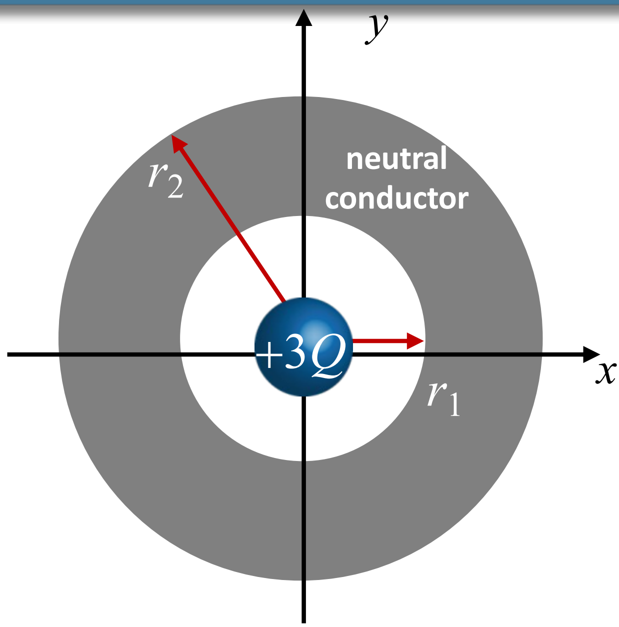
Electricity & Magnetism
Lecture 6

Today's Concepts:

A) Using Gauss' Law

Calculation



Point charge $+3Q$ at center of neutral conducting spherical shell of inner radius r_1 and outer radius r_2.

What is $\vec{E}$ everywhere?

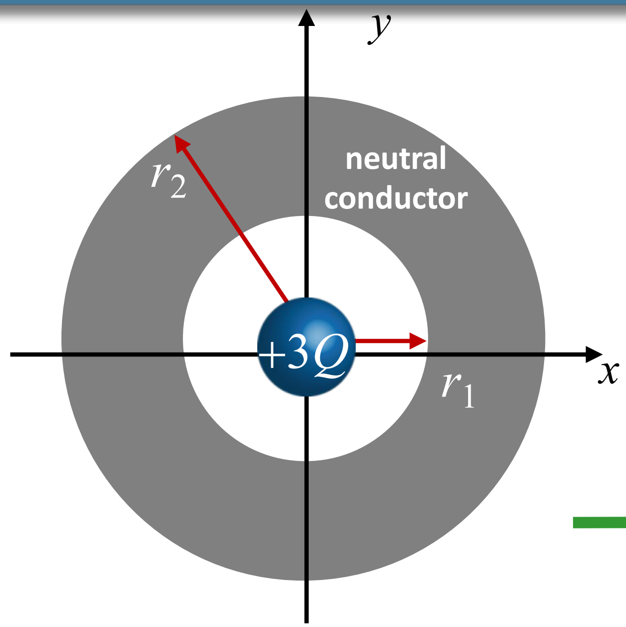
First question: Do we have enough symmetry to use Gauss' Law to determine $\vec{E}$?

Yes, Spherical Symmetry (what does this mean???)

A) Magnitude of $\vec{E}$ depends on r only
B) Magnitude of $\vec{E}$ depends on x only
C) Magnitude of $\vec{E}$ depends on y only
D) None of the above

A) Direction of $\vec{E}$ is along $\hat{x}$
B) Direction of $\vec{E}$ is along $\hat{y}$
C) Direction of $\vec{E}$ is along $\hat{r}$
D) None of the above

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.
What is $\vec{E}$ everywhere?

We know:

magnitude of $\vec{E}$ depends on r only
direction of $\vec{E}$ is along $\hat{r}$

We can use Gauss' Law to determine E
Use Gaussian surface = sphere centered on origin

