Electricity & Magnetism
Lecture 8: Capacitors

Today's Concept:

Capacitors

(Capacitors in a circuits, Dielectrics, Energy in capacitors)

Alternate terms: "condensors", "capacitators", ...

Stuff you asked about:

- "the formula for calculating capacitance in parallel and series are the opposite of the formula for resistance "
- "Please go over the Capactitors with and without a dielectric from checkpoint."
- "this stuff is a lot harder than what we've been doing before, one thing i don't understand is when you disconnect a battery from a capacitor what exactly happens, how come it still has a charge?? "

Capacitors, connected and unconnected

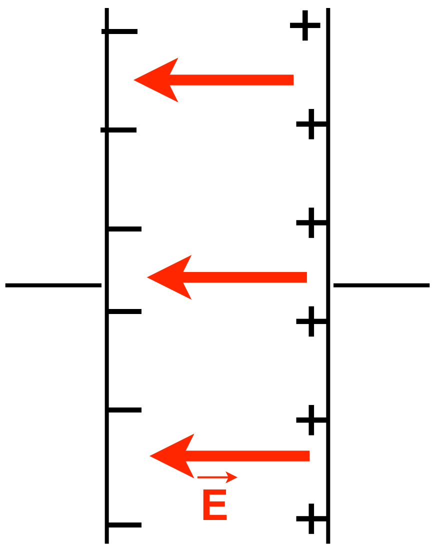
- A capacitor that is unconnected to a battery has constant charge: $V = Q/C$ (V is determined by Q)
- Capacitors in parallel have the same voltage. Charge may redistribute among them: $Q_1/C_1 = Q_2/C_2 = Q_2/C_2$ ...
- A capacitor connected to a battery has a constant voltage. $Q = CV$ (Q is determined by V)

