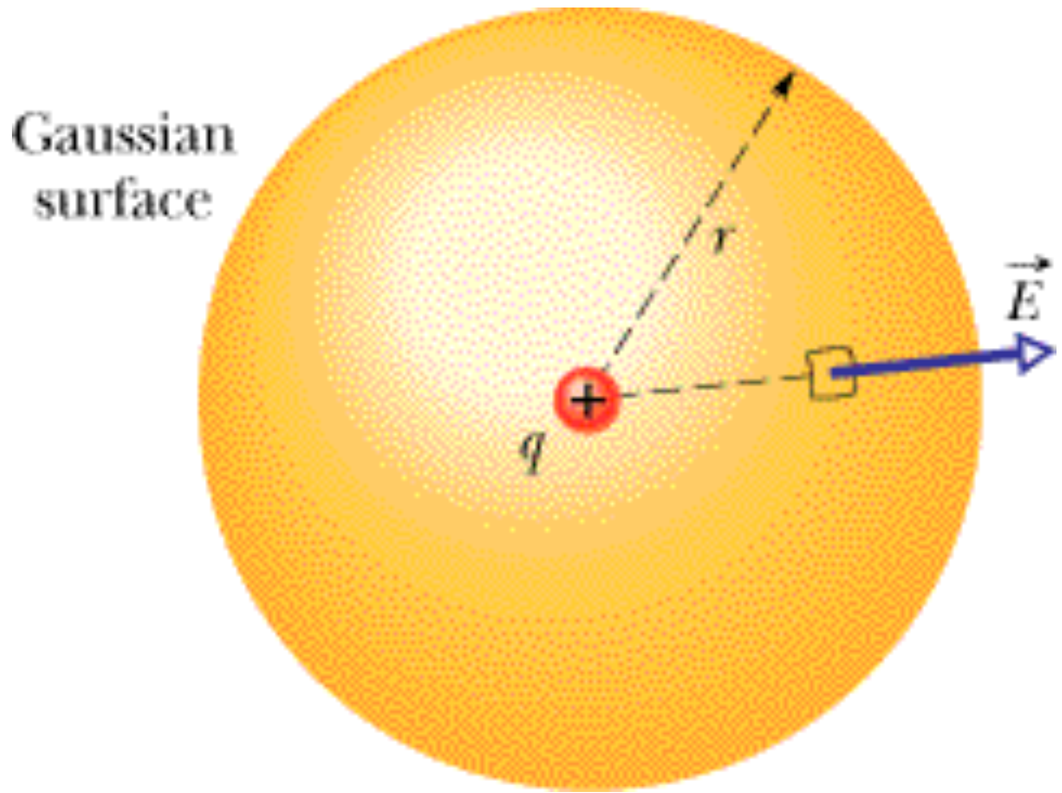
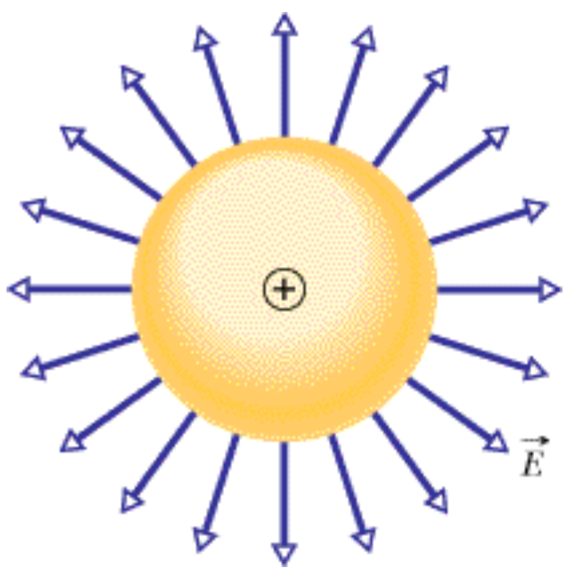
Gauss’ Law



Imagine a charge q and a spherical Gaussian surface of radius r with q at the centre.

