Electricity & Magnetism
Lecture 7

Today's Concept:

Electric Potential Energy

Electrostatic and Gravitational Forces

- Coulomb's Law:

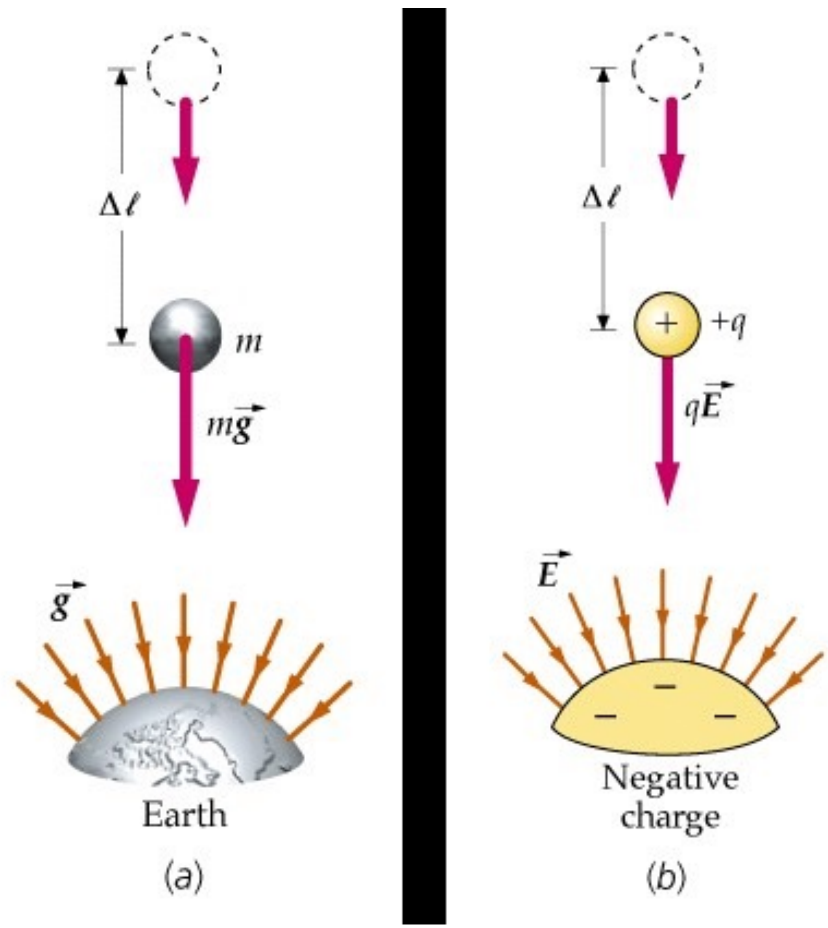
$$|F| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$$

- Newton's Law of Gravitation:

$$|F| = G\frac{|m_1||m_2|}{r^2}$$

- Identical except for:

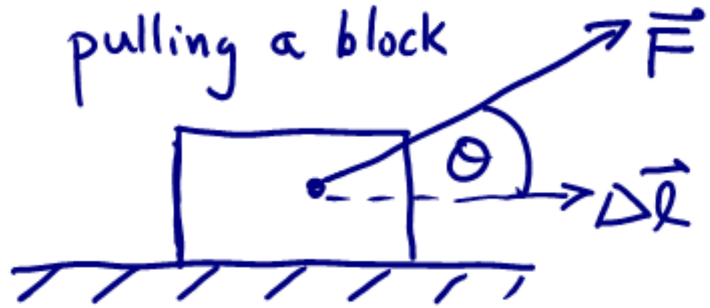
- overall constants
- existence of negative charge

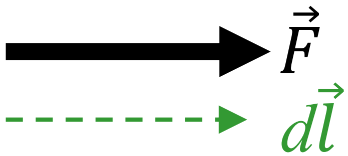


Work done by a force

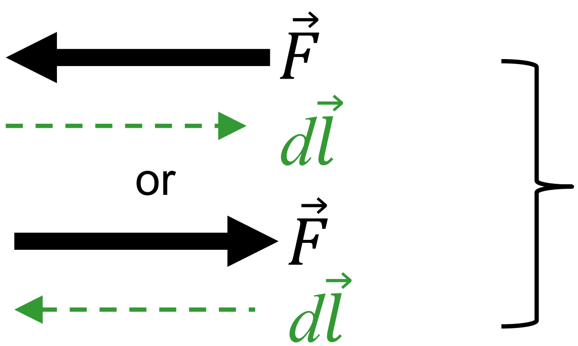
Remember:

$$W_F = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l}$$

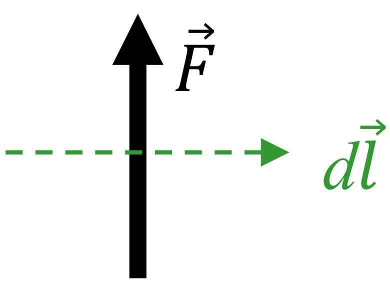




$W > 0$



$W < 0$



$W = 0$

$$W_{Net} = \Delta K$$

And work can
speed up/slow down an object
(Work-Energy Theorem)