Dielectric

- Charged capacitor, not connected to battery
- Dielectric makes ΔV smaller

air capacitor

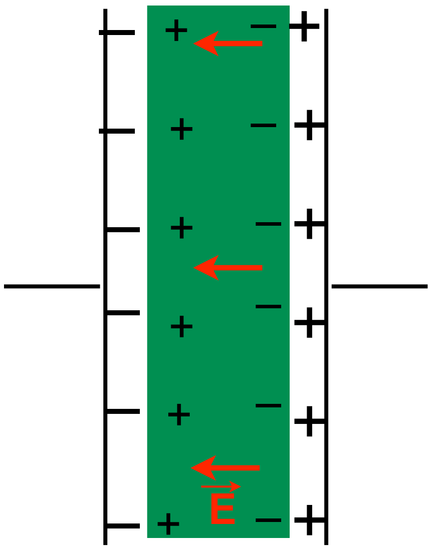
$\kappa = 1$



$C = \varepsilon_0 A/L$

dielectric capacitor

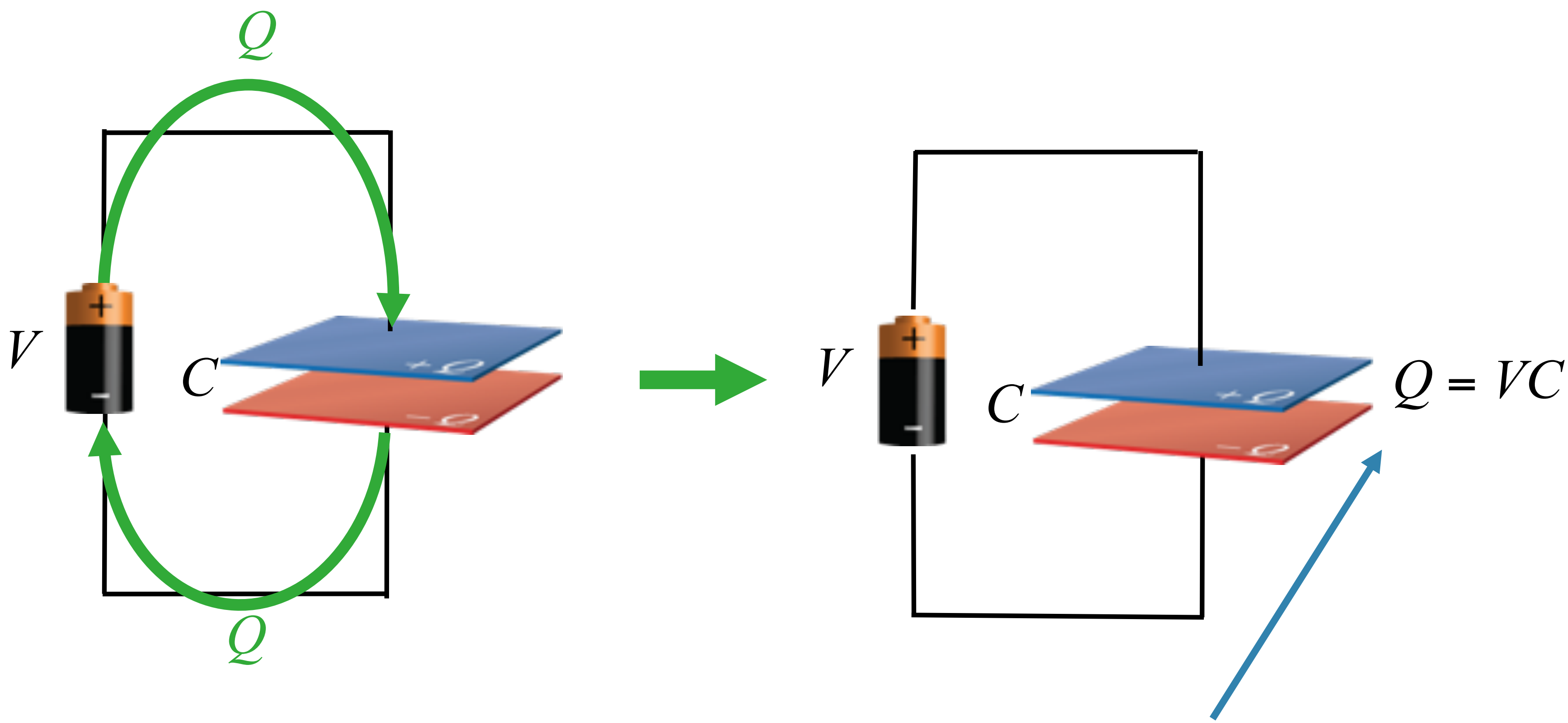
$\kappa > 1$



$C = \kappa\varepsilon_0 A/L$

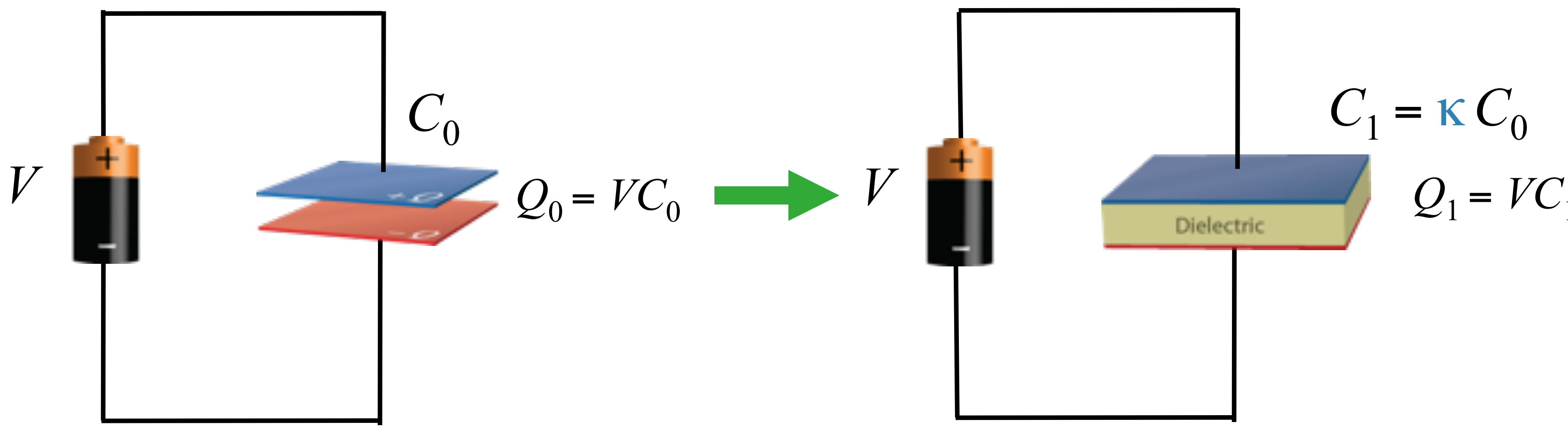
E inside dielectric is smaller than E outside.
The charge on the surface of the dielectric partially cancels the E field from the charge on the plates.
Therefore the ΔV between the plates is less.

Simple Capacitor Circuit



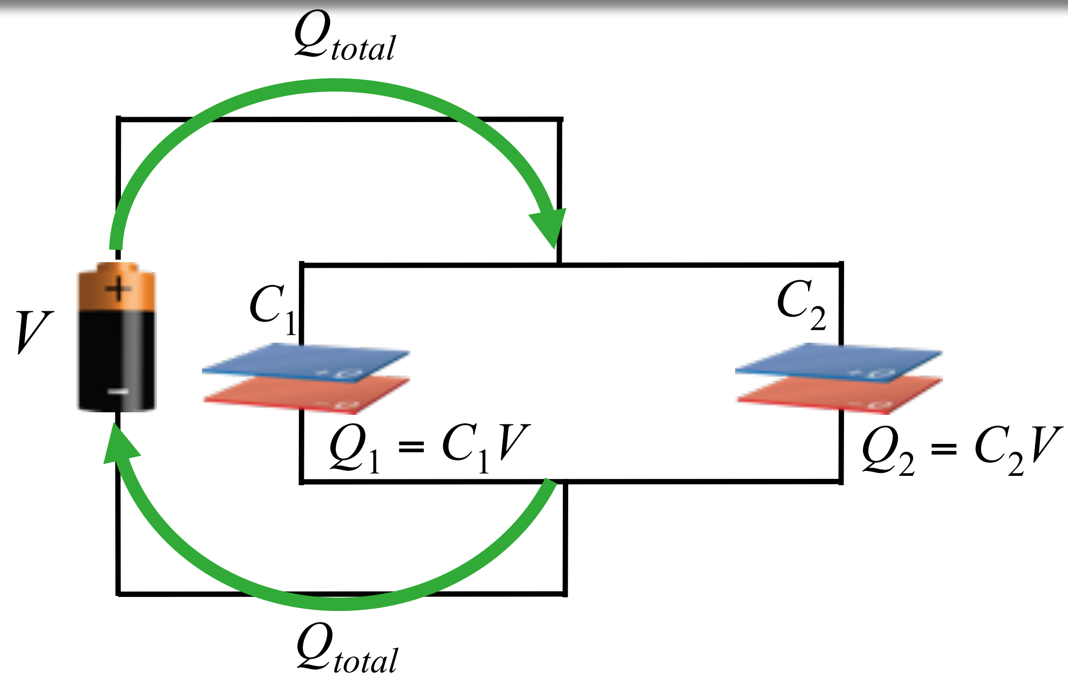
This “Q” really means that the battery has moved charge Q from one plate to the other, so that one plate holds $+Q$ and the other $-Q$.