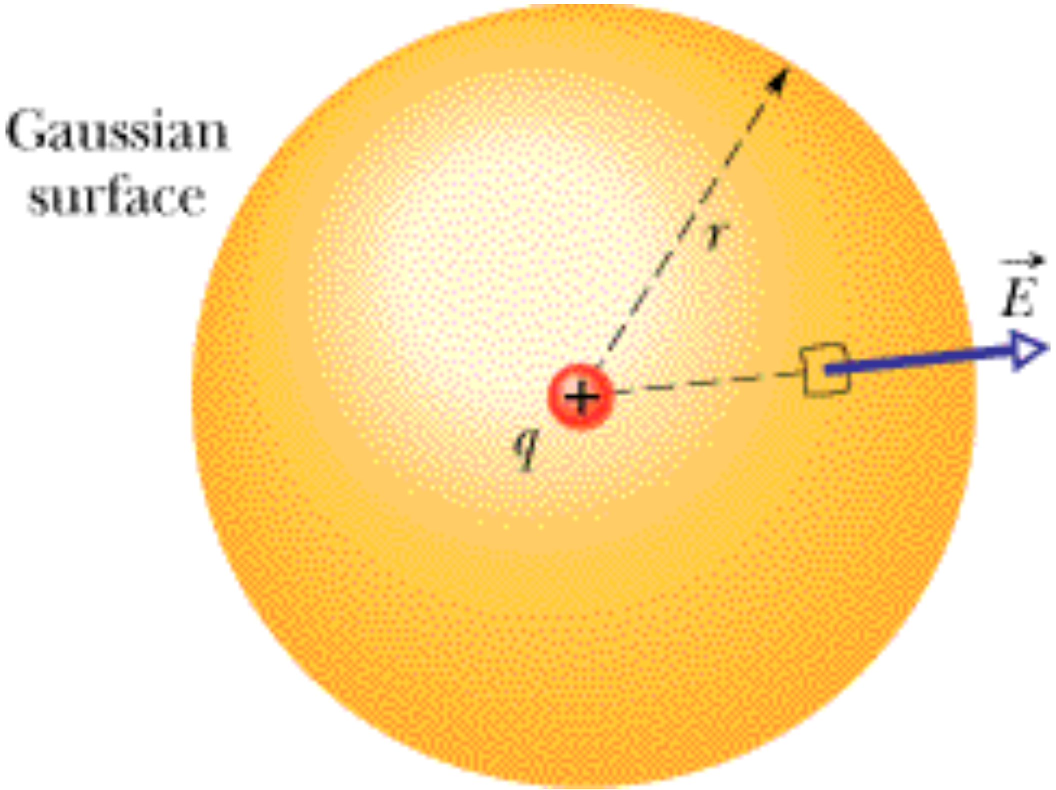
The E field at the surface is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$



E is the same over the entire area.
The area is

$$A = 4\pi r^2$$



$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

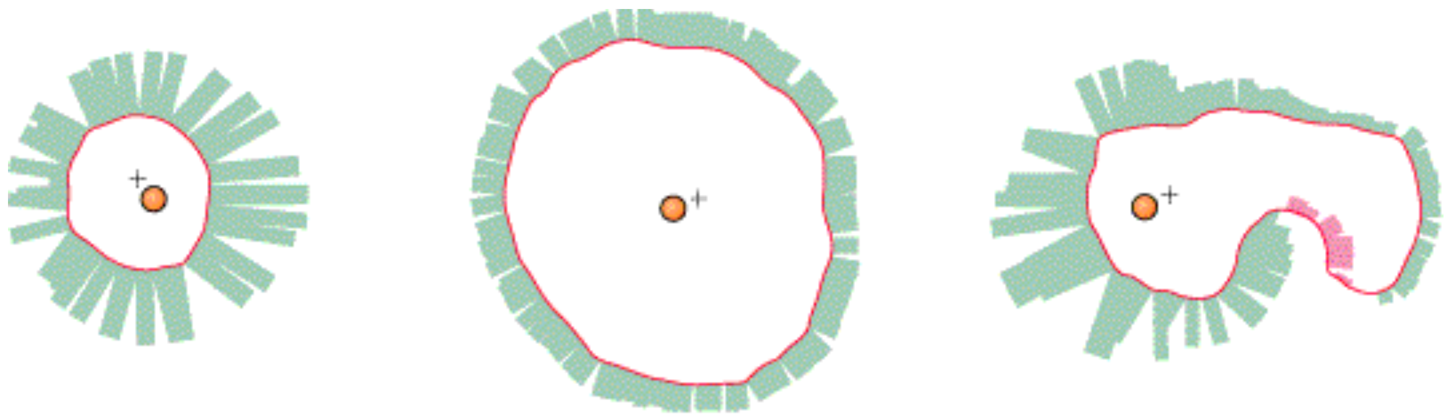
$$A = 4\pi r^2$$

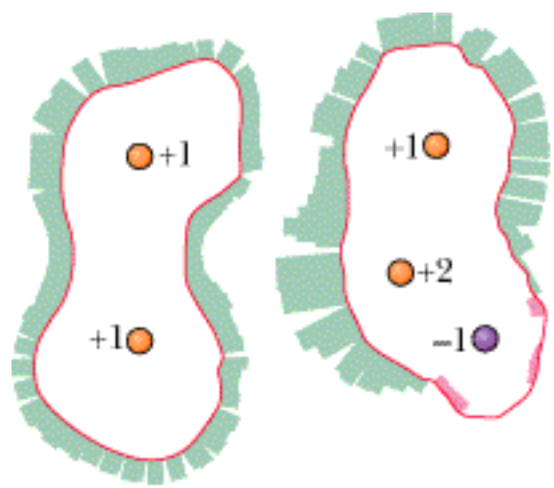
$$\Phi^{\text{net}} = EA = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)4\pi r^2$$

$$\Phi^{\text{net}} = \frac{q}{\epsilon_0}$$

Gauss proved this is true in general for all distributions of charges surrounded by any surface shape.