Potential Energy

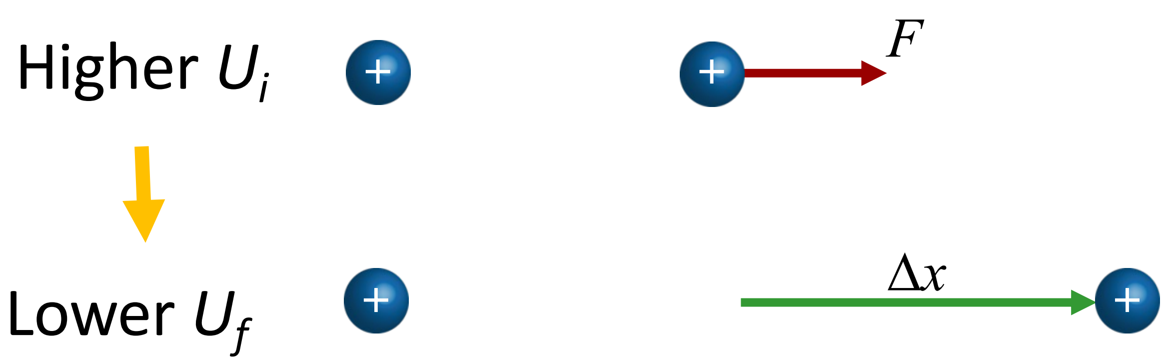
Conservative forces (e.g. Gravity, Coulomb, spring force) - special class of forces for which we can define a potential energy function U.

$$\Delta U = U_f - U_i \equiv -W_{conservative}$$

Work independent of path $i \rightarrow f$

If gravity does positive work, potential energy decreases.

Same idea for Coulomb force: if Coulomb force does positive work, potential energy decreases.



$$\vec{F} \cdot \vec{\Delta x} > 0$$

$$W > 0$$

$$U_f - U_i = -W < 0$$

$$U_f < U_i$$

Bridge 4

A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.

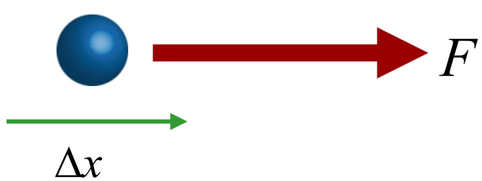
B) In a direction that makes its potential energy decrease.

C) Along a path of constant potential energy.

It will move in the same direction as $\vec{F}$

Work done by force: $W = \vec{F} \cdot \Delta\vec{x} > 0$

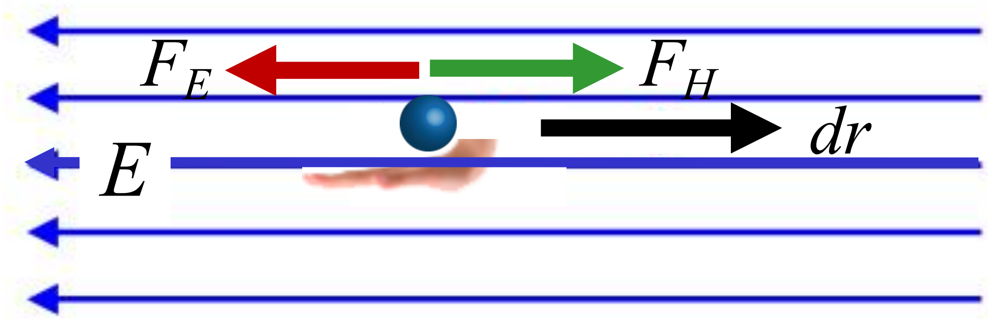
Change in potential energy
$\Delta U = -W < 0$ negative.



Intuition: particles tend to move from a location with higher potential to lower potential

Clicker Question

You hold a positively charged ball and walk rightward in a region that contains an electric field directed leftward.