Dielectrics



By adding a dielectric you are just making a new capacitor with larger capacitance (factor of κ)

Parallel Capacitor Circuit

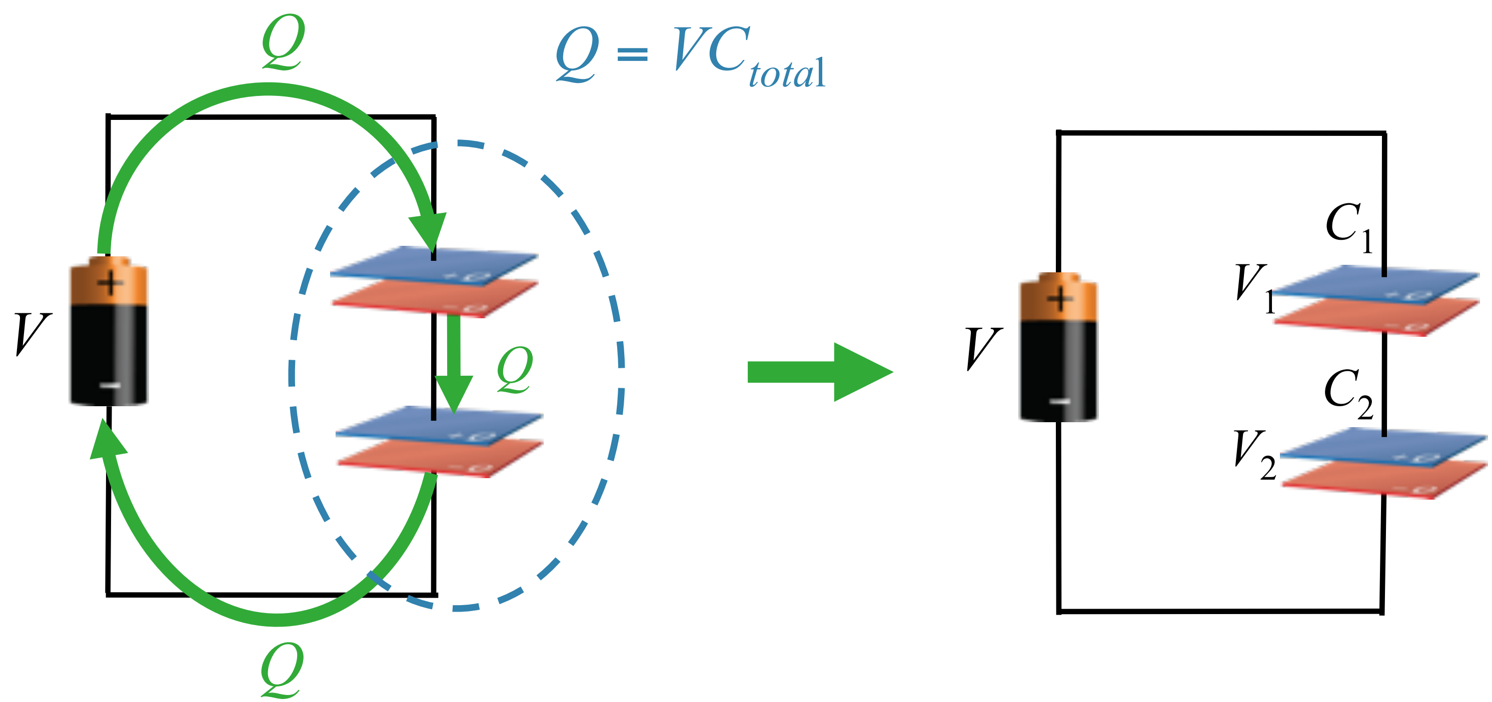


Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V(C_1 + C_2)$

$C_{total} = C_1 + C_2$

Series Capacitor Circuit



Key point: Q is the same for both capacitors

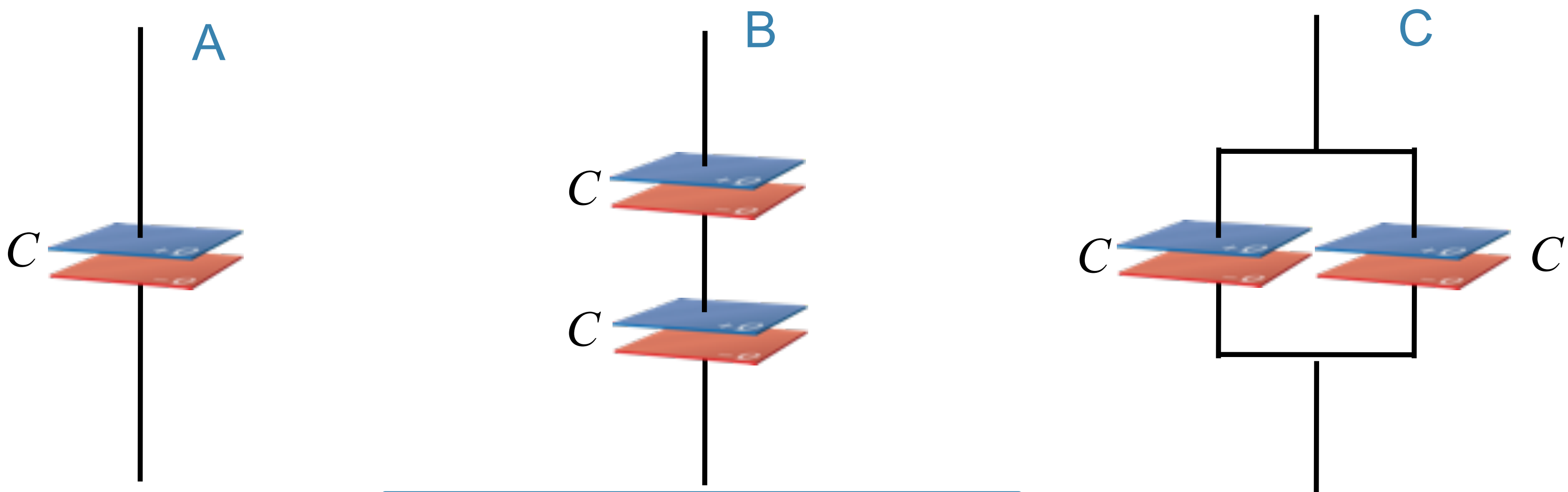
Key point: $Q = VC_{total} = V_1C_1 = V_2C_2$

Also: $V = V_1 + V_2$ → $Q/C_{total} = Q/C_1 + Q/C_2$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

CheckPoint: Three Capacitor Configurations

The three configurations shown below are constructed using identical capacitors. Which of these configurations has lowest total capacitance?