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$r < r_1$

$$\oint \vec{E} \cdot d\vec{A} = E 4\pi r^2$$

$$Q_{enc} = +3Q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$$

$r_1 < r < r_2$

A) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r_1^2}$

C) $E = 0$

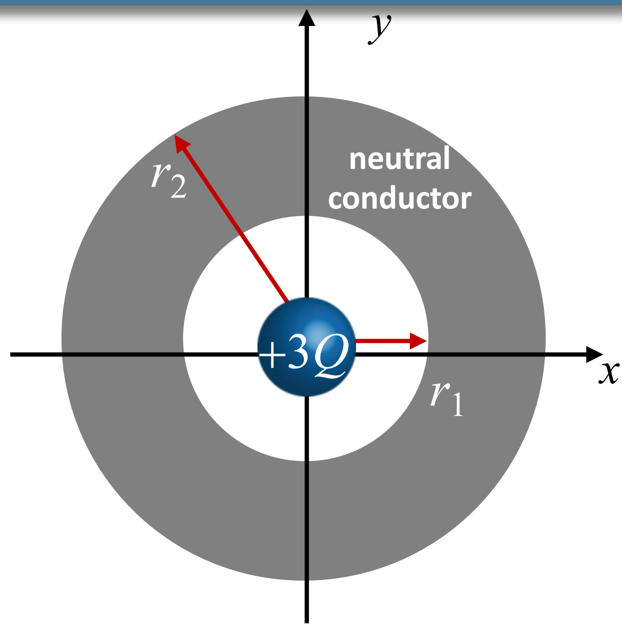
$r > r_2$

A) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{(r - r_2)^2}$

C) $E = 0$

Calculation



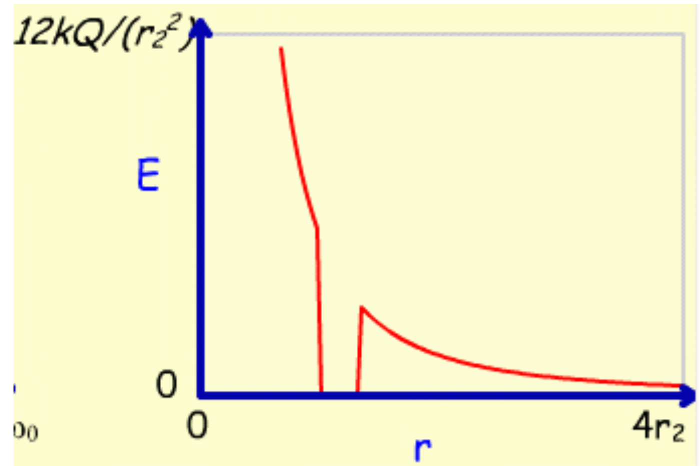
Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.
What is $\vec{E}$ everywhere?

We know:

$r < r_1$
$r > r_2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2 \quad E = 0$

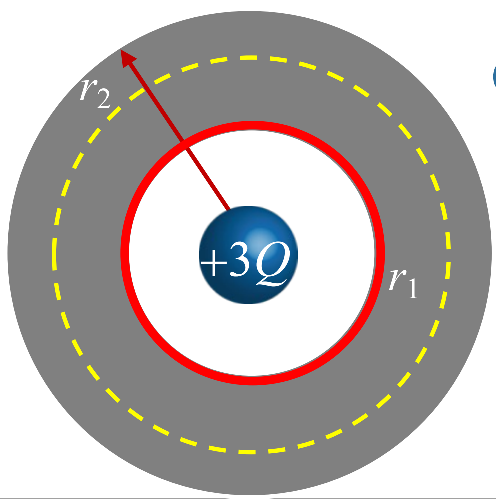


B) What is surface charge density at r_1?

A) $\sigma < 0$

B) $\sigma = 0$

C) $\sigma > 0$



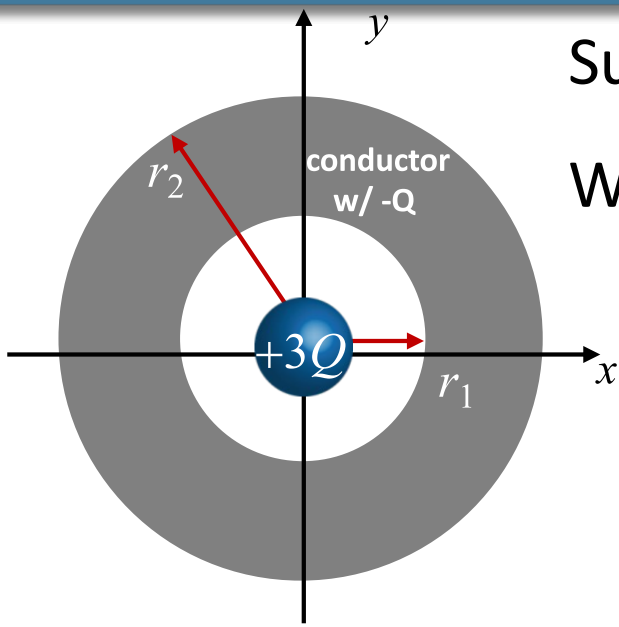
Gauss' Law:

$$E = 0 \rightarrow Q_{enc} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

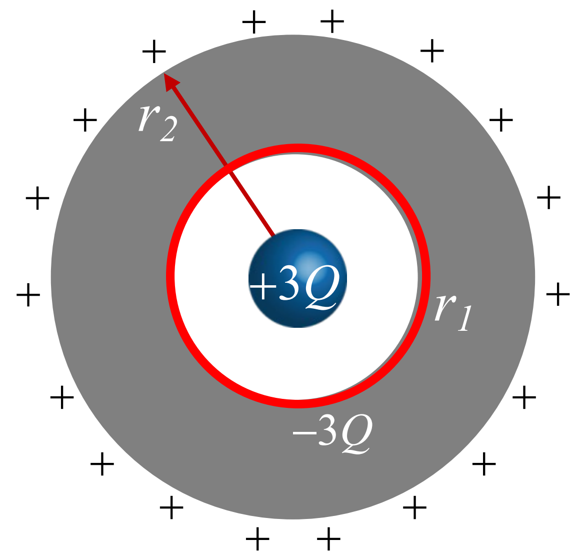
Calculation



Suppose we give the conductor a charge of $-Q$.

