Introduction

0.1 What's Happening?

Physics is an experimental science. Everything we know about the world comes from observing it. That's why laboratory experience is an important aspect of learning elementary physics. Obviously you will not have time to discover and verify fundamental laws of physics in these introductory labs. Nevertheless, the laboratory serves to emphasize that contact with nature is the ultimate basis of physics.

Suppose you went to a physics lecture and you heard statements like the following:

> “A motionless object will stay at rest until a force is applied. When a force is applied to an object it will move in the direction of the applied force at a velocity related to its mass. The object will remain in motion until the force is removed. Motion ceases when the applied force ceases.”

> “Bodies which are unsupported above a horizontal surface will fall along the line between the initial position of the body and the centre of the Earth. Heavy bodies fall very rapidly and their velocities while falling are difficult to measure. However, the nature of the fall may be determined by observing less massive bodies which fall more slowly. These considerations lead us to conclude that such objects fall at a constant rate with the velocity being larger for bigger masses but slower for smaller sized objects.”

Would you believe these statements? They both seem reasonable and agree with most of our casual observations. In fact, they were accepted wisdom throughout ancient times and during the middle ages. Now, we consider these ideas to be false. Why?

The task of physics is to describe. But people describe things everyday and we don't call it physics. There are several things that distinguish the descriptions of physics from the common descriptions of everyday happenings.

First of all *physical descriptions are quantitative.* In the physics laboratory you won't just say the ball fell quickly. You will try to measure the speed as accurately as you can. Is the ball heavy or light? You must state the mass in grams and avoid vague, qualitative or subjective statements.

Secondly, *technical words we use in physics have precise definitions* and will be used to mean the same thing every time. Words like “force,” “energy,” "work," and "potential," are commonly used in English but their meanings are vague and vary from time to time. These words when used in the physics context always have very precise meanings and these meanings are tied to measurable quantities. Beginning physics students often confuse common meanings of words like "energy" with their precise physical definition. (An advertisement for a candy says it is "High in Energy—Low in Calories." A physicist thinks of calories as a measure of energy. The everyday idea of energy is much broader.)

Finally, physics strives to *describe as much as possible with one law*. Many things that seem completely different are actually aspects of the same principle. The law of gravitation, for example, describes how objects are attracted together.

$$F = \frac{GMm}{r^2} .$$

This one law encompasses events as different as a pebble falling to the ground and the motion of the moon around the earth. The connection between these phenomena can be obtained by logical reasoning. That's why physical laws are stated in mathematical form—mathematics is a language that includes the rules of logic. If the methods of algebra, trigonometry and calculus let you use one physical law to describe a lot of different phenomena, then the law is useful. On the other hand, a physical law that only describes one particular event and doesn't relate to anything else is bad because it does not simplify our view of the world.

The two false laws quoted above are bad for all three reasons: Terms like "force" are used vaguely. The descriptions are not quantitative. Worst of all, they describe some particular observations, but do not apply in other cases—the rules have too many exceptions. For example, if things move only when a force is applied—that's fine for ox carts—then why doesn't a boat stop moving when the motor stops. What about the trajectory of a cannon ball? Contemporary physics describes all these situations in a unified way and does not require a separate explanation for each one.

Physics is a science of observation. Find out what's happening, understand qualitatively, figure out the important quantities, and measure them. Formulate a law to describe what happened accurately and quantitatively in a general way. Find the logical implications of the law for other situations. Then test to see if it works. If the descriptive law misses the mark, that is, it doesn't predict properly, go back and try to find one that does work. This is the scientific method that has allowed us to view nature as unified and ordered rather than as a collection of independent and unrelated events. The essence here is direct experimentation and testing.

0.2 The Notebook

Your lab notebook is a record of your explorations. Everything you do during an experiment should be written down here in the form of a reliable and honest record. Use notebooks with bound pages. Loose-leaf notebooks are not acceptable. Paper with squared lines is helpful for diagrams and data tables. In the notebook explain what you were trying to do. Write down all measurements, what equipment you used and how you did them. **Write all data directly into the notebook in ink. Do not use whiteout for corrections.** Do not first put data on a scrap of paper and later copy it into the notebook. This risks introducing errors when you copy and you may even lose the data. All problems you encountered or mistakes you made should also be noted. Write down any unusual observations or thoughts that occur to you even if you can't fully explain them at the time. The notebook is not a formal report so it doesn't need to be exquisitely neat. The guiding rule in keeping a notebook is to *write down* *enough detail so that you could reconstruct several months or years later what you did during the experiment.*