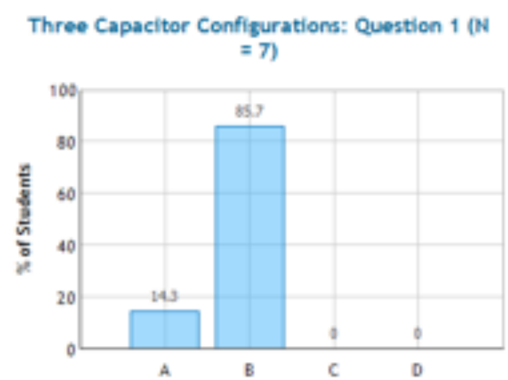
$$C_{total} = C$$

$$1/C_{total} = 1/C + 1/C$$
$$= 2/C$$

$$C_{total} = C/2$$

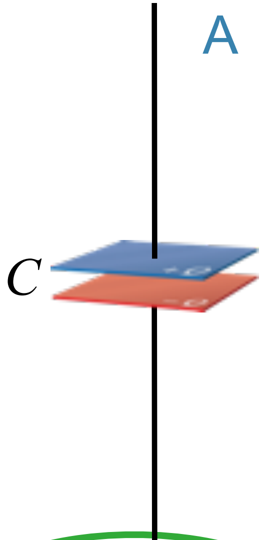
$$C_{total} = 2C$$

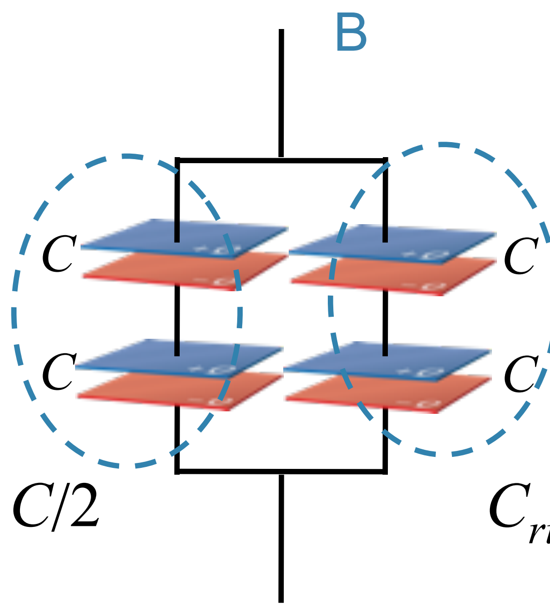
D: All 3 are the same



CheckPoint: Two Capacitor Configurations

The two configurations shown below are constructed using identical capacitors. Which of these configurations has the lowest overall capacitance?