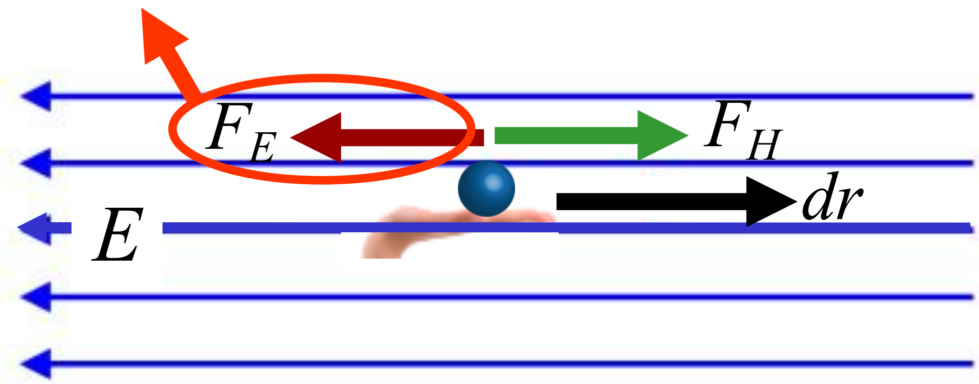
W_H is the work done by the hand on the ball
W_E is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$
B) $W_H > 0$ and $W_E < 0$
C) $W_H < 0$ and $W_E < 0$
D) $W_H < 0$ and $W_E > 0$

Clicker Question

Conservative force: $\Delta U = -W_E$

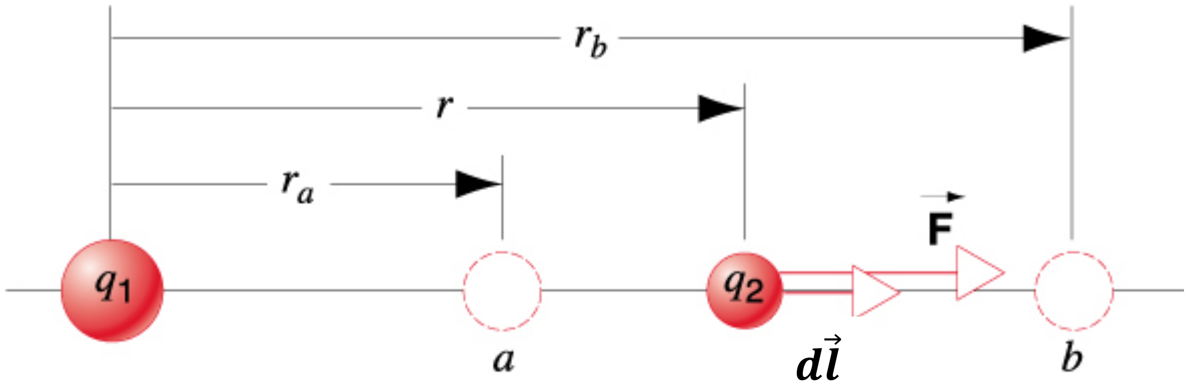


Is ΔU positive or negative?

A) Positive

B) Negative

Potential Energy of 2 Point Charges

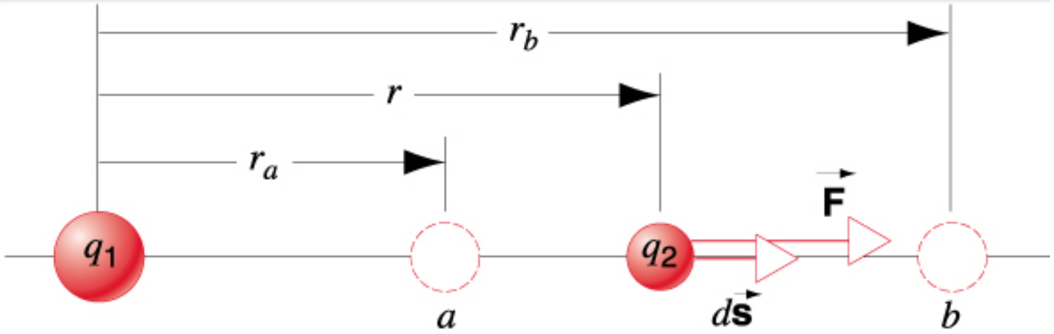


- Two charges, q_1 (fixed) and q_2.
- q_2 moves from r_a to r_b subject to the force due to charge q_1.
- The work done by the electric field on q_2 is

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{l}$$

Potential Energy of 2 Point Charges

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{l} = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} dr$$



$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \left(-\frac{1}{r} \right)_{r_a}^{r_b}$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

$$\Rightarrow \Delta U = U(r_b) - U(r_a) = -W_{ab} = \frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

Electric Potential Energy at a Single Point

- We have so far been talking about the difference in potential energy between two points.

It is also useful to define the potential energy at a point. All this means is that we choose a common reference point for the energy.