What is E everywhere?

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$



$r < r_1$

A) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}$

C) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$r > r_2$

A) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}$

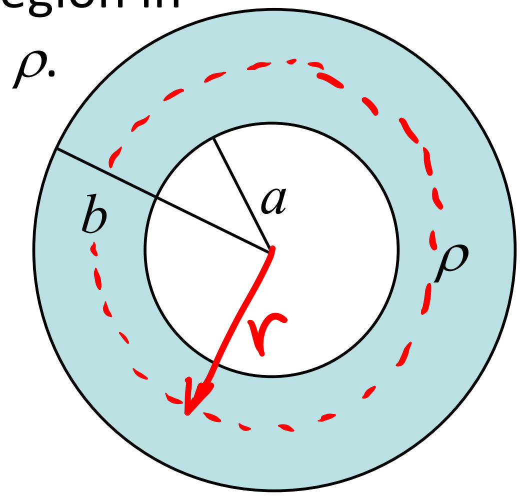
C) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$r_1 < r < r_2$

$E = 0$

Calculation

Consider a **non-conducting** cylinder of infinite length with a hollow core. The inner radius is a, the outer radius is b, and the solid region in between carries a uniformly-distributed volume charge density ρ.



Using Gauss' Law, calculate the electric field at a radius of r from the axis of the cylinder, where $a < r < b$.

Cylindrical symmetry $\bar{E} = E(r)\hat{r}$

∴ use Gauss' Law.

Gaussian surface: cylinder radius $a<r<b$, length ℓ.

$$\oint \bar{E}\cdot d\bar{A} = E\,2\pi r\ell = \frac{Q_{enc}}{\epsilon_0} = \rho\frac{(\pi r^2 - \pi a^2)\ell}{\epsilon_0}$$

$$\therefore \quad E = \frac{\rho}{2\epsilon_0}\frac{(r^2 - a^2)}{r}$$

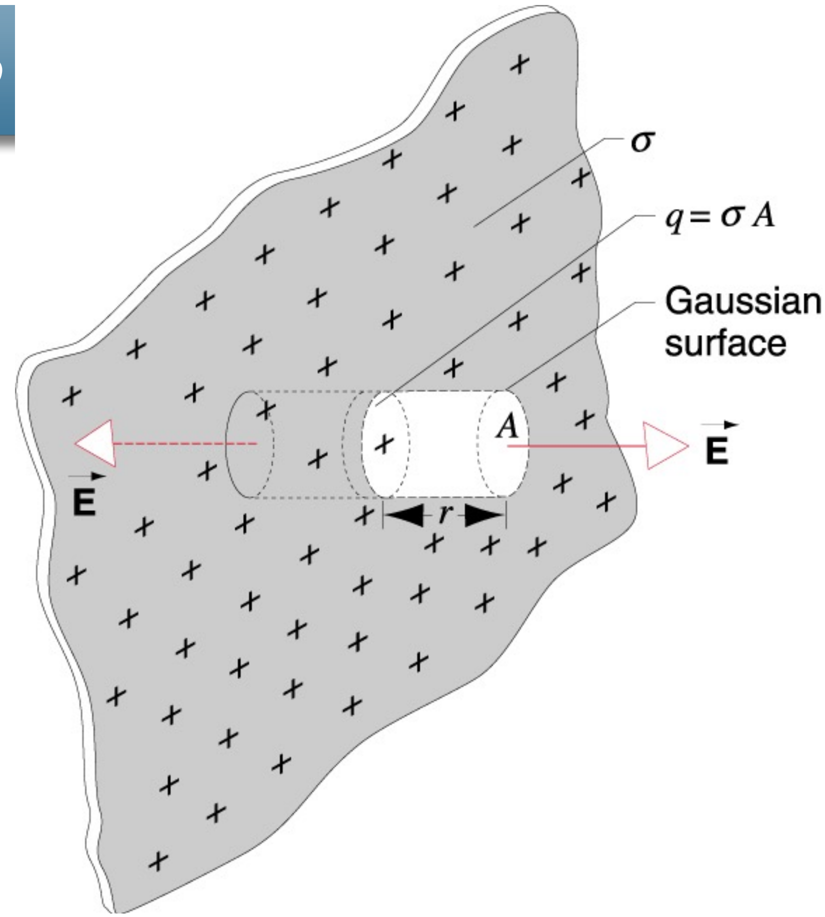
$$\bar{E} = \frac{\rho}{2\epsilon_0}\frac{(r^2-a^2)}{r}\hat{r} \quad a<r<b$$

Infinite Sheet of Charge

Planar symmetry

→ E direction perpendicular to surface.
E independent of lateral position.