Here are some important items to include in the notebook for each experiment:

- Date
- Your name and your partner's name
- Title of the experiment
- Brief sentence listing what you hope to accomplish
- Sketch of apparatus with labels
- All data stating the instruments used to measure them and error estimates.
  Put data in tables. Think carefully how to arrange the table. Try to place derived quantities beside the raw data from which they are calculated. It is usually better to put different quantities in different columns with its label and units at the top of the column as shown below.

distance (m)	time (s)	velocity (m/s)
0.25 ± 0.002	1.1 ± 0.1	0.23 ± 0.02
0.31	1.7	0.18 ±.0.01
0.39	1.9	0.21 ± 0.01

- Sample calculations of derived quantities.
  If you do the same calculation repeatedly you don't need to display each calculation in detail.
- Graphs made on separate pieces of paper should be stapled or glued into the book.
- Conclusions and discussion.
  A few brief sentences are usually enough. These introductory experiments are not of an extremely profound nature and the graders don't like to read unnecessary verbiage.

0.3 Measurements

0.3.1 What is Truth?

The instructor asks you to measure a distance on the table as accurately as you can with the meter stick. You take the metre stick, place it along the length to be measured, and tilt the stick on edge so that the millimeter marks touch the table. After very carefully positioning one end of the stick at the beginning of the distance you squint diligently at the end point. The nearest centimeter before the point is easily seen: 24 cm. Then you count the number of millimeters past the centimeter mark and get 7 mm. Now you notice that the end point is somewhere between two millimeter marks. It looks like a little less than halfway—maybe 0.3 or 0.4 millimeters. So you split the difference and write

24.735 cm.

Confident of deserving praise you give the result to the instructor who emits a barely audible moan. What went wrong?

When you measure a value you are trying to estimate something that nobody will ever know: the exact value. No instrument is perfect. For example, the marks on the meter stick might not be in exactly the right place, or the end might be a little worn making it difficult to align with the beginning point. There is always some judgment involved in reading the value like guessing between 0.3 or 0.4 mm. Even very precise instruments with digital readouts are limited. If there is a five-digit reading of 24.734 the real value could be anywhere between 24.7335 to 24.7345.

In addition to the limitation of the measuring instrument most things we measure are not exactly fixed and definite. On a gross scale the length may seem constant. But as we try to refine the measurement to microscopic scale it becomes apparent that the length is fluctuating because of vibrations or temperature changes. Irregularities in size become evident as the measurement accuracy increases.

To be honest we must acknowledge these limitations. In lining up the beginning of a metre stick there is the possibility of at least a 0.5 mm error one way or the other. This is written as ±0.5 mm. When we cannot really decide where between the two millimetre marks the end point lies, we should decide that it is most probably between 0.2 and 0.4 mm: another ±0.2 mm uncertainty. In total there is an uncertainty of at least ±0.7 mm just in reading the meter stick. If you add in the other sources of uncertainty that gives at least ±1 mm or ±0.1 cm. You could write

$$24.734 \pm 0.1 \text{ cm.}$$

The hundredth and thousandth of a cm are not significant when you are uncertain about the tenth of a cm. Therefore, you should write

$$24.7 \pm 0.1 \text{ cm.}$$

Including an uncertainty estimate and rounding off to the correct number of significant digits like this is simply being honest about how well you can estimate the value. Realistically, you probably should allow a little more uncertainty because of meter stick inaccuracy or irregularities at the ends of the object being measured.

Try this test. Take a length of string or wire. Give it to five people to measure and note the variation of length measurements. If you have estimated the uncertainty well, most of the values should lie within your uncertainty range. In making any measurement always try to estimate the range of values five different people would get (assuming they are competent) and use this to guide your estimate of the uncertainty.

0.3.2 Scale-Reading Uncertainties

One source of uncertainty comes from reading a scale on a meter or dial. Here are some guidelines when estimating such reading inaccuracies.

If the space between marks is about the same as the width of the marks then just estimate to the nearest mark and the uncertainty is ± one-half the smallest division. For example, if the mm marks on a meter stick are 0.5 mm wide and there is 0.5 mm space between them then you can’t get much closer than ±0.5 mm (the nearest mm).