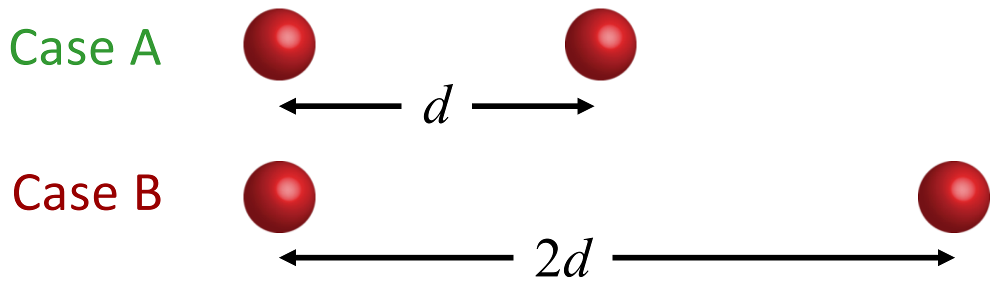
- Pick $r = \infty$ as the reference point and define $U(\infty) \equiv 0$.

$$U(r) - \underbrace{U(\infty)}_{\equiv 0} = \frac{q_1 q_2}{4\pi\epsilon_0}\left(\frac{1}{r} - \cancel{\frac{1}{\infty}}\right)$$

$$\Rightarrow U(r) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- Note: potential energy is negative when q_1 and q_2 have opposite charges.

Clicker Question



In case A two negative charges which are equal in magnitude are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A