Use cylindrical Gaussian pillbox straddling surface



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

$$\int_{\text{cap}\,1} + \int_{\text{cap}\,2} + \int_{\substack{\text{barrel}\\\text{surface}}} = \frac{\sigma A}{\varepsilon_o}$$

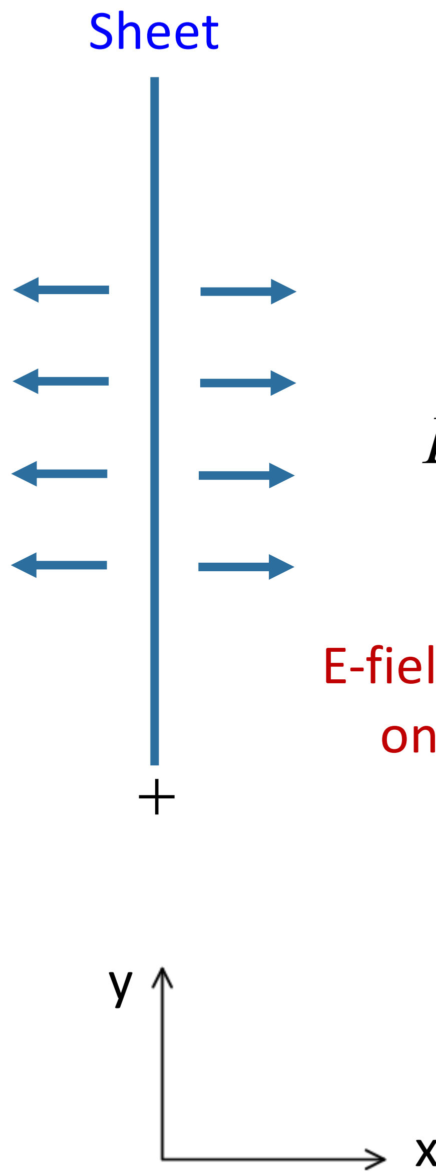
$$EA + EA + 0 = \frac{\sigma A}{\varepsilon_o} \Rightarrow 2EA = \frac{\sigma A}{\varepsilon_o} \Rightarrow E = \frac{\sigma}{2\varepsilon_o}$$

$\vec{E} \cdot d\vec{A} = 0$ on barrel surface

points away from surface for $\sigma > 0$

Electric field for charged insulating sheet

Cross-sectional view:



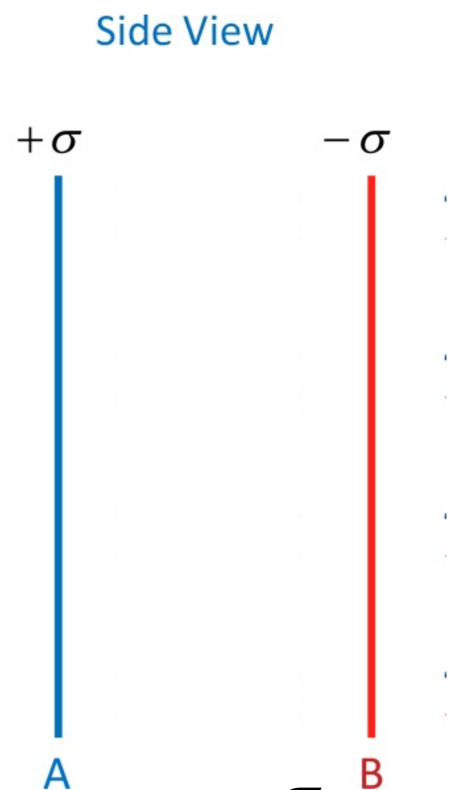
$$E_x = -\frac{\sigma}{2\varepsilon_0}$$

$$E_x = +\frac{\sigma}{2\varepsilon_0}$$

E-field strength does not depend on distance from the sheet!

