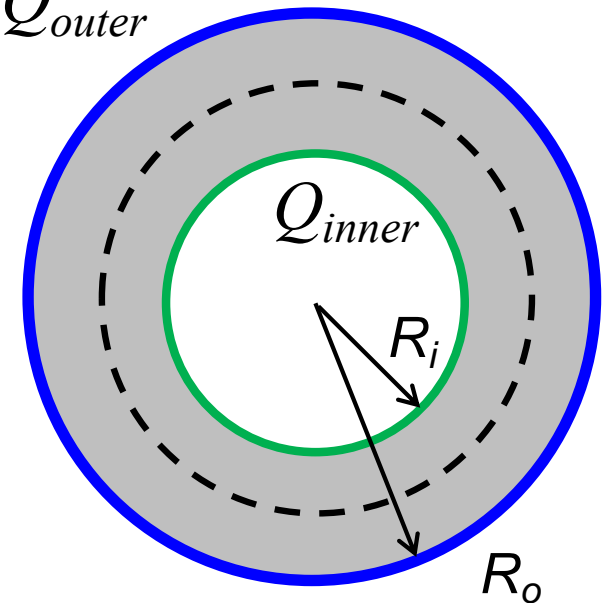
A hollow conducting sphere has charge  $+Q$ .

$Q_{outer}$

Q1: Where does the charge reside?

A: On the **outer** surface.

Why? By Gauss' Law!



$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

➤ Choose a spherical Gaussian surface inside the shell:  $R_i < r < R_o$

➤  $E=0$  inside conductor  $\therefore \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{enc}}{\epsilon_0} = \frac{Q_{inner}}{\epsilon_0}$

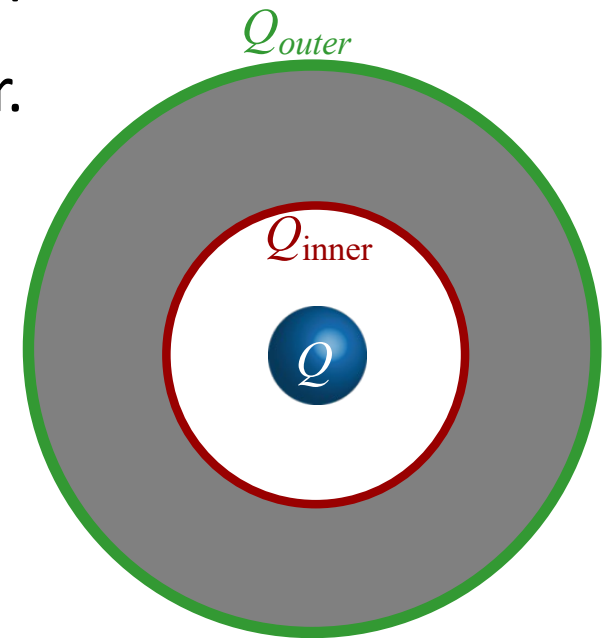
$$\Rightarrow Q_{inner} = 0 \Rightarrow Q_{outer} = +Q$$

# Gauss' Law + Conductors + Induced Charges

Now imagine an uncharged conducting hollow sphere.

A particle with charge  $+Q$  is placed in the center.

What happens?



Charges in conductor move to surfaces to make  $E=0$  everywhere inside the conducting material.

We say charge is induced on the surfaces of conductors.

# Charge in Cavity of Conductor



A particle with charge  $+Q$  is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?

A) inner =  $-Q$ , outer =  $+Q$

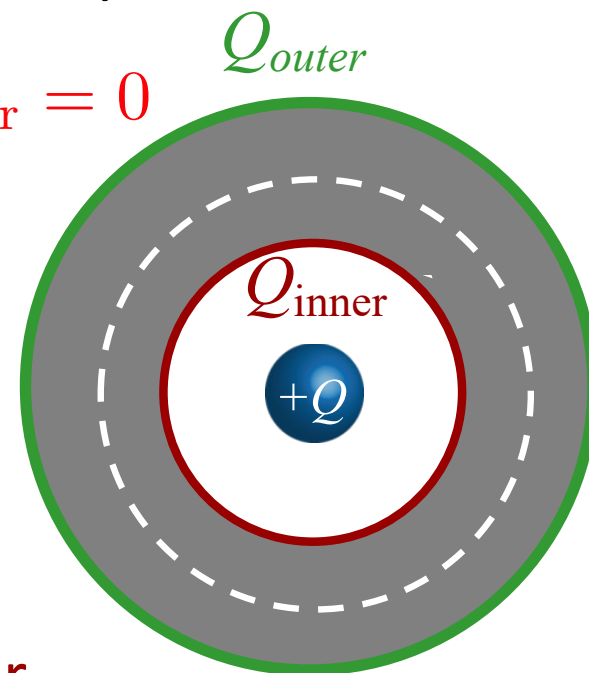
B) inner =  $-Q/2$ , outer =  $+Q/2$

C) inner = 0, outer = 0

D) inner =  $+Q/2$ , outer =  $-Q/2$

E) inner =  $+Q$ , outer =  $-Q$

$$Q_{\text{inner}} + Q_{\text{outer}} = 0$$



Since  $E = 0$  in conductor

➤ Gauss' Law:  $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \rightarrow Q_{\text{enc}} = 0$

$Q_{\text{enc}} = Q + Q_{\text{inner}}$

# Infinite Cylinders

A long thin wire has a uniform positive charge density of  $2.5 \text{ C/m}$ . Concentric with the wire is a long thick conducting cylinder, with inner radius  $3 \text{ cm}$ , and outer radius  $5 \text{ cm}$ . The conducting cylinder has a net linear charge density of  $-4 \text{ C/m}$ .