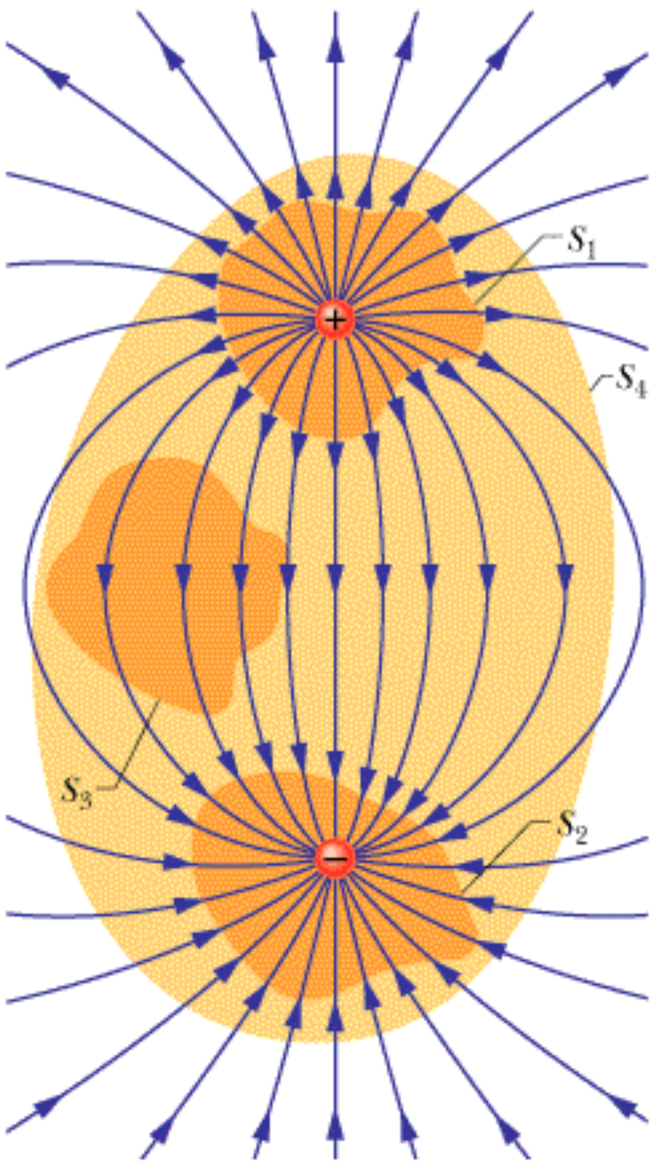
$$\Phi^{\text{net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q^{\text{encl}}}{\epsilon_0}$$





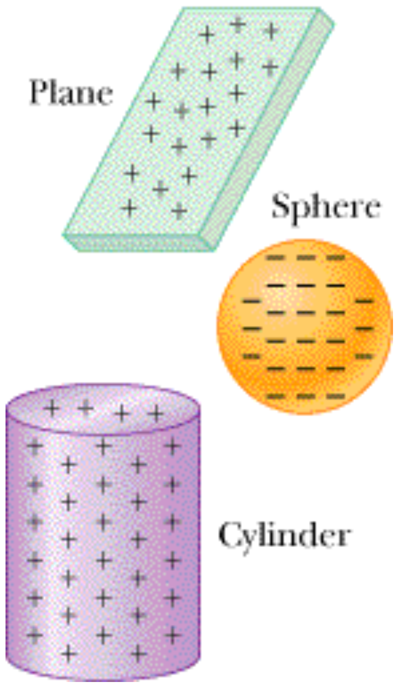
More complex distributions inside the surface.

Here the charge is outside the surface, so net flux is zero.

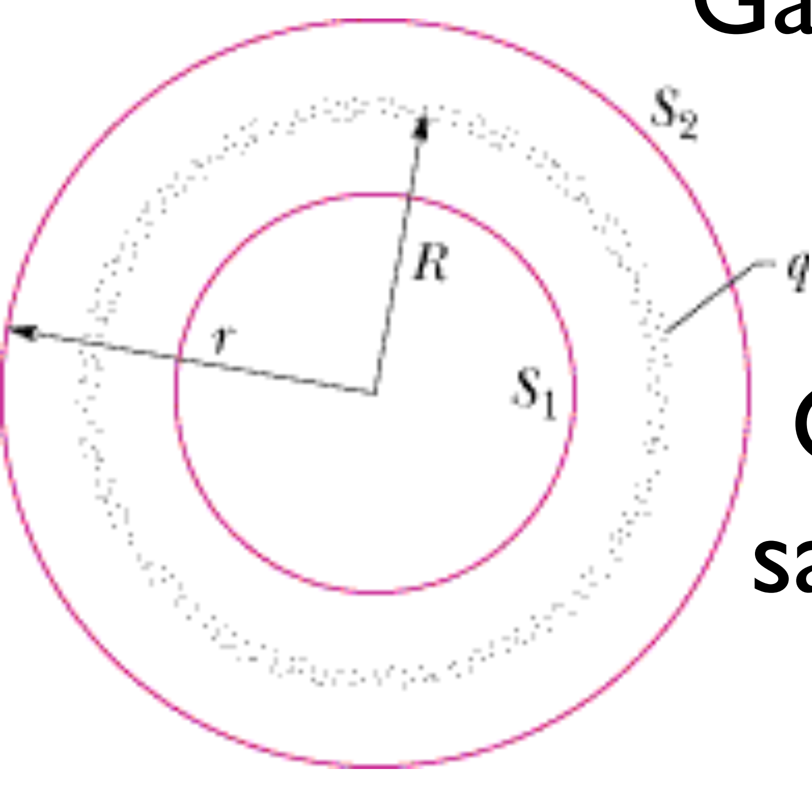


The number of electric field lines passing through a surface can serve as a proxy for the net flux.