Superposition

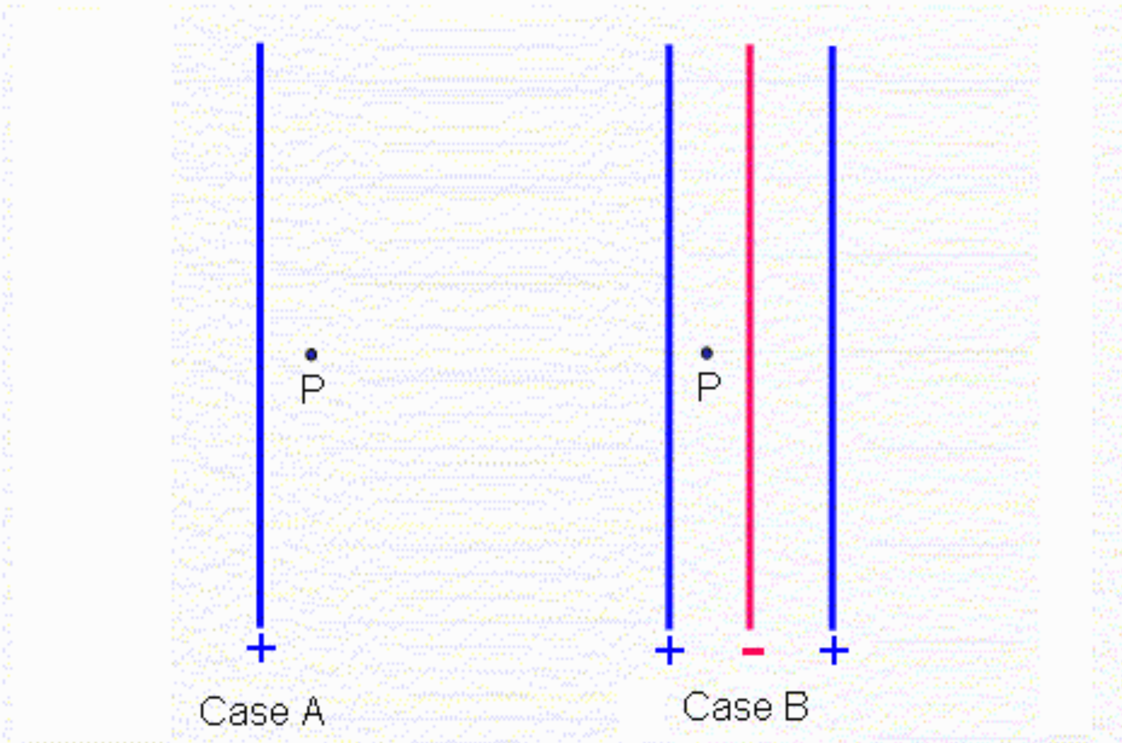
- Suppose now that we have two adjacent infinite sheets of equal but opposite surface charge. How to figure out the E-field in space?
- Superposition!
  - We already used symmetry and Gauss' Law to solve for each sheet independently. The net field is the sum of the two.



$E_x = 0$ $\quad E_x = 2\dfrac{\sigma}{2\varepsilon_0}$ $\quad E_x = 0$

Bridge 4

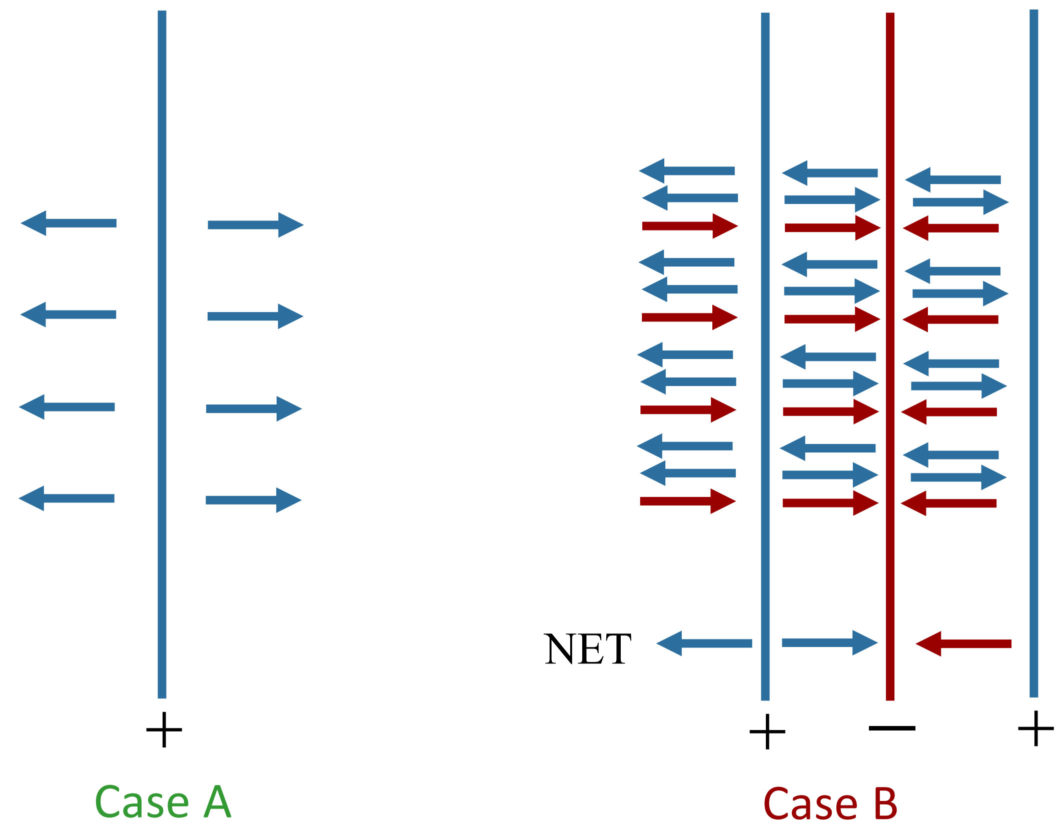
10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.