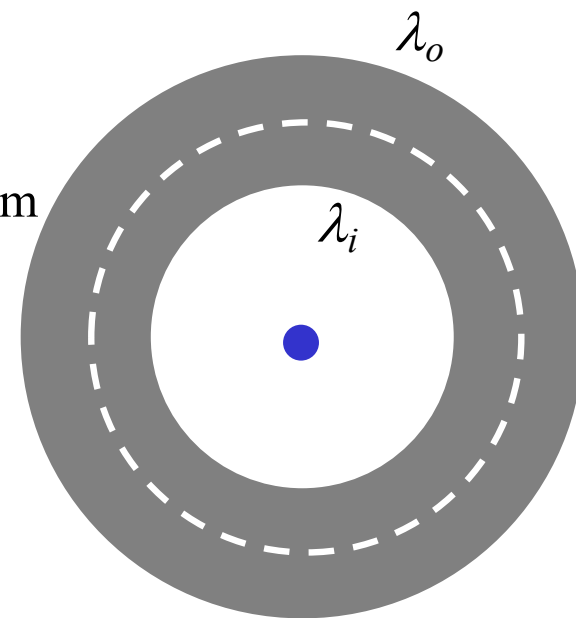
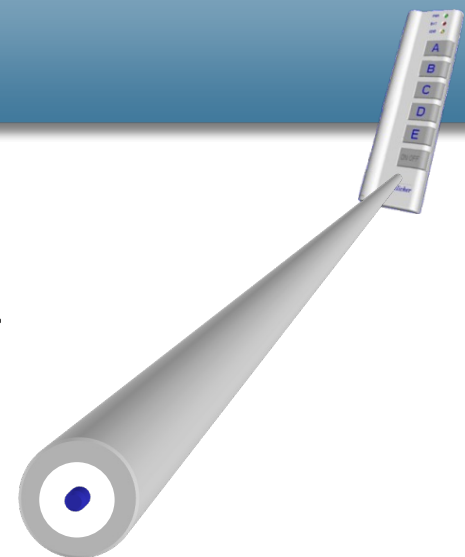
What is the **linear charge density** of the induced charge on the inner surface of the conducting cylinder ( $\lambda_i$ ) and on the outer surface ( $\lambda_o$ )?

$\lambda_i$ :	+2.5 C/m	-4 C/m	-2.5 C/m	-2.5 C/m	0
$\lambda_o$ :	-6.5 C/m	0	+2.5 C/m	-1.5 C/m	-4 C/m
	A	B	C	D	E

➤  $E = 0$  inside the conductor material  $\rightarrow Q_{enc} = 0$

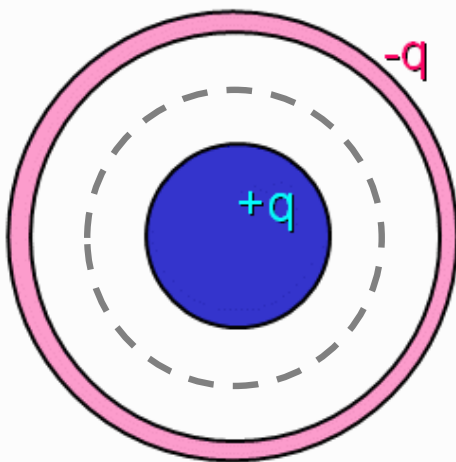
$\rightarrow \lambda_i = -2.5 \text{ C/m}$

➤ And  $\lambda_i + \lambda_o = -4 \text{ C/m} \rightarrow \lambda_o = -1.5 \text{ C/m}$



## Bridge 3.1

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.



Which of the following statements best describes the electric field in the region between the spheres?

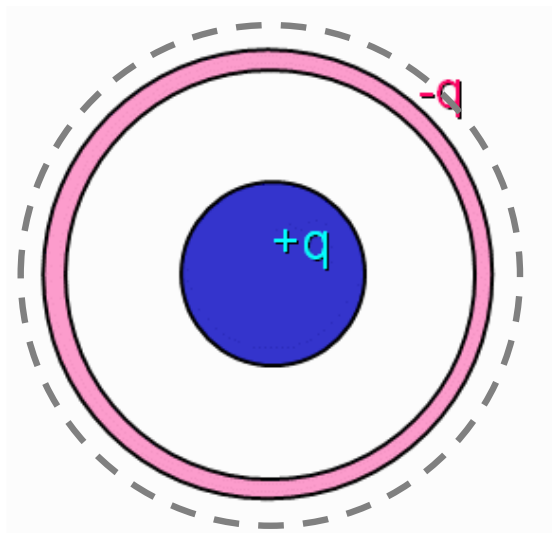
- The field points radially outward
- The field points radially inward
- The field is zero

Outward regardless of the charge on the outer shell

Generally, due to spherical symmetry, the charge of an outer shell contributes no electric field within the shell.

## Bridge 3.2

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.



What is direction of field  
OUTSIDE the red sphere?

- A  The field points radially outward
- B  The field points radially inward
- C  The field is zero

Total charge is  $+q - q = 0$

# *Final Thoughts*

1. **Online homework** due this **Saturday** (May 16) 11:59pm.
2. If you have tutorial on next Tuesday, it would be helpful to go through **Ch 33 Prelecture and Bridge** before attending tutorial.
3. **More examples at 1130am**
4. **Open class** at 230pm **today** in P9412