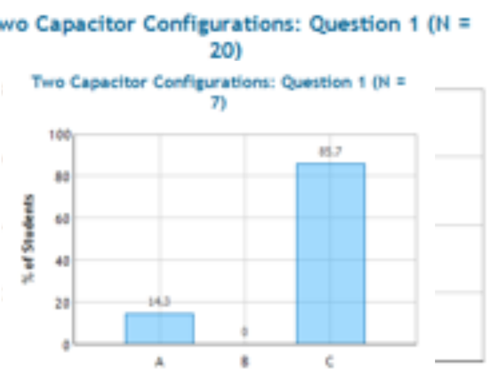


$C_{left} = C/2$ $\qquad$ $C_{right} = C/2$

$C_{total} = C$

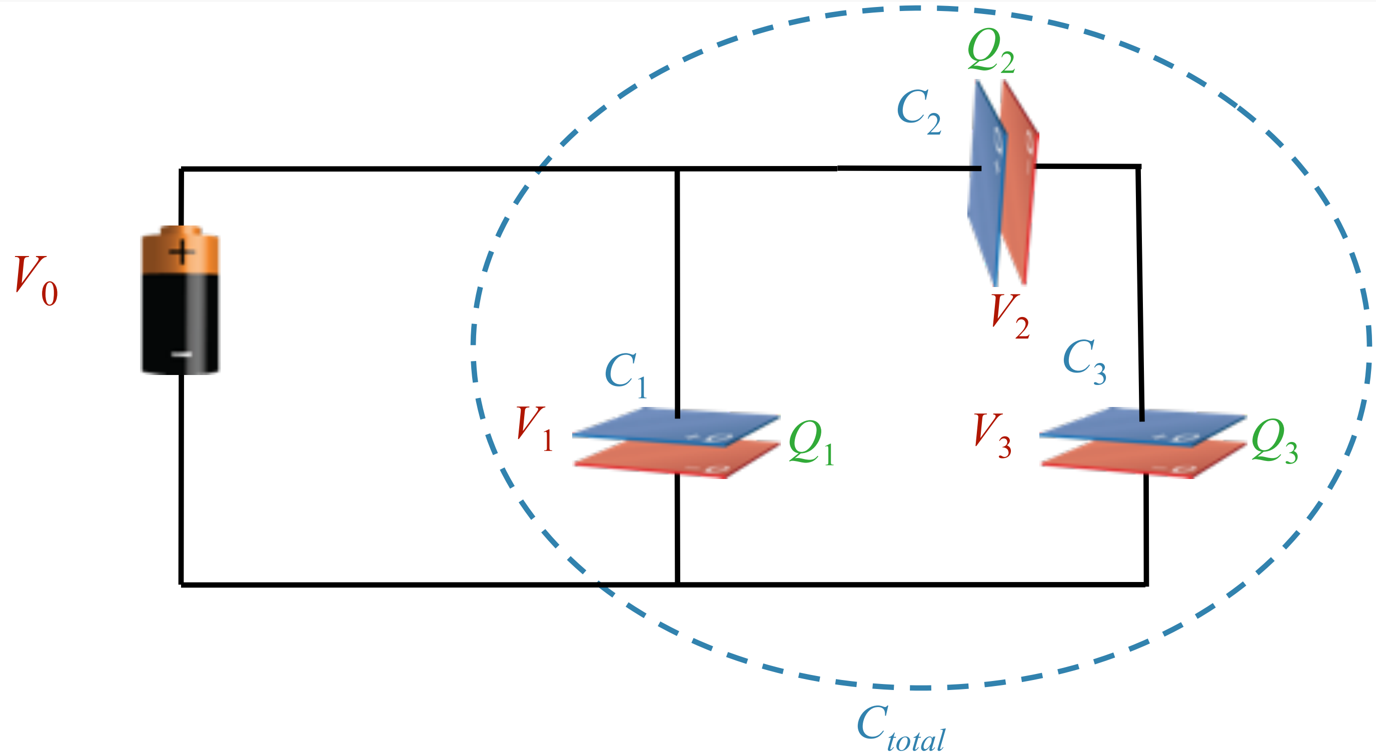
$C_{total} = C_{left} + C_{right}$

$C_{total} = C$



- ○ A
- ○ B
- ○ Both configurations have the same capacitance

Clicker Question: Capacitor Network



Which of the following is NOT necessarily true:

A) $V_0 = V_1$

B) $C_{total} > C_1$

C) $V_2 = V_3$

D) $Q_2 = Q_3$

E) $V_1 = V_2 + V_3$

CheckPoint: Capacitor Network

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of voltage V_0. The capacitance of C_2 is twice that of C_1. The capacitance of C_3 is three times that of C_1. The capacitors obtain charges Q_1, Q_2, and Q_3.

Compare Q_1, Q_2, and Q_3.