B) Case B

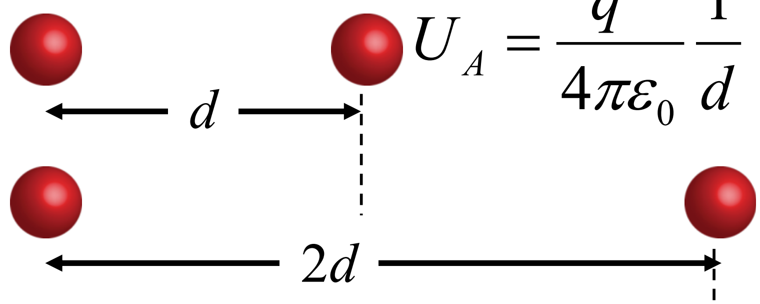
Clicker Question Discussion

As usual, choose $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$

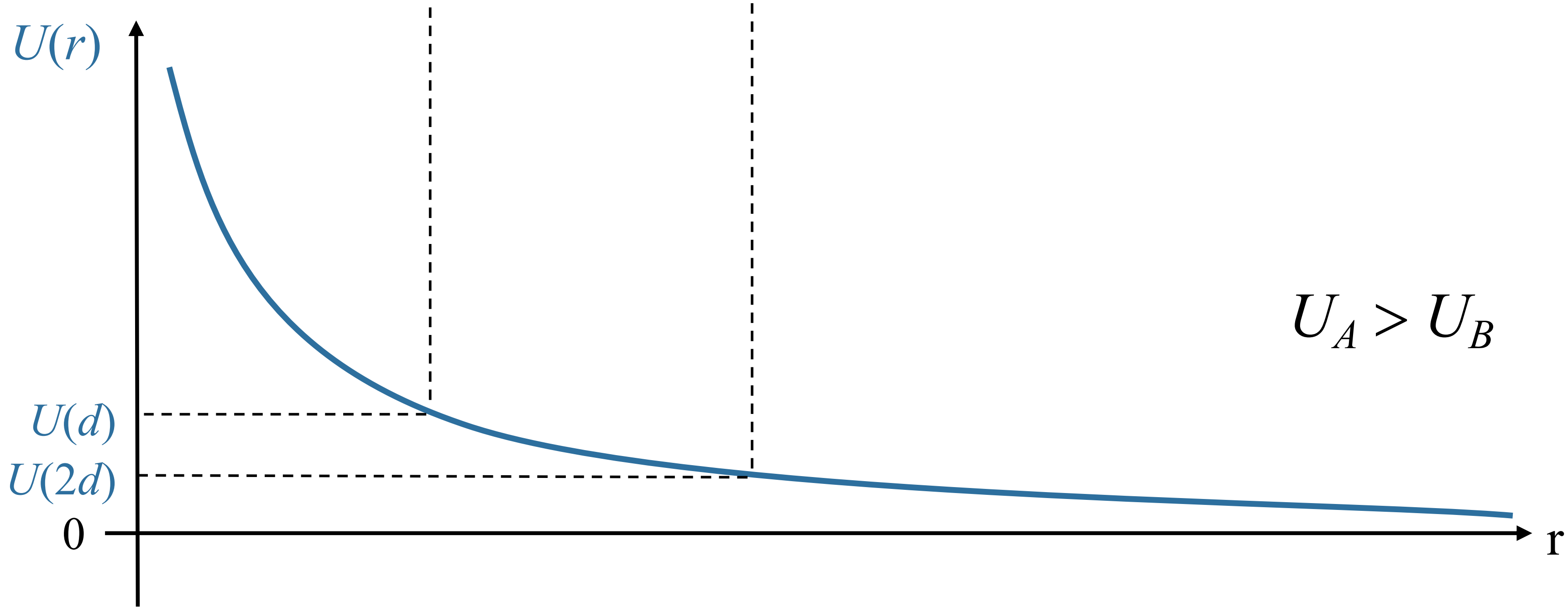
Case A

Case B



$$U_A = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{d}$$

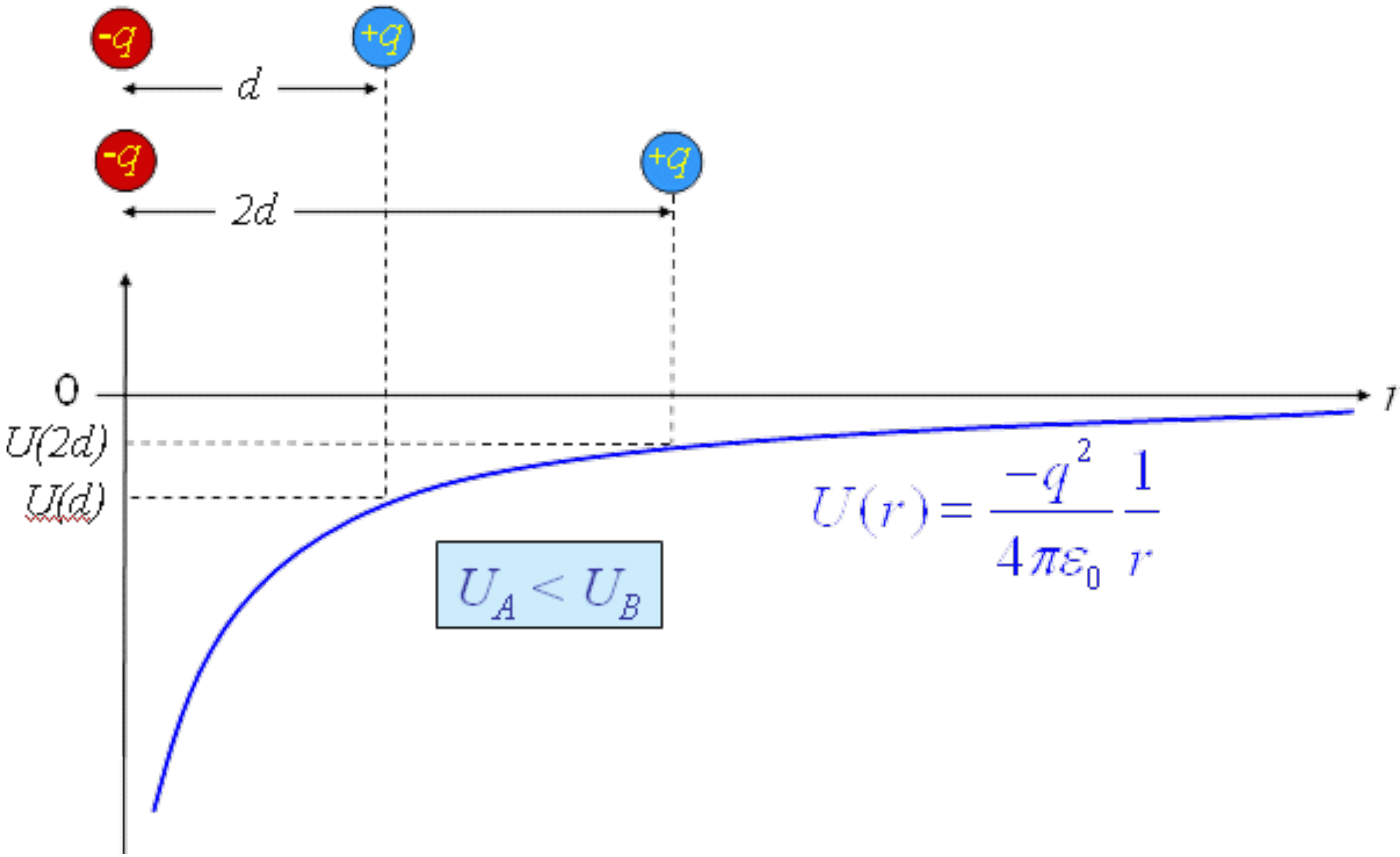
$$U_B = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{2d}$$



$$U_A > U_B$$

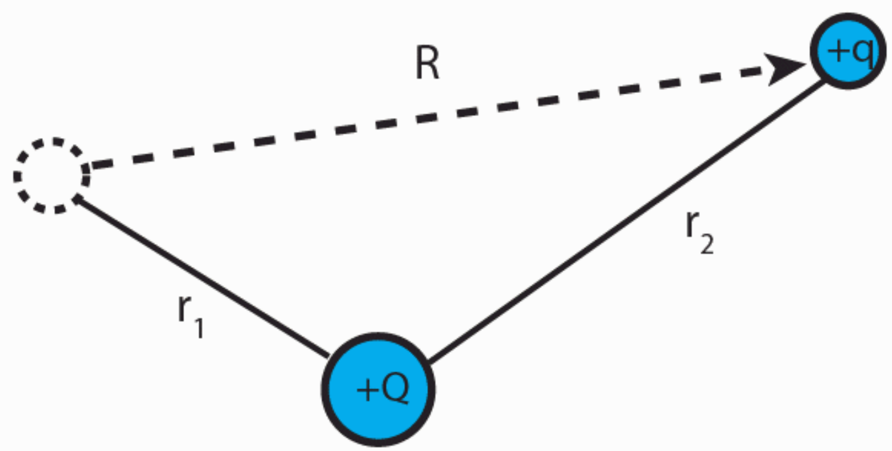
Prelecture Question

Which configuration has the highest potential energy?