Lines going from in to out are + flux and from out to in are – flux


Plane
Sphere
Cylinder

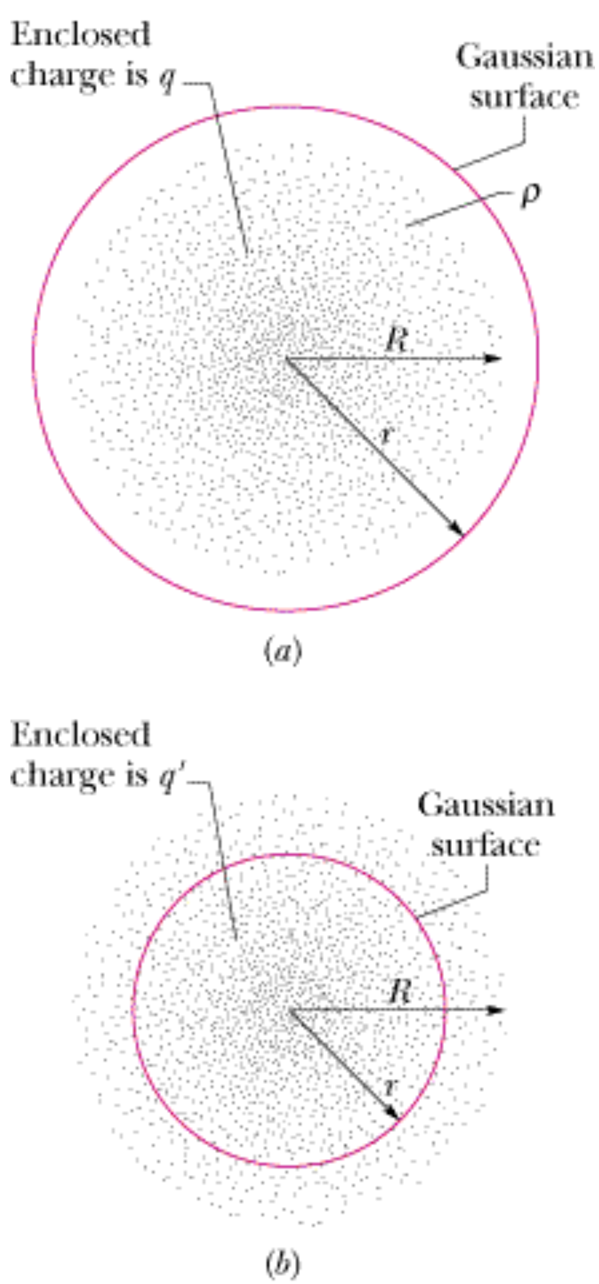
Consider a thin shell of charge.
Imagine a spherical Gaussian surface inside, S_1
Then a spherical Gaussian surface outside, S_2



Gauss' law tells us that E at S_1 is zero!
Because no charge is inside.

Gauss' law also tells us that E at S_2 is the same as for a point charge q at the centre.

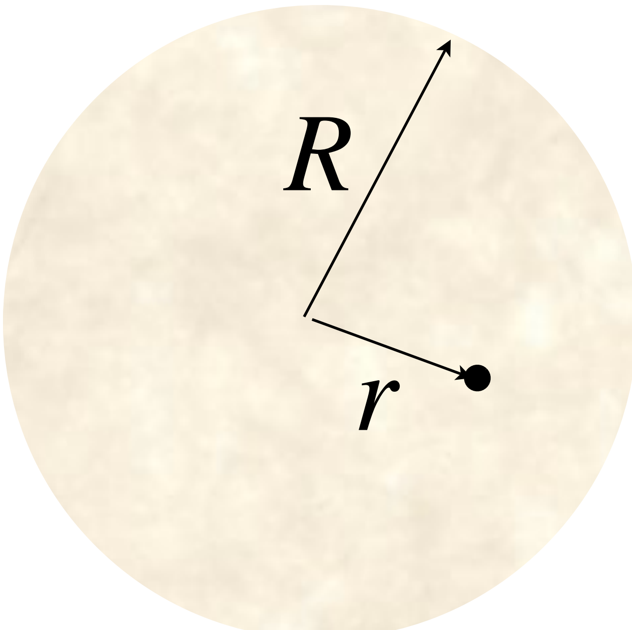
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$



In fact *E* outside any *spherical* charge distribution is that same as it would be if all the charge were concentrated at the centre.

Inside a spherical distribution, a distance *r* from the centre, the *E* field only depends on the charge within the sphere of radius *r*.

charge density: $\rho = \frac{q}{V}$



Inside a uniform sphere of charge of density ρ and radius *R* the *E* field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{R^3}r$$