If the space between marks is several times the width of the marks then try to guess whether the reading is more than half way or less than half way. For example you could write 0.2 or 0.3 mm if it’s less than halfway, 0.7 or 0.8 mm if it is more than halfway, 0.5 mm if it looks like it is exactly halfway, 0.1 or 0.9 mm if it is very near a line. Give yourself an uncertainty of ±0.2 or ±0.3 mm.

If the distance between the marks seems very wide then try to mentally divide the interval into ten divisions. For example, if there are only cm markings on the meter stick, try to estimate the nearest millimeter and allow ±1 or ±2 mm uncertainty.

Reading a scale is only one source of inaccuracy in the value. Add more if there is a possibility of calibration error, or if the quantity you measure is irregular or fluctuates. It is difficult to list all the things that can add to the unreliability of a measured quantity because every situation is a little different.

For example, in the case of a needle on a dial or meter you must consider parallax: the relative positions of the needle and tick marks move when you move your head from side to side.

Digital readings seem more certain. However, the accuracy is usually at least ±1 digit plus a certain percentage of the value. Meters sometimes carry an accuracy note on them. (Look at the bottom of the digital volt meters.) If the accuracy is indicated as ±(2% + 1 digit) this means that you have to add ±2% of the reading to ±1 in the position of the last digit of the reading to get the total error. For example, a three-digit meter reads 25.3 mV then the accuracy is

$$\pm(0.02 \times 25.3 + 0.1) = \pm(0.5 + 0.1) \text{ mV} = 0.6 \text{ mV}.$$

Your estimate of the uncertainty of a value may well be different from someone else's estimate. Generally there is no exact value. Therefore you only need to keep one significant figure for the uncertainty estimate unless its first digit is 1 when you should keep two digits. If your estimate is a factor two different from the instructor's there should be no problem. However, if your estimate differs from an instructor's by a factor of ten it will probably be questioned.

0.3.3 Significant Figures and Precision

When you write values make sure that the number of significant figures corresponds roughly with the error estimate. Even without an explicit error estimate, the number of significant figures implies an uncertainty. For example writing 24.73 cm implies 24.73 ± 0.05 cm. For intermediate values that will be used in calculation, it's a good idea to keep an extra significant figure to maintain accuracy: 24.734 ± 0.05 cm. But final results should not keep unnecessary figures.

Sometimes for large numbers you may need to write zeros that are not significant, e.g., 24000 ± 400 gm. In this case it is better to write (24.0 ± 0.4) x 10^3 gm to avoid insignificant zeros.

0.4 Graphical Analysis

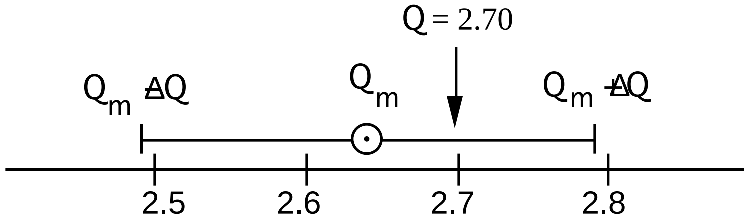
How do we interpret experimental results given that they have uncertainties? Have you disproved conservation of linear momentum if you find that the initial momentum is 0.105 kg-m/s and final momentum is 0.114 kg-m/s? Suppose you determine the value of g in Lab 2 to be 9.93 m/s^2 while the accepted value is 9.8 m/s^2. Is the deviation significant? The important points are:

1. you must consider the experimental uncertainties in order to decide these questions and
2. you can only give an argument based on probability.

This section shows some standard graphical techniques that will help you interpret experimental results. In the first part (0.4.1) we consider the case where your result is a single quantity, for instance the density of aluminum, which will be compared with its accepted value. In the next part (0.4.2) we assume that you study the relationship between two quantities by varying one of them systematically, for instance measuring the vertical velocity v_y of a falling object at various times t.

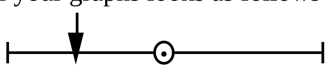
0.4.1 Graphical analysis of a single quantity.

Suppose Q_m is the value you measured for some quantity or calculated from several measured quantities. Let $\pm\Delta Q$ be the best estimate of the uncertainty of Q_m and let Q be the accepted or assumed value. Draw a (horizontal) number line marked in appropriate, easy-to-read units, choosing a scale such that ΔQ measures from 1 cm to 5 cm. Locate the three points Q_m, $Q_m - \Delta Q$, and $Q_m + \Delta Q$ and plot them just above the number line as shown.