In which case is E at point P the biggest?

A) A　　B) B　　C) the same

Superposition:



Charged conducting metal plate

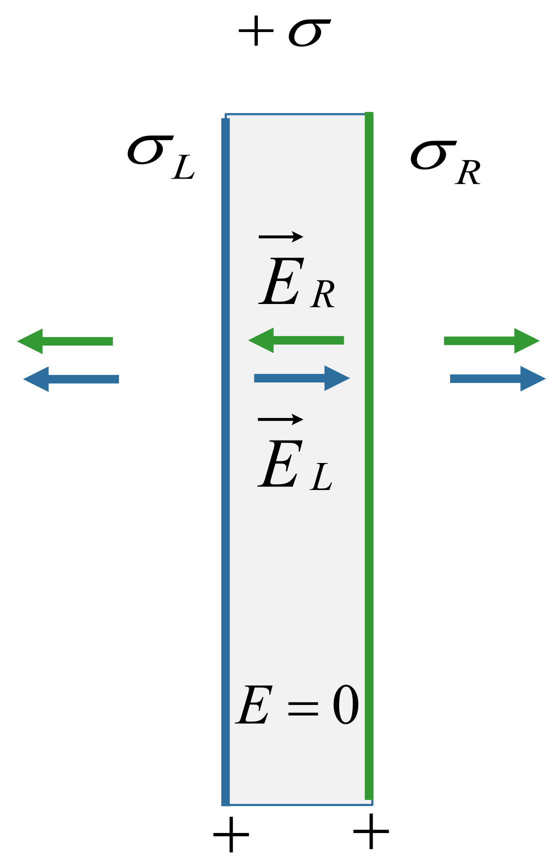
An infinite **conducting slab** in free space has a charge σA.
Question: How is the charge distributed?

Ans: At the surface

By symmetry: $\sigma_L = +\frac{\sigma}{2} = \sigma_R$

Inside: $E_x = \frac{\sigma/2}{2\varepsilon_0} - \frac{\sigma/2}{2\varepsilon_0} = 0$ Yes!

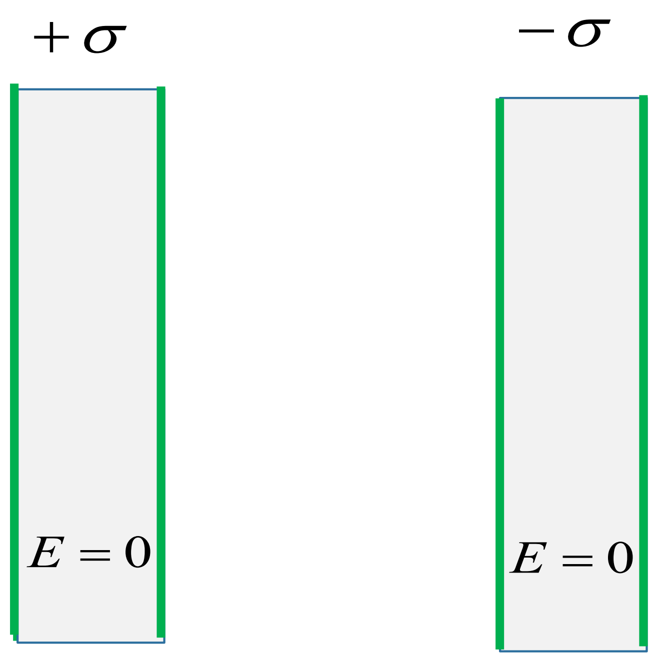
Outside: $E_x = \pm 2 \cdot \frac{\sigma/2}{2\varepsilon_0} = \pm \frac{\sigma}{2\varepsilon_0}$



just like thin sheet w/ σ.

Two charged metal plates

Now add a second infinite conducting slab with negative uniform charge density -σ. How is the charge distributed on the slabs now?