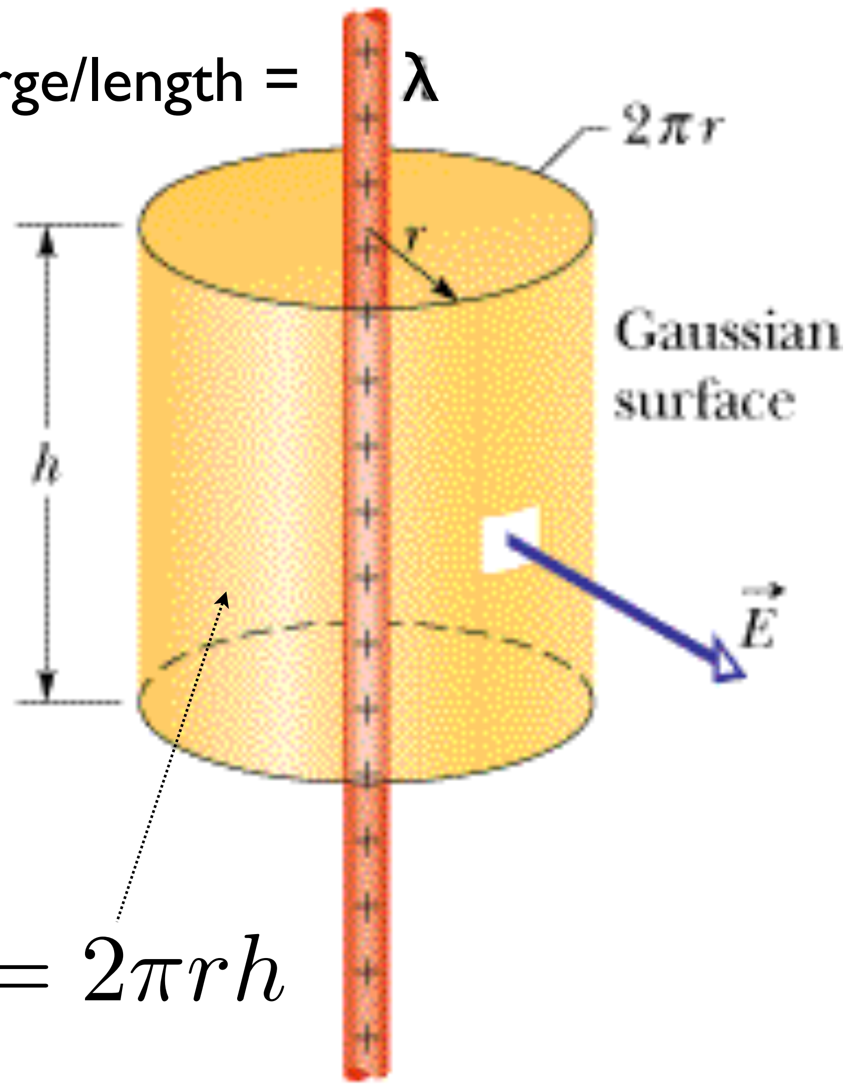
Thus of we have 1 C spread over a sphere of radius 1 m and we are 0.3 m from its centre:

$$E = (9\times10^9\ \mathrm{Nm^2/C^2})(1\mathrm{C})(0.3\mathrm{m})/(1\mathrm{m})^3$$
$$= 0.27\times10^9\ \mathrm{N/C}$$

What would be the force on 1 electron there?

What is *E* from a long line or cylinder of charge?

charge/length = λ



$A = 2\pi r h$

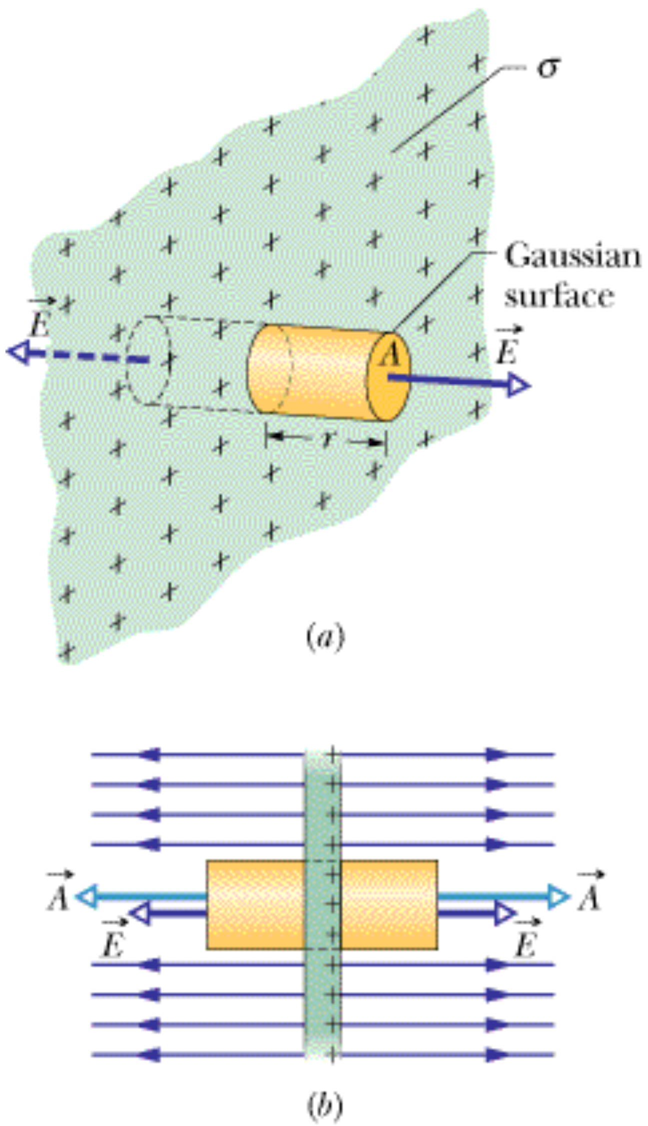
$$\Phi^{\text{net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q^{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{q^{\text{encl}}}{\epsilon_0}$$

$$E(2\pi r h) = \frac{\lambda h}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

