Propagation of Errors

Sum of two measurements

$$\text{width } w = 0.24 \pm 0.03 \text{ m}$$
$$\text{length } l = 0.89 \pm 0.04 \text{ m}$$

$$\text{sum } s = w + l = 0.24 + 0.89 \text{ m} = 1.13 \text{ m}$$

possible error of the sum is

$$\Delta s = \sqrt{\Delta w^2 + \Delta l^2} = \sqrt{0.03^2 + 0.04^2} = 0.05 \text{ m}$$

The perimeter is twice the sum.

$$p = 2(w + l) = 2s = 2.26 \text{ m}$$

possible error of perimeter is

$$\Delta p = 2\Delta s = 0.1 \text{ m}$$

In conclusion write the perimeter as

$$\boxed{p = 2.3 \pm 0.1 \text{ m}}$$

Product of two measurements

$$\text{width } w = 0.24 \pm 0.03 \text{ m} = \frac{0.03}{0.24} = 12.5\ \%$$

$$\text{length } l = 0.89 \pm 0.04 \text{ m} = \frac{0.04}{0.89} = 4.5\ \%$$

Area is the product of width and length

$$A = wl = (0.24 \text{ m})\ (0.89 \text{ m}) = 0.2136 \text{ m}^2$$

Possible error of area is given by

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

(add percentage errors)

$$\frac{\Delta A}{A} = \sqrt{12.5\%^2 + 4.5\%^2} = 13\ \%$$

$$\Delta A = (0.13)(0.2136 \text{ m}^2) = 0.028 \text{ m}^2$$

So one should write

$$A = 0.21 \pm 0.03 \text{ m}^2$$

What do you do with wierd functions?

For example what is the possible error of

$$x = \cos(\theta)$$

when $\theta = 21° \pm 2°$?

$$x = \cos(21°) = 0.9335$$

The easiest way to find Δx is to substitute for the minimum and maximum values:

$$\Delta\cos(\theta) = \frac{1}{2}\left|\cos(23°) - \cos(19°)\right|$$

$$= \frac{1}{2}\left|0.9205 - 0.9455\right| = \frac{0.024}{2}$$

$$= 0.012$$

so write

$$\boxed{x = 0.934 \pm 0.012}$$

Using Calculus to find $\Delta\cos(\theta)$:

$$\Delta x = \Delta\cos(\theta) = \left|\frac{d\cos(\theta)}{d\theta}\right| \Delta\theta$$

Caution:
If you use calculus, $\Delta\theta$ must be in **radians.**

$$\pm 2° = \pm 2\frac{\pi}{180} = 0.035 \text{ radians}$$

$$21° = 0.367 \text{ radians}$$

$$\Delta x = \Delta\cos(\theta) = |\sin(0.367)\ (0.035)|$$
$$= |(0.359)(0.035)|$$

$$= 0.0125$$

write 0.012 or 0.013 as you wish

Question:
This is slightly different from the substitution result of 0.12. Which is more accurate?