Bridge 1

A charge of +Q is fixed in space. A second charge of +q was first placed at a distance r_1 away from +Q. Then it was moved along a straight line to a new position at a distance R away from its starting position. The final location of +q is at a distance r_2 from +Q.



What is the **change** in the potential energy of charge +q during the process?

A) kQq/R

B) $kQqR/r_1^2$

C) $kQqR/r_2^2$

D) $kQq((1/r_2)-(1/r_1))$

E) $kQq((1/r_1)-(1/r_2))$

Final - Initial

$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} \qquad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2}$$

$$\Delta U \equiv U_2 - U_1 = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Note: $+q$ moves AWAY from $+Q$.

Its Potential energy MUST DECREASE

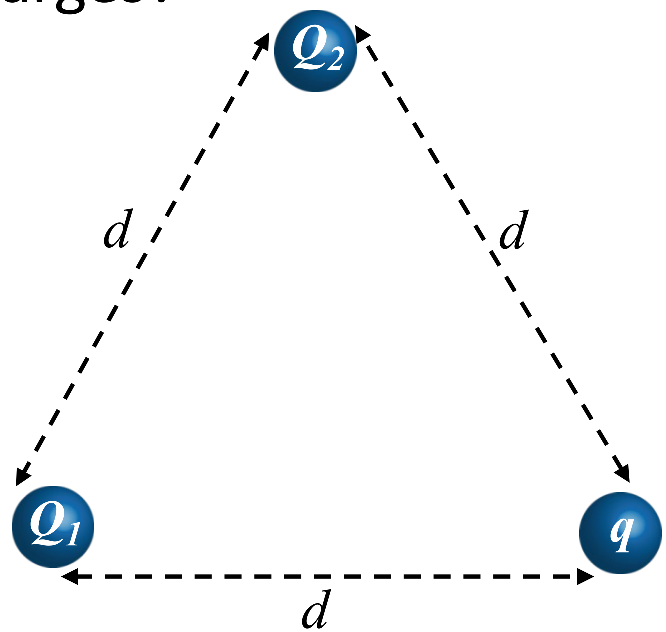
$$\Delta U < 0$$

Potential Energy of a System of Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{1}{4\pi\varepsilon_0}\frac{qQ_1}{d} + \frac{1}{4\pi\varepsilon_0}\frac{qQ_2}{d}$$

(superposition)



Sometimes it is useful to think of ΔU as the work done by an external agent to bring in the third charge starting from infinity.

Potential Energy of Many Charges

What is the total energy required to bring in three identical charges, from infinitely far away to the points on an equilateral triangle shown.

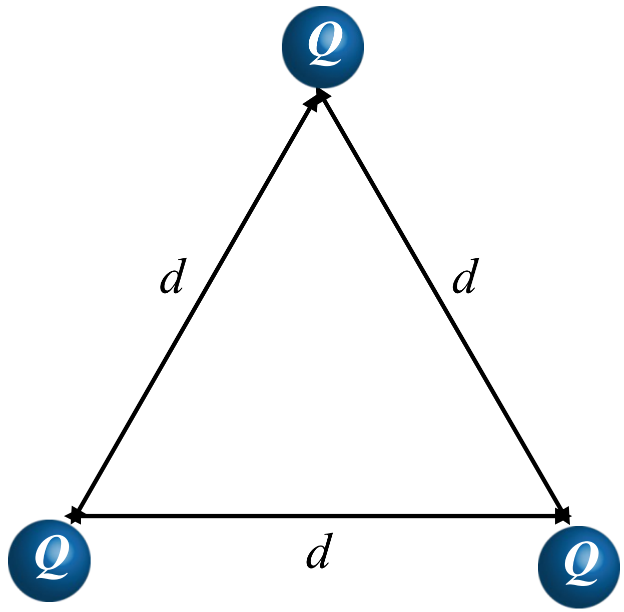
A) 0

B) $\Delta U = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = 2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = 3\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = 6\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



Potential Energy of Many Charges

Suppose one of the charges is negative. Now what is the total energy required to bring the three charges in from infinitely far away?