Now you have a visual comparison of Q with your result that can be interpreted more easily than the set of numbers Q, Q_m, and ΔQ. There is a predictable probability that a measured result Q_m will deviate by more than $\pm\Delta Q$ from the "true" or accepted value if certain assumptions hold about the randomness of the measurement errors. At this point of your scientific career you need no probability theory, but use the following rules of thumb.

If your graphs looks as follows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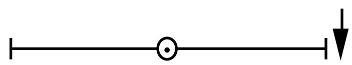
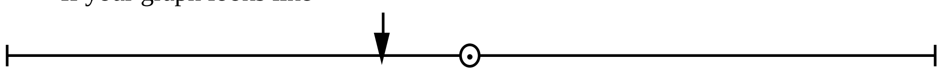


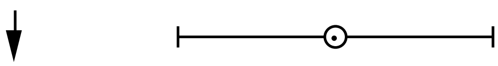
then you have agreement, or your disagreement is not significant.

If your graph looks like



then you were very lucky, or you overestimated your errors. The latter may seem to be a safe thing to do, but it renders your results less meaningful. So be sure to estimate errors more carefully next time.

If you get



where the deviation is as large as two times your error bar ($2\Delta Q$) then there is a question as to whether the disagreement is real or not. Further measurements may be useful.

But if you get

then either

1) you made a blunder, or

2) you underestimated the errors involved, or

3) you have discovered a significant departure from the accepted or assumed value. This condition (3) is the most interesting and requires the most careful scrutiny. You are not likely to discover significant departures from accepted laws in this course. In your later research, however, you may find such cases, and if you do, make sure you avoid pitfalls (1) and (2).

Sometimes you must compare two values Q_1 and Q_2 which both have significant uncertainties. Asking the question "do they differ?" is equivalent to forming a new variable $D = Q_1 - Q_2$ and asking does D differ significantly from zero? See section 0.5 for how to calculate ΔD from ΔQ_1 and ΔQ_2. Alternatively you could form a ratio, $R = Q_1/Q_2$ and compare R to one. There are some examples of this method in Lab 1.

0.4.2 Graphical analysis of two related quantities

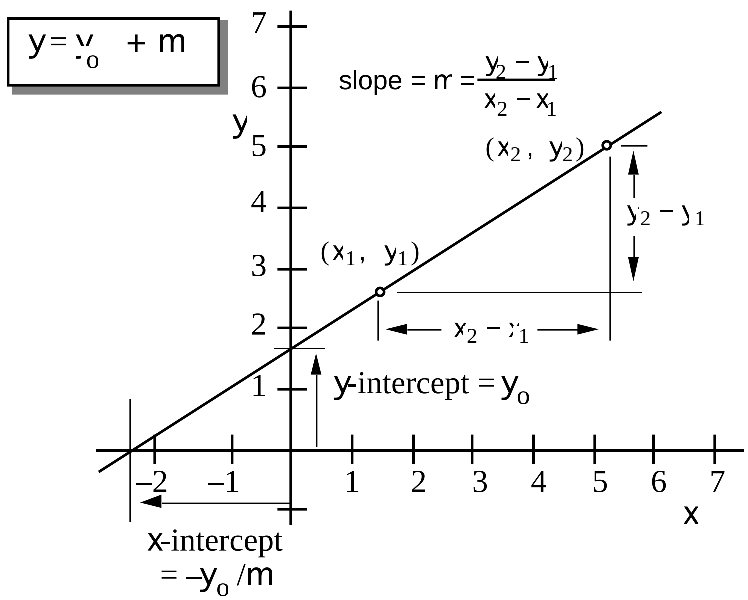
In several labs you are asked to graph one measured quantity, say y, against some other, say x, and (i) to decide if there is a straight-line relationship between y and x, and if this is the case, to determine (ii) the slope ("rise over run") and, possibly, (iii) the intercept of the straight line with the x- or y-axis. You remember from math courses that a relationship between sets of numbers y and x

of the form $y = y_o + mx$ leads to a straight-line graph as shown in the figure. The slope is defined as

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

for any pair of numbers (x_1, y_1) and (x_2, y_2) on the line. The y-intercept is the value of y for $x = 0$, and the x-intercept is the value of x for $y = 0$. The slope, m, can be positive, negative or zero, and the same is true for the intercept, y_o.