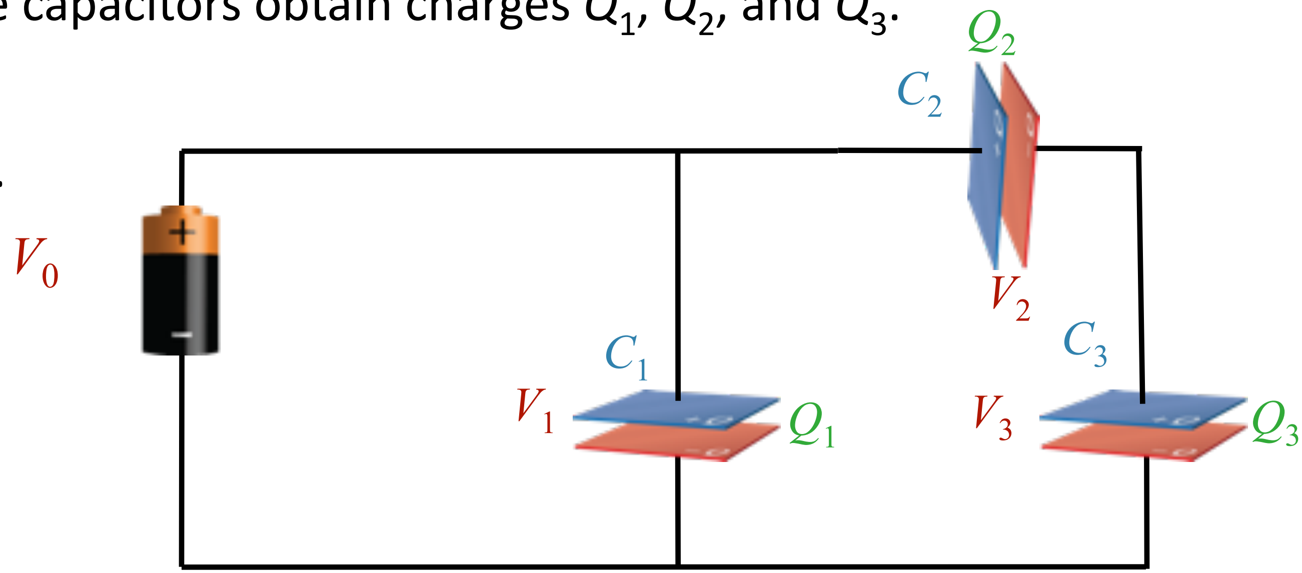
A. $Q_1 > Q_3 > Q_2$

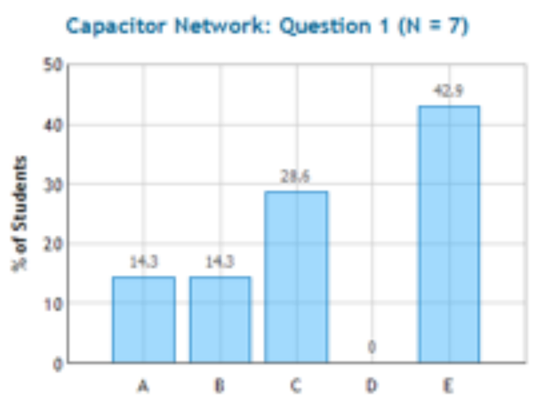
B. $Q_1 > Q_2 > Q_3$

C. $Q_1 > Q_2 = Q_3$

D. $Q_1 = Q_2 = Q_3$

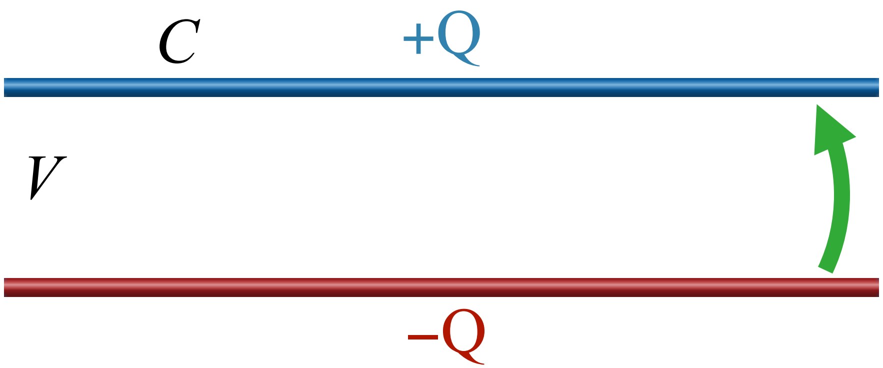
E. $Q_1 < Q_2 = Q_3$





Energy in a Capacitor

In Prelecture 7 we calculated the work done to move charge Q from one plate to another:



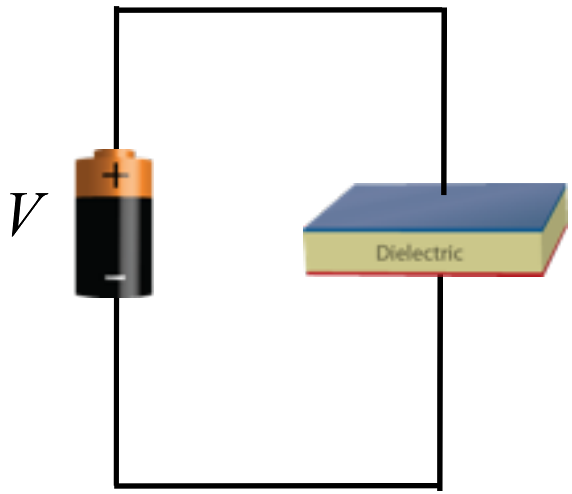
$$U = {}^{1}\!/_{2}QV$$
$$= {}^{1}\!/_{2}CV^2$$
$$= {}^{1}\!/_{2}Q^2/C$$

Since $Q = VC$

This is potential energy waiting to be used…

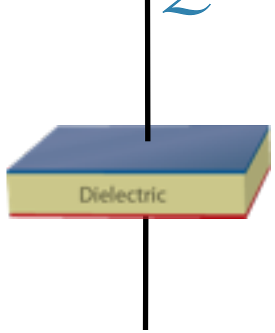
Messing with Capacitors

If connected to a battery V stays constant



$$V_1 = V$$
$$C_1 = \kappa C$$
$$\rightarrow Q_1 = C_1 V_1 = \kappa C V = \kappa Q$$

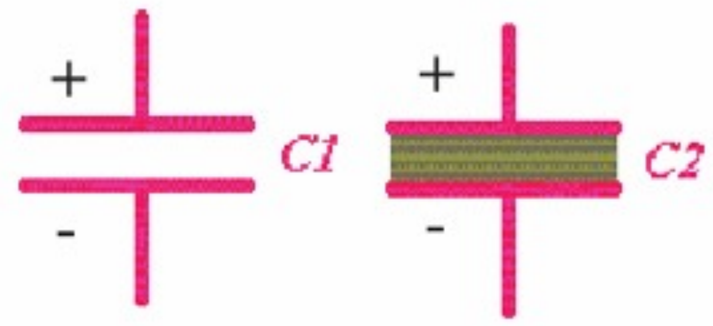
If isolated then total Q stays constant



$$Q_1 = Q$$
$$C_1 = \kappa C$$
$$\rightarrow V_1 = Q_1 / C_1 = Q / \kappa C = V / \kappa$$

CheckPoint: Capacitors and Dielectrics 1

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric.



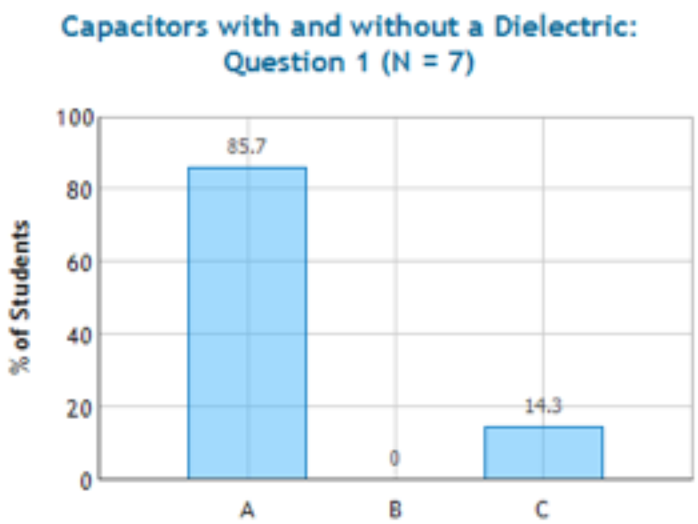
Compare the voltages of the two capacitors.

A. $V_1 > V_2$
B. $V_1 = V_2$
C. $V_1 < V_2$

"The electric field decreases on the second due to the dielectric, so its voltage must decrease."
"The little brown thing is stopping the pikachus from reaching the other side."