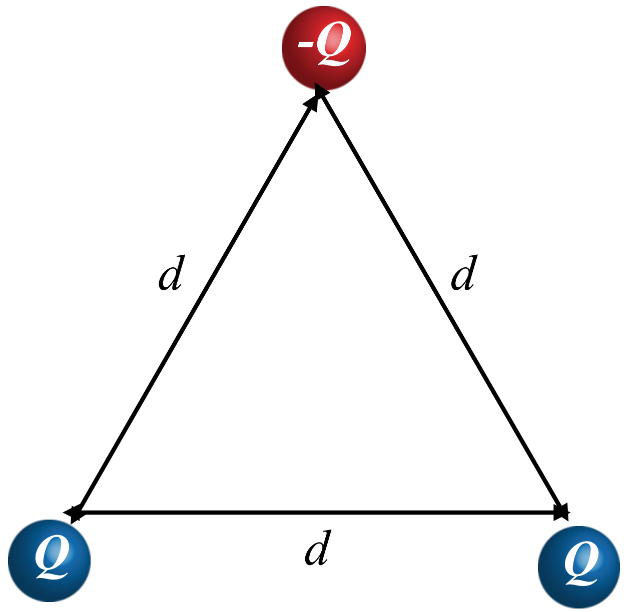
A) 0

B) $\Delta U = +1 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

C) $\Delta U = -1 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

D) $\Delta U = +2 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

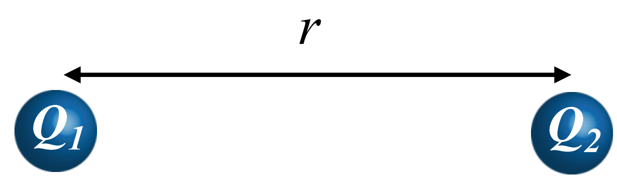
E) $\Delta U = -2 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$



Potential energy of many charges

For a pair of charges:

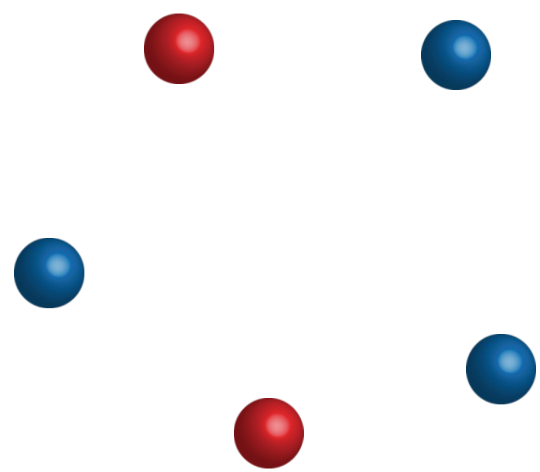
Just evaluate $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$



(We usually choose $U = 0$ to be where the charges are far apart)

For a collection of charges:

Sum up $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$ for all pairs

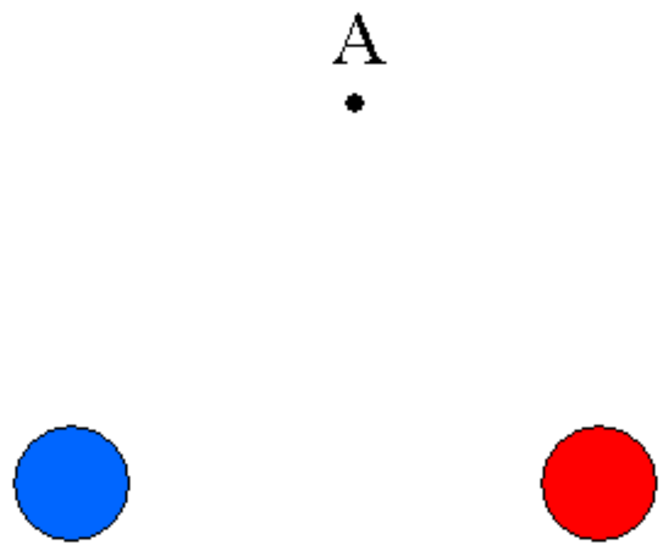


Be aware of double-counting,
each pair should count only once

$$\sum_{i<j} \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

Bridge 2

4) Two charges which are equal in magnitude, but opposite in sign are placed at equal distances from point A.



If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

- ○ increases
- ○ decreases
- ○ doesn't change
- ○ the answer depends on the sign of the third charge

Potential energy gain due to positive charge $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$

Potential energy gain due to negative charge $-\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$

Total potential energy change: 0 (both changes cancel each other)

Bridge3

6) You start with two point charges separated by some distance. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first.