The figure shows a mathematically perfect straight line where there is no ambiguity about the relationship between x and y. In an experiment, however, we have a small set of measured values (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) with finite uncertainties Δx and Δy respectively. As a result, it is not always obvious that y and x obey a straight-line relationship. Even if they do neither the slope nor the intercepts are accurately determined. How do we decide and how do we find the best values and their uncertainties? The following rules should help you prepare graphs and analyse your data.



1. Find the smallest and largest x- and y-values that you want to plot, including uncertainties, and consider the size of graph paper. Choose a convenient scale for the x- and y-axes that accommodates all the relevant points, spreading them out as far as possible but is easy to read.

   Note: If you are checking a relation of the form $y = mx$, then the origin is a relevant point and should be on your graph even if you did not measure that point explicitly, as x=0 implies y=0 .

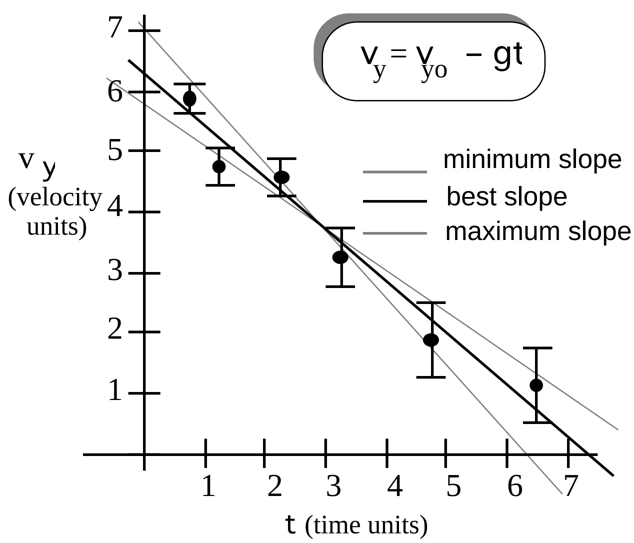
   In another situation you may have to determine the x-intercept by extrapolation to the value y=0 which could be unmeasureable. In this case too, the graph must include the point (0,0).

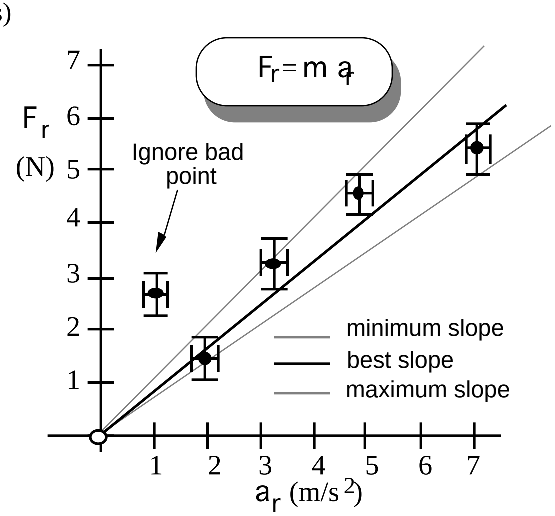
2. Label the two axes: Mark the scale, indicate the quantity plotted and the units used.

3. Plot your data points (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) with their uncertainties $\pm\Delta x_i$, $\pm\Delta y_i$ shown as error bars as in part 0.4.1. If Δx or Δy is too small to be plotted say so on the graph.

4. Now you are ready to check if a straight line fits your data. Use a transparent ruler to find the "best line" that passes closest to all your data points. Remember that your "best line" must pass through the origin if you test a relationship of the form $y = mx$. You can claim that a straight line fits your data if it intersects at least 2/3 of your error bars. If a data point is far off the line check for errors in plotting it or recalculate it—chances are that 95% of all points are within $\pm\Delta y$ of the best line. If you find no error in the odd data point, ignore it and check if a straight line fits the rest of the points.

5. Having found the best straight line determine the slope from well-separated points on the line. (Use points on the straight line that you draw. Don't use raw data points unless they are exactly on the line.) To find the uncertainty in the slope, Δm, draw the steepest and shallowest lines that still pass through more than half of the error bars. Let these lines pass through the origin if you check a relation of the form $y=mx$ and you are sure there is no zero-point offset. Pick two well-separated points on the steepest line and two well-separated points on the shallowest line in order to calculate the maximum and minimum slopes m_{max} and m_{min}. Define Δm as $(m_{max} - m_{min})/2$. For a typical set and a reasonable choice of steepest, best and shallowest line you expect that $m \approx (m_{max} + m_{min})/2$. If this is not the case, reconsider your choice of lines. The next figures show two hypothetical examples.