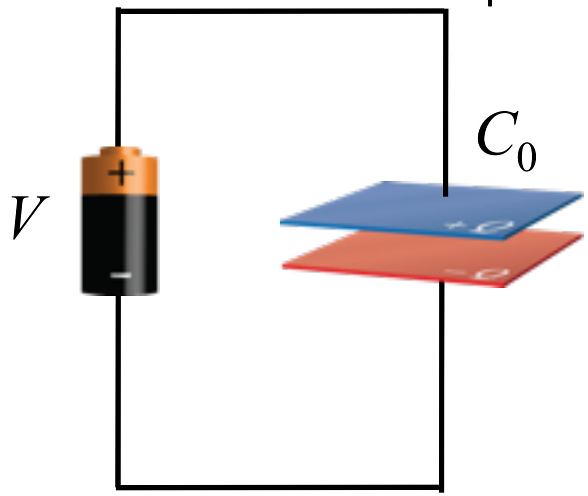
" Dielectrics don't change voltages. "

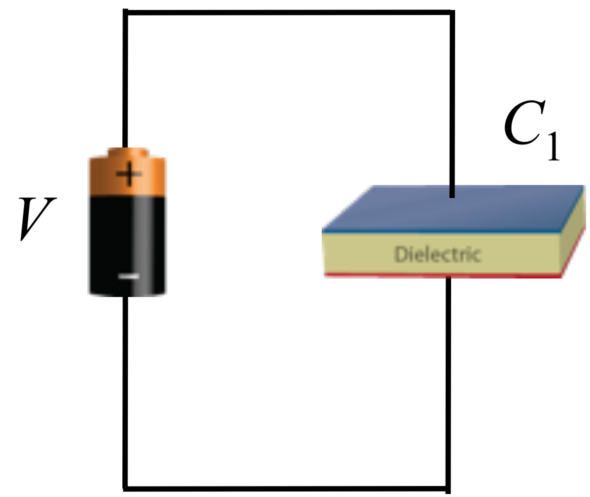
"Capacitance lowers in case 2, meaning V increases in case 2. "



Clicker Question: Messing with Capacitors

Two identical parallel plate capacitors are connected to identical batteries. Then a dielectric is inserted between the plates of capacitor C_1. Compare the energy stored in the two capacitors.





A) $U_1 < U_0$

B) $U_0 = U_1$

C) $U_1 > U_0$

Compare using $U = {}^1/_2 CV^2$

$$U_1/U_0 = \kappa$$

→ Potential Energy goes UP

CheckPoint: Capacitors and Dielectrics 2

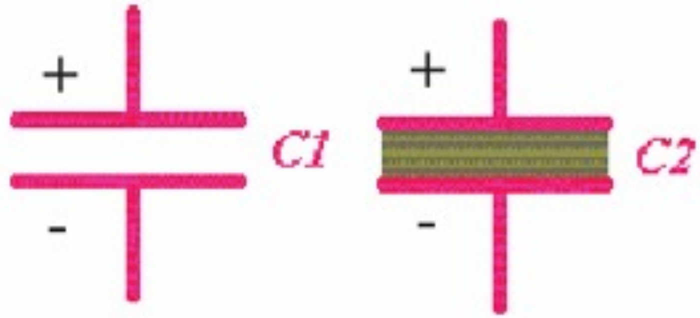
Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric.

Compare the potential energy stored by the two capacitors.

A. $U_1 > U_2$

B. $U_1 = U_2$

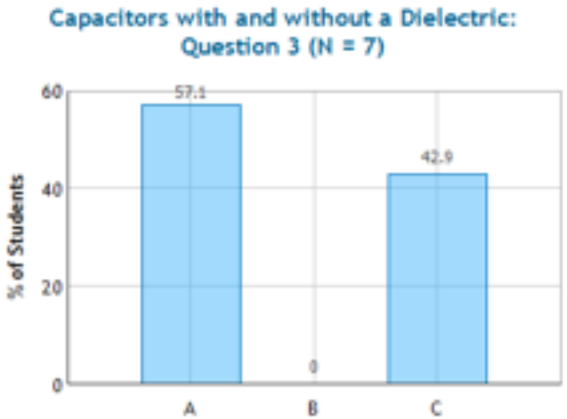
C. $U_1 < U_2$



" If the electric field goes down, then so does the potential energy of the second system.."

"Dielectrics don't affect potential."

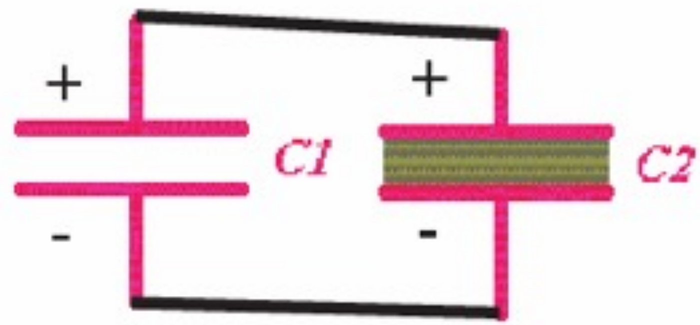
"U = QV. Since voltage remains constant and charge increases, potential energy must also increase."



CheckPoint: Capacitors and Dielectrics 3

The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?

V must be the same !

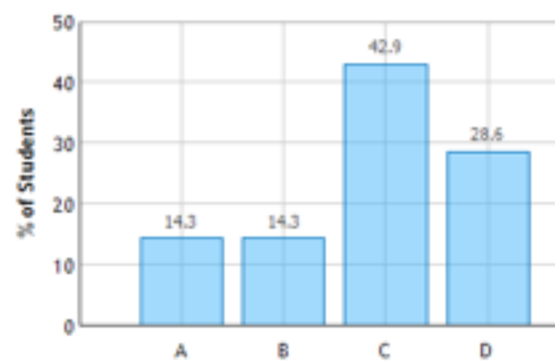


- The charges will flow so that the charge on C1 will become equal to the charge on C2.
- The charges will flow so that the energy stored in C1 will become equal to the energy stored in C2
- The charges will flow so that the potential difference across C1 will become the same as the potential difference across C2.
- No charges will flow. The charge on the capacitors will remain what it was before they were connected