A) $+\sigma/2$ $+\sigma/2$ $-\sigma/2$ $-\sigma/2$

B) 0 $+\sigma$ $-\sigma$ 0

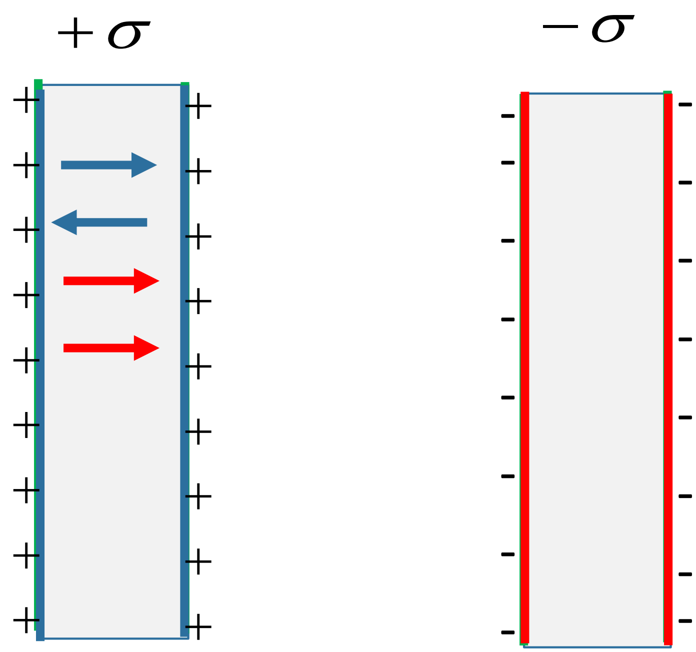
C) $+\sigma$ 0 0 $-\sigma$

D) Cannot be determined without further information

Two charged metal plates

Now add a second infinite conducting slab with negative uniform charge density -σ. How is the charge distributed on the slabs now?

Non-vanishing E inside conductors

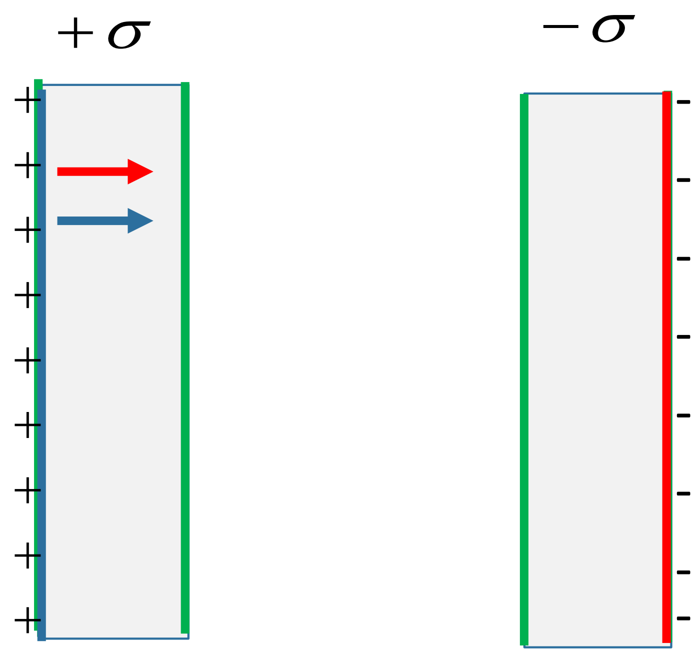


~~A) $+\sigma/2$ $+\sigma/2$ $-\sigma/2$ $-\sigma/2$~~

Two charged metal plates

Now add a second infinite conducting slab with negative uniform charge density -σ. How is the charge distributed on the slabs now?

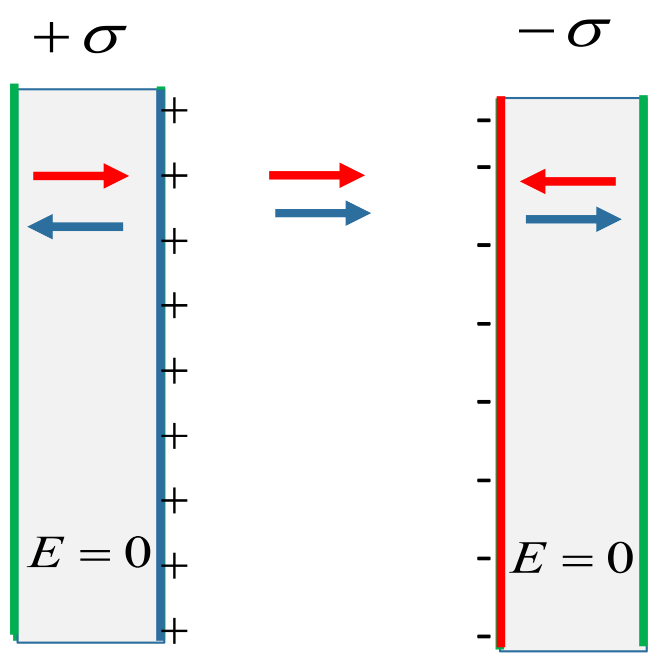
Non-vanishing E inside conductors



C) $+\sigma$ 0 0 σ

Two charged metal plates

Now add a second infinite conducting slab with negative uniform charge density -σ. How is the charge distributed on the slabs now?