E for a sheet of charge



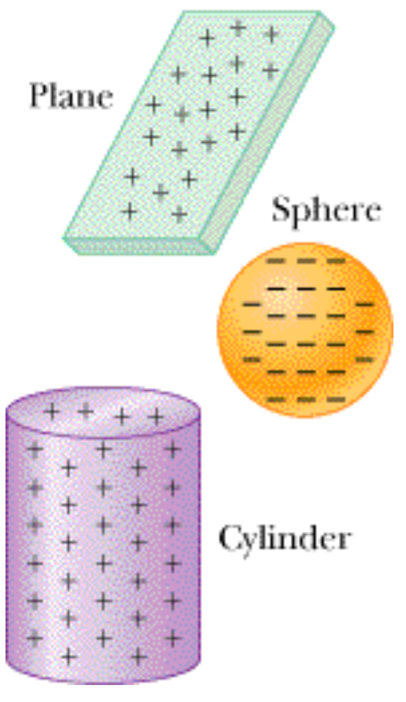
$$\Phi^{net} = 2EA = \frac{2\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

There is no *r* dependence of *E*!

Summary

$$E = \frac{\sigma}{2\epsilon_0}$$



$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

True if the plane and cylinder extend to ∞.
Approximately true for positions near the surface.

Example:

Assume an infinite sheet with charge density of 1 C/m^2.

What is the E-field at 1 m?

at 2 m?….

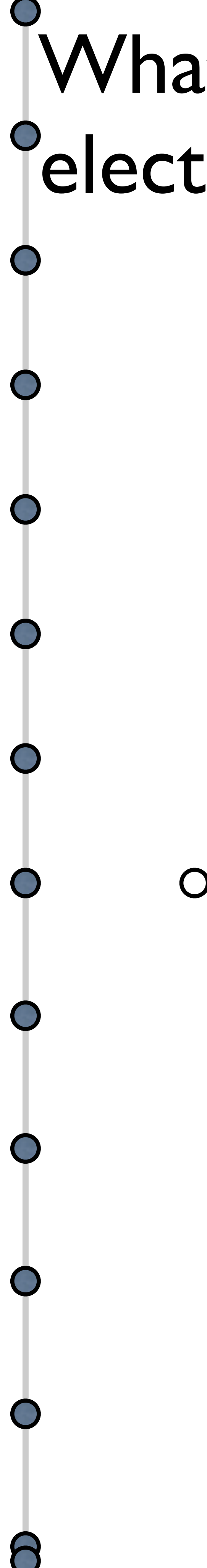
$$E = \frac{1}{2\epsilon_0} = \frac{1.13 \times 10^{11} \mathrm{N/C}}{2}$$

$$E = 0.565 \text{ x } 10^{11} \text{ N/C}$$

What would be the E for an infinite sheet charged with one electron/mm^2?

about –0.01 N/C

What is *E* for a line of charge with 1 electron/mm at 1 mm from the line?



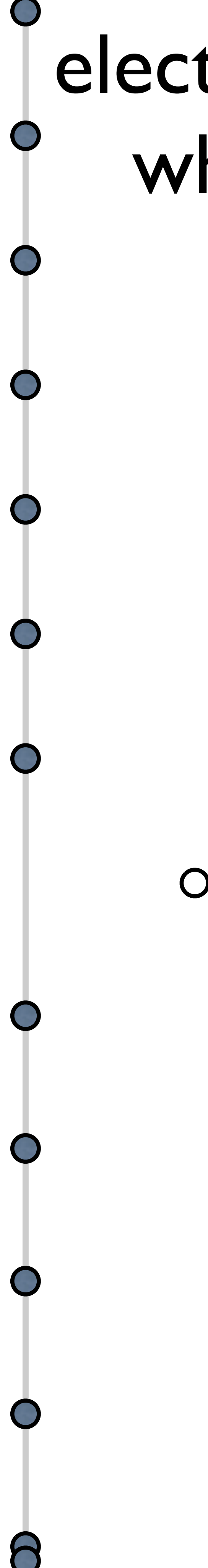
–0.0028 N/C

What is *E* at a position 1 mm from an electron?