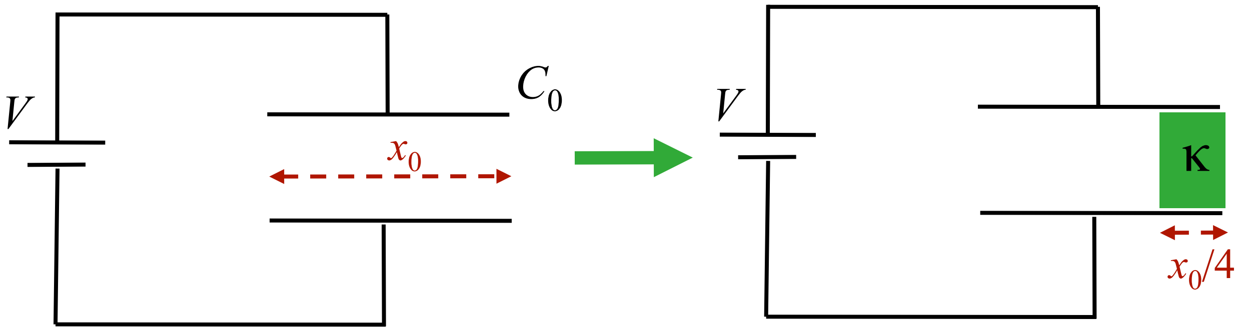
Q: $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow Q_1 = \frac{C_1}{C_2} Q_2$$

U: $$U_1 = \tfrac{1}{2} C_1 V^2 \qquad U_2 = \tfrac{1}{2} C_2 V^2 \rightarrow U_1 = \frac{C_1}{C_2} U_2$$

Capacitors with and without a Dielectric: Question 5 (N = 7)



Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

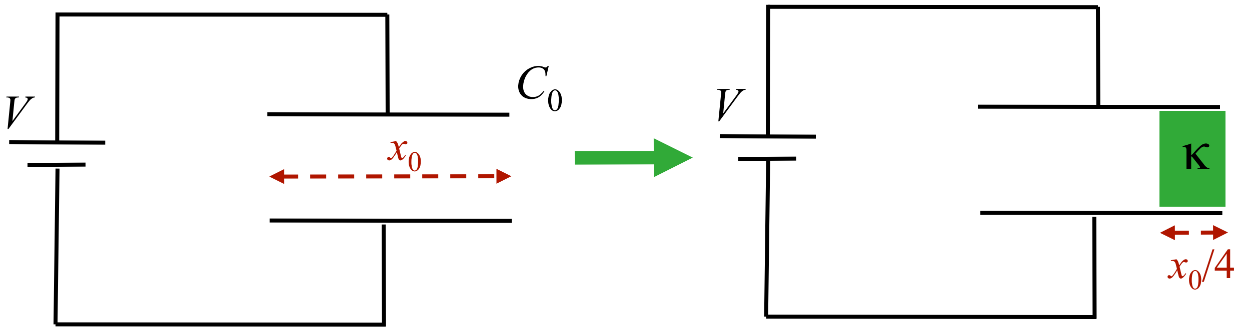
What changes when the dielectric added?

A) Only C B) only Q C) only V D) C and Q E) V and Q

Adding dielectric changes the physical capacitor → C changes

V does not change and C changes → Q changes

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

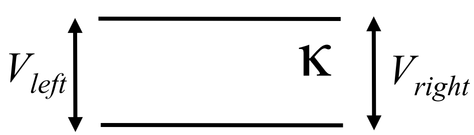
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

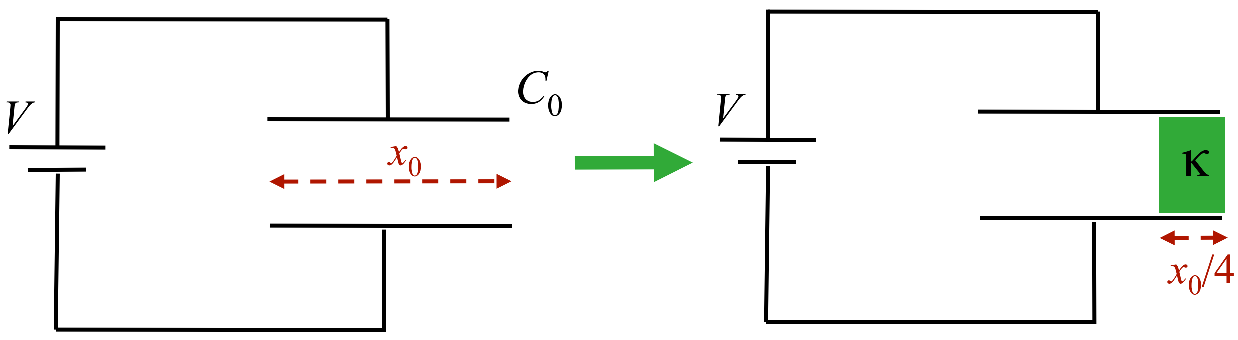
To calculate C, let's first look at:



A) $V_{left} < V_{right}$ B) $V_{left} = V_{right}$ C) $V_{left} > V_{right}$

The conducting plate is an equipotential !

Calculation

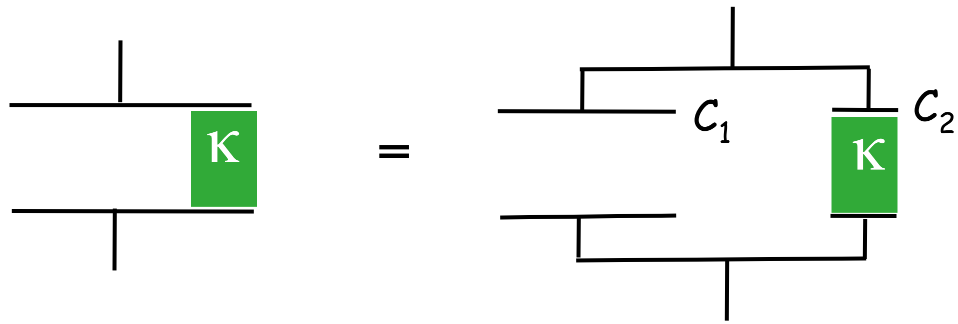


An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