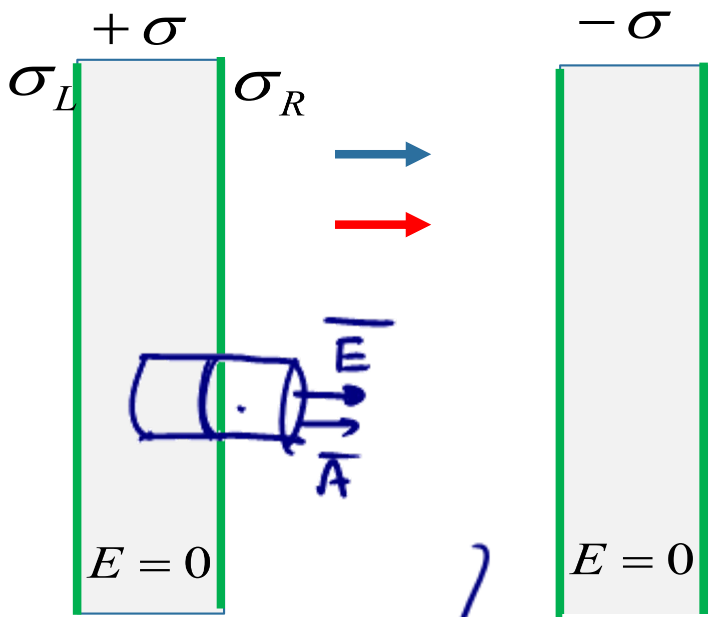


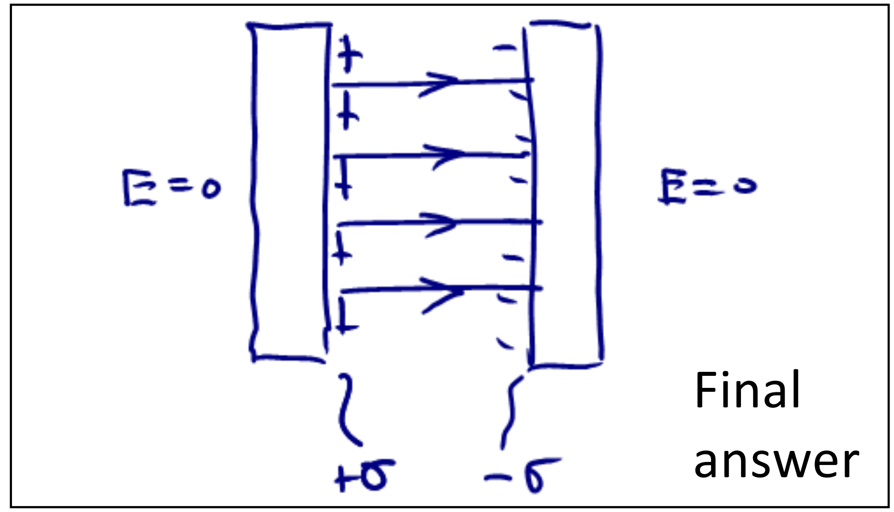
A)	$+\sigma/2$	$+\sigma/2$	$-\sigma/2$	$-\sigma/2$
B)	0	$+\sigma$	$-\sigma$	0
C)	$+\sigma$	0	0	$-\sigma$

Only one that makes E=0 inside conductors

D) Cannot be determined without further information

Detailed Explanation I (Extra)





we know: $E = 2\left(\frac{\sigma}{2\epsilon_0}\right) = \frac{\sigma}{\epsilon_0}$

Apply Gauss' Law to pill-box straddling inner surface on one plate:

$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma_R A}{\epsilon_0}$$

$$\frac{\sigma}{\epsilon_0} A = \frac{\sigma_R A}{\epsilon_0} \quad \therefore \quad \boxed{\sigma_R = \sigma}$$

Final Thoughts

1. Online homework due this Saturday (May 16) 11:59pm.
2. Next Online homework due next Thursday (May 20) 11:59pm.
3. Open class at 230pm today in P9412
4. If you have tutorial on next Tuesday, it would be helpful to go through Ch 33 Prelecture and Bridge before attending tutorial.