0.5 Propagation of Errors: examples

Sum of two measurements

width $w = 0.24 \pm 0.03$ m
length $l = 0.89 \pm 0.04$ m

sum $s = w + l = 0.24 + 0.89$ m $= 1.13$ m

possible error of the sum is

$$\Delta s = \sqrt{\Delta w^2 + \Delta l^2} \qquad = \sqrt{0.03^2 + 0.04^2} = 0.05 \text{ m}$$

The perimeter is twice the sum.

$$p = 2(w + l) = 2s = 2.26 \text{ m}$$

possible error of perimeter is

$$\Delta p = 2\Delta s = 0.1 \text{ m}$$

In conclusion write the perimeter as

$$\boxed{p = 2.3 \pm 0.1 \text{ m}}$$

Product of two measurements

$$\text{width } w = 0.24 \pm 0.03 \text{ m} = \frac{0.03}{0.24} = 12.5\ \%$$
$$\text{length } l = 0.89 \pm 0.04 \text{ m} = \frac{0.04}{0.89} = 4.5\ \%$$

Area is the product of width and length

$$A = wl = (0.24 \text{ m})\ (0.89 \text{ m}) = 0.2136 \text{ m}^2$$

Possible error of area is given by

(add percentage errors)

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

$$\frac{\Delta A}{A} = \sqrt{12.5\%^2 + 4.5\%^2} = 13\ \%$$
$$\Delta A = (0.13)(0.2136 \text{ m}^2) = 0.028 \text{ m}^2$$

So one should write

$$\boxed{A = 0.21 \pm 0.03 \text{ m}^2}$$

What do you do with wierd functions?

For example what is the possible error of

$$x = \cos(\theta)$$

when $\theta = 21° \pm 2°$?

$$x = \cos(21°) = 0.9335$$

The easiest way to find Δx is to substitute for the minimum and maximum values:

$$\Delta\cos(\theta) = \frac{1}{2}\left|\cos(23°) - \cos(19°)\right|$$

$$= \frac{1}{2}\left|0.9205 - 0.9455\right| = \frac{0.025}{2}$$

$$= 0.0125$$

so write

$$\boxed{x = 0.934 \pm 0.012}$$

Using Calculus to find $\Delta\cos(\theta)$:

$$\Delta x = \Delta\cos(\theta) = \left|\frac{d\cos(\theta)}{d\theta}\Delta\theta\right|$$

Caution:
If you use calculus, $\Delta\theta$ must be in **radians.**

$$\pm 2° = \pm 2\frac{\pi}{180} = 0.035 \text{ radians}$$

$$21° = 0.367 \text{ radians}$$

$$\Delta x = \Delta\cos(\theta) = \left|\sin(0.367)(0.035)\right|$$
$$= \left|(0.359)(0.035)\right|$$

$$= 0.0125$$

write 0.012 or 0.013 as you wish.

0.6 Propagation of Uncertainties: General Rules

The uncertainty of calculated quantities depends directly on the uncertainties of the variables used in the calculation. For brevity we simply state the rules for commonly encountered situations here. Later some of these rules will be justified but a complete understanding needs statistical methods which are too advanced for this course.

In the following let $A, B, C, \ldots$ stand for *independent* quantities going into a calculation with uncertainties ΔA, ΔB, ΔC, ... Let $Y = f(A,B,C, \ldots)$ be the calculated quantity of interest.

0.6.1 **Rule 1: A constant multiple**

If

$$Y = k\,A$$

where k is a constant, then

$$\Delta Y = k\,\Delta A$$

0.6.2 **Rule 2: Addition and Subtraction**

If

$$Y = A \pm B \pm C$$

then

$$\Delta Y = \sqrt{(\Delta A)^2 + (\Delta B)^2 + (\Delta C)^2}$$

The generalization to four or more addends should be obvious. The reason for taking the root-squared sum instead of just adding the uncertainties is that we are not certain whether the errors will cancel or add. If there are many terms in the sum, there will typically be some cancellation and the combined error will not likely be as large as the error given by the sum $|\Delta A| + |\Delta B| + |\Delta C| + \ldots$.