–0.0014 N/C

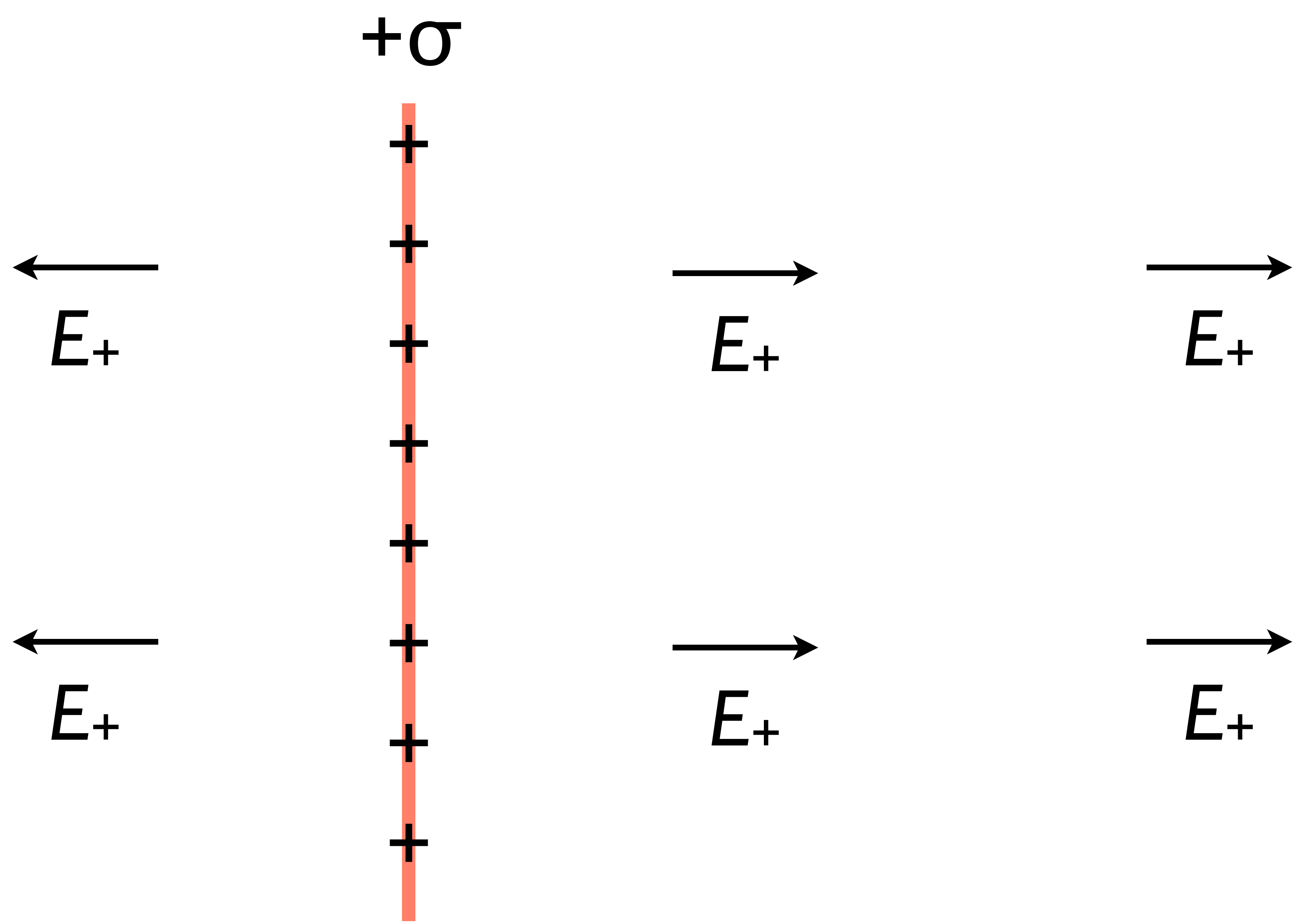
That’s exactly ½ the previous answer.

electron/mm at 1 mm from the line when we take away the closest electron?