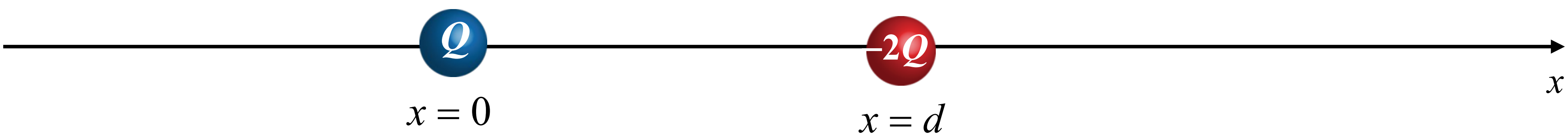
Is it possible find a place to which you can bring a third charge in from infinity without changing the total potential energy of the system?

- ○ YES, as long as the third charge is positive
- ○ YES, as long as the third charge is negative
- ○ YES, no matter what the third charge is
- ○ NO

Why? Let's do some calculations

Example

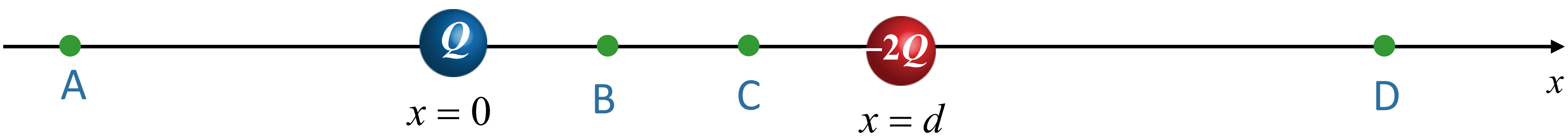
A positive charge Q is placed at $x = 0$ and a negative charge $-2Q$ is placed at $x = d$. At how many different places along the x axis could another positive charge $+q$ be placed without changing the total potential energy of the system?



A) 0
B) 1
C) 2
D) 3

Example

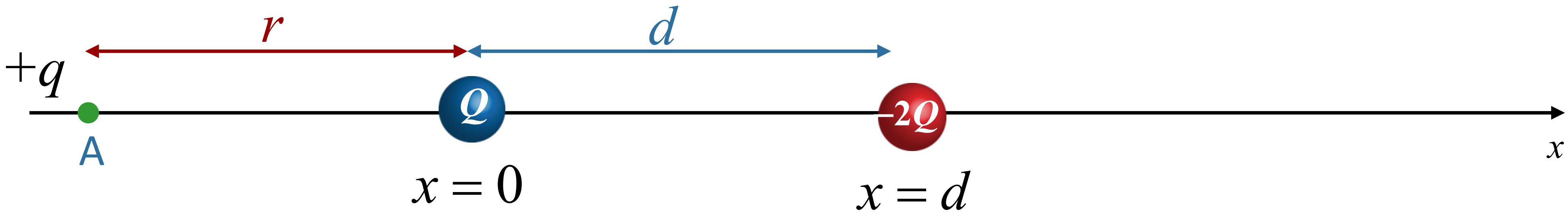
At which two places can a positive charge be placed without changing the total potential energy of the system?



A) A & B
B) A & C
C) B & C
D) B & D
E) A & D

Let's calculate the positions of A and B

Lets work out where A is



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

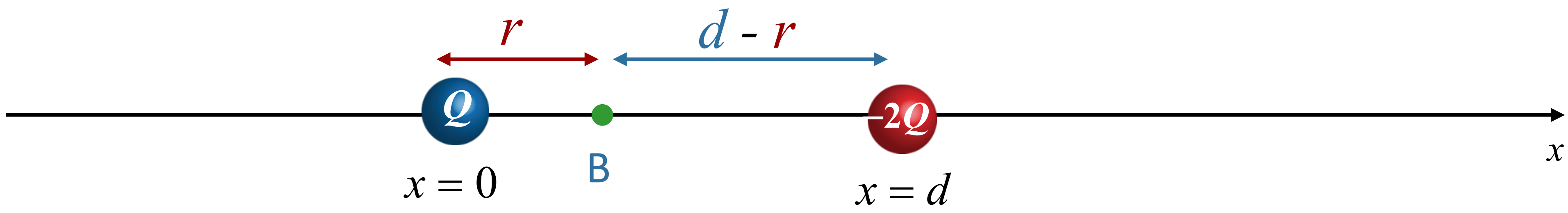
$$\frac{1}{r} = \frac{2}{r+d}$$

$$r = d$$

Makes Sense!

q is twice as far from $-2Q$ as it is from $+Q$

Lets work out where B is



Setting $\Delta U = 0$ → $$\frac{1}{r} = \frac{2}{d-r}$$

$$2r = d - r$$

$$r = \frac{d}{3}$$

Makes Sense!

q is twice as far from $-2Q$ as it is from $+Q$

Final Thoughts

1. Online homework due today 11:59pm
2. Pre-lecture and Bridge before next class: Ch23: Electric Potential
3. Instructor office hour today 2:30pm
4. Tutorials, iClicker, Achieve Prelecture and Bridge start to be counted this week