0.6.3 **Rule 3: Multiplication and Division**

If

$$Y = ABC,\ Y = ABC^{-1},\ Y = AB^{-1}C^{-1},\ \text{or}\ Y = A^{-1}B^{-1}C^{-1}$$

then

$$\frac{\Delta Y}{Y} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2 + \left(\frac{\Delta C}{C}\right)^2}$$

For multiplication and division we add the fractional (or percentage) errors.

0.6.4 **Rule 4: Powers**

If

$Y = A^{\alpha}$, where α is arbitrary: integer, fraction, positive or negative

then

$$\frac{\Delta Y}{Y} = |\alpha|\frac{\Delta A}{A}$$

0.6.5 **Examples**

(1) $Y = AB^2$

Let $C = B^2$ so that $Y = AC$.

Then according to rule 3 $\quad \dfrac{\Delta Y}{Y} = \sqrt{\left(\dfrac{\Delta A}{A}\right)^2 + \left(\dfrac{\Delta C}{C}\right)^2}$.

According to rule 4 $\quad \dfrac{\Delta C}{C} = 2\dfrac{\Delta B}{B}$

Therefore $\quad \dfrac{\Delta Y}{Y} = \sqrt{\left(\dfrac{\Delta A}{A}\right)^2 + 4\left(\dfrac{\Delta B}{B}\right)^2}$.

Example (2) $Y = \frac{1}{A} + \frac{1}{B}$

Let $C = A^{-1}$ and $D = B^{-1}$.

According to rule 4 $\frac{\Delta A}{A} = \frac{\Delta C}{C}$ and $\frac{\Delta B}{B} = \frac{\Delta D}{D}$.

However $C = A^{-1}$ and $D = B^{-1}$ so these latter two expressions can be written as

$$\Delta C = \Delta A/A^2 \text{ and } \Delta D = \Delta B/B^2.$$

Hence

$$\Delta Y = \sqrt{\left(\frac{\Delta A}{A^2}\right)^2 + \left(\frac{\Delta B}{B^2}\right)^2}.$$

0.6.6 **The General Case**

The four rules and the rule for the general case can be derived with the help of calculus. In this discussion we will assume a function of the form

$$Y = f(A,B) \tag{1}$$

Generalization to functions of more variables is easy.

Calculus tells us that if we change A by a small amount dA and B by dB then the change in Y is give by

$$dY = \left(\frac{\partial f}{\partial A}\right)_B dA + \left(\frac{\partial f}{\partial B}\right)_A dB \tag{2}$$

where the subscript A or B on the partial derivatives has the conventional meaning that the quantity A or B is to be held fixed while taking the derivative. (That is the definition of partial derivative.) For convenience these subscripts are omitted from now on.

This equation tells us how fast the function Y changes when we change its inputs A and B by some small amounts dA and dB. We can identify these small changes with small errors in our measurements, $\pm\Delta A$ and $\pm\Delta B$. These errors can have either algebraic sign and so can the derivatives $(\partial f/\partial A)$ and $(\partial f/\partial B)$. In the worst case both terms in (2) are positive or both negative in which case you have

$$\Delta Y_{\text{worstcase}} = \left|\left(\frac{\partial f}{\partial A}\right)\Delta A\right| + \left|\left(\frac{\partial f}{\partial B}\right)\Delta B\right| \tag{3}$$

On the other hand it could turn out that you are lucky and the two terms in equation (2) tend to cancel. Then you would have

$$\Delta Y_{\text{bestcase}} = \left|\left|\left(\frac{\partial f}{\partial A}\right)\Delta A\right| - \left|\left(\frac{\partial f}{\partial B}\right)\Delta B\right|\right| \tag{4}$$

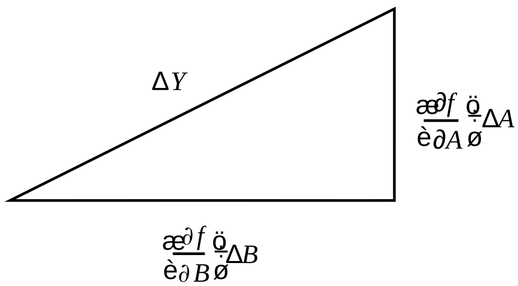
In practice there is no way of knowing if the errors are going to cancel or add in the final answer. Probability theory says that in this case, if the errors are independent and have a normal distribution then we should add the individual errors "in quadrature," i.e., form the root-squared sum as follows

$$\Delta Y = \sqrt{\left(\left(\frac{\partial f}{\partial A}\right)\Delta A\right)^2 + \left(\left(\frac{\partial f}{\partial B}\right)\Delta B\right)^2} \qquad (5)$$

Note that ΔY has the property

$$\Delta Y_{\text{worst case}} \geq \Delta Y \geq \Delta Y_{\text{best case}}.$$

You can visualize this way of adding errors by means of a right triangle.