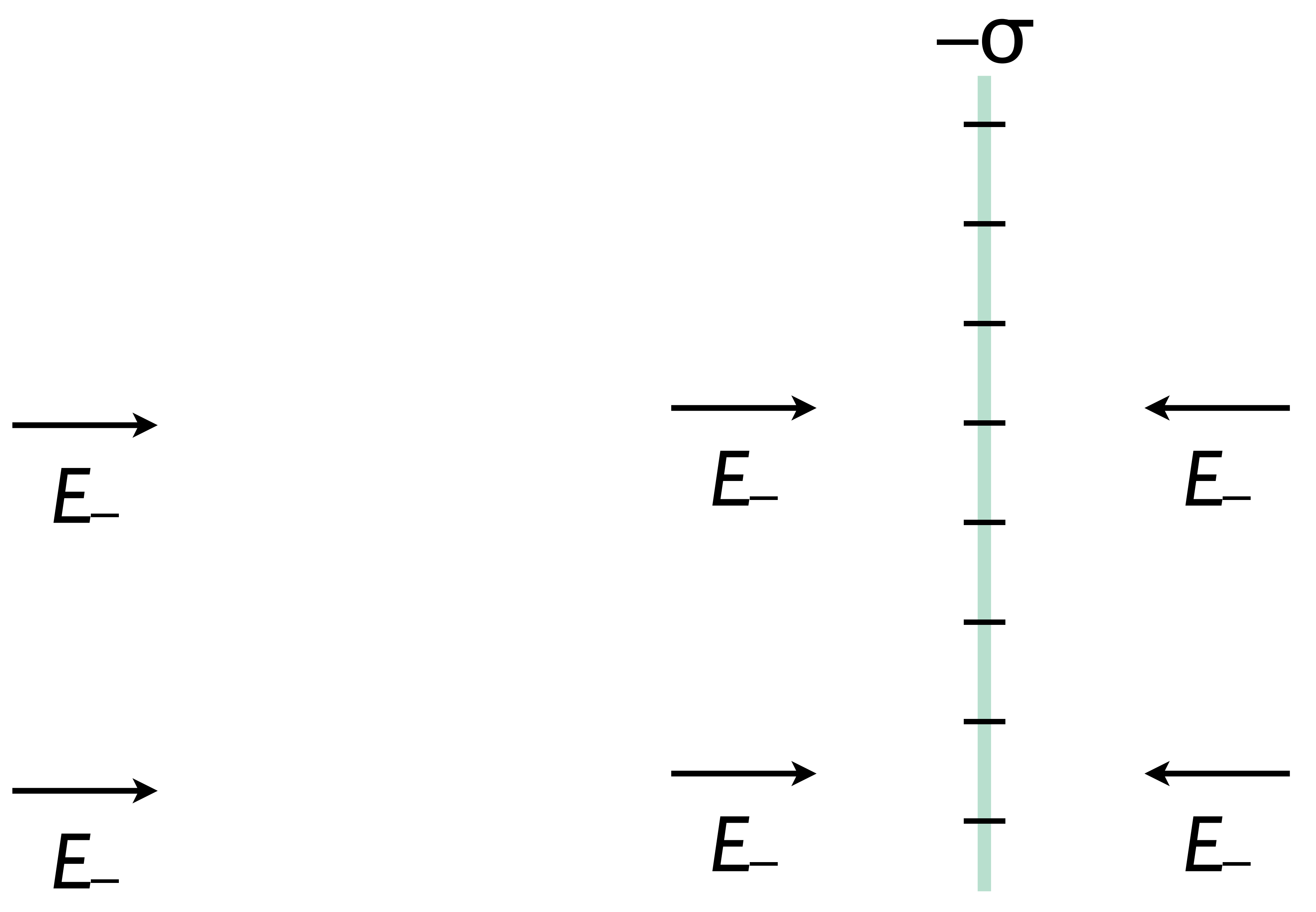


–0.0014 N/C

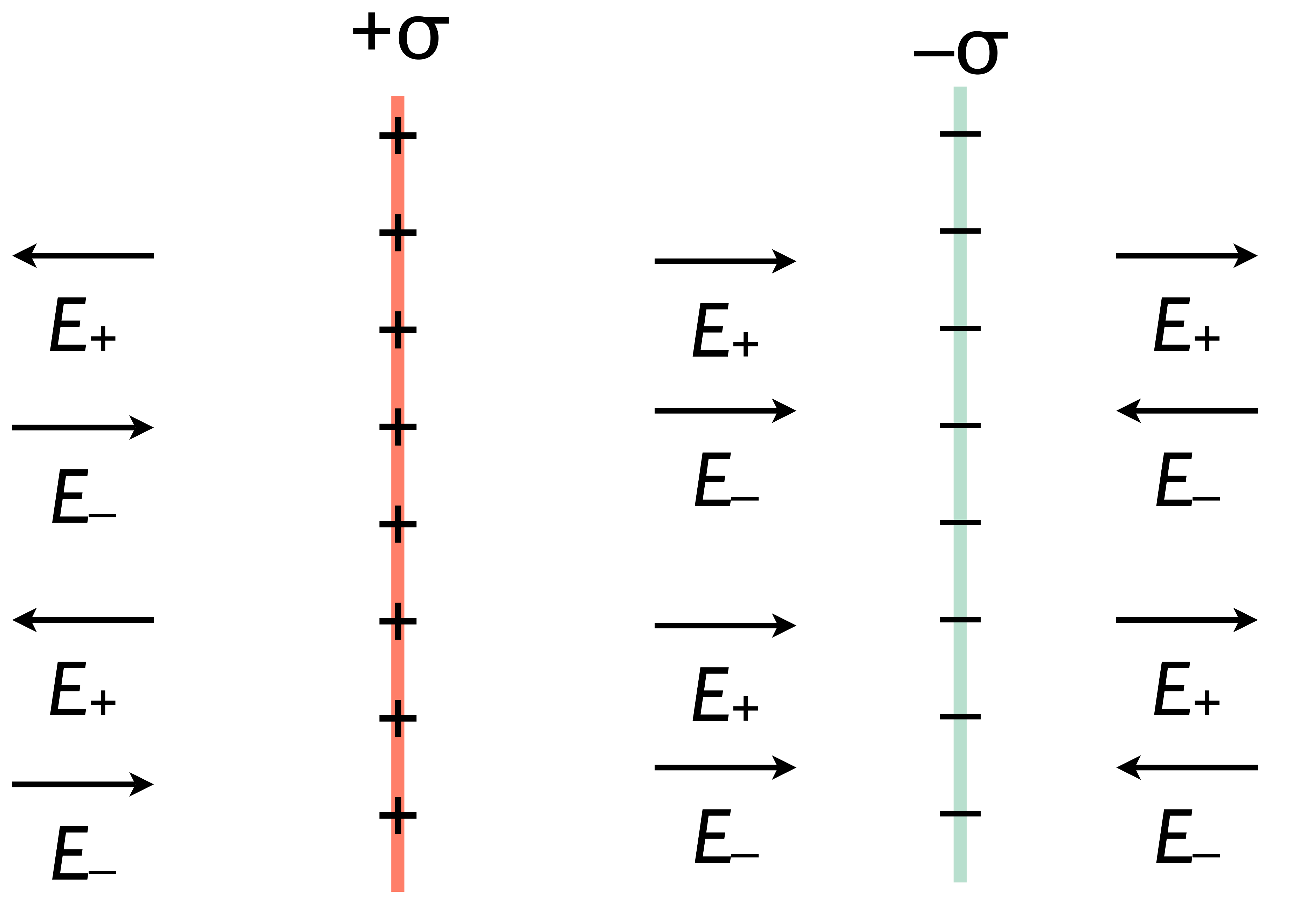
Cross section of an infinite sheet of charge



$E_+ = \sigma/2\varepsilon_0$ in magnitude



$E_{-} = \sigma/2\varepsilon_0$ in magnitude



$E_{net} = 0$ $E_{net} = \sigma/\varepsilon_0$ $E_{net} = 0$