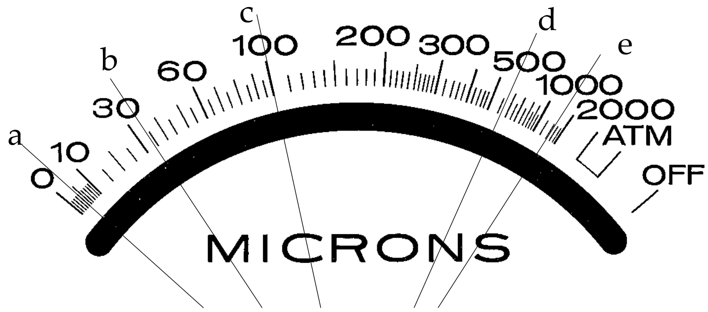
The generalization of equation (5) to an arbitrary number of variables $Y = f(A,B,C...)$ is

$$\Delta Y = \sqrt{\left(\left(\frac{\partial f}{\partial A}\right)\Delta A\right)^2 + \left(\left(\frac{\partial f}{\partial B}\right)\Delta B\right)^2 + \left(\left(\frac{\partial f}{\partial C}\right)\Delta C\right)^2 + \ldots} \qquad (6)$$

With a little effort you should be able to convince yourself that rules 1 through 4 are but special cases of equation (5). Equation (5) can also be used where the rules do not apply such as trigonometric or exponential functions.

Error Analysis Problems

1. The scale shown is from a pressure gauge. The reading is in microns (μm) of mercury. State the value and uncertainty limits of the reading at positions a, b, c, d and e.



2. Specifications on the bottom of a Fluke digital multimeter say:

ACCURACY: ±(% OF READING + DIGITS)
DC VOLTAGE: (0.1 + 1)
DC CURRENT: (0.3 + 1) EX (0.5 + 1 @ 10A)

Decipher this inscription and rewrite the following readings with the correct number of significant figures and state the uncertainty. a) 2.125 V, b) 0.002 mV, c) 0.615 A, d) 21.21 V, e) 9.892. A

3. Determine the values and uncertainties of the following derived quantities from the stated values and uncertainties of the respective measured quantities.

a) voltage, V = 5.6±0.1 V, current I = 0.125 ±0.02 A,	power, P = ? ± ?
b) time, t = 3.67±0.05 s, distance x = 4.52±0.01 m,	velocity, v = ? ± ?
c) distances: x = 6.24±0.05 m, y = 7.8±0.1 m, z = 4.03±0.01m	area, $A = x^2 + yz$ = ? ± ?
d) distances: x = 6.24±0.05 m, y = 7.8±0.1 m, z = 4.03±0.01m	geometric mean $\langle x \rangle = (xyz)^{1/3}$ = ? ± ?
e) amplitude, A = 9.000± 0.001 m, time, t = 26.0 ± 0.5 s	distance, $d = A \sin(t/16)$ = ? ± ? (angle in radians)
f) amplitude, A = 9.000± 0.001 m, time, t = 51.0 ± 0.5 s	distance, $d = A \sin(t/16)$ = ? ± ? (angle in radians)

4. a) Five measurements of diameter are made with an estimated reading accuracy of ±0.3mm:
3.20, 3.19, 3.21, 3.22, 3.20 cm. What is the average and its uncertainty?

b) Six measurements are made on another sample with the same reading accuracy:
3.21, 3.68, 2.69, 3.43, 3.52, 3.54 cm. What is the average and its uncertainty?

5. Gold is denser than lead and more expensive. You wish to determine if a wafer of gold has been diluted with lead by measuring its density. The wafer is supposed to be 25 g of pure gold and you can measure its volume with an uncertainty of ±0.2 ml and its mass within ±0.1 g. What is the largest admixture of lead that can escape detection expressed as a fraction of total mass?

Appendix A:

Excerpts Adapted from *PSSC Physics, 2nd Ed.,*
D.C. Heath Co., 1965

Appendix B:

Excerpts from *Physics for the Inquiring Mind* by Eric M. Rogers
Princeton University Press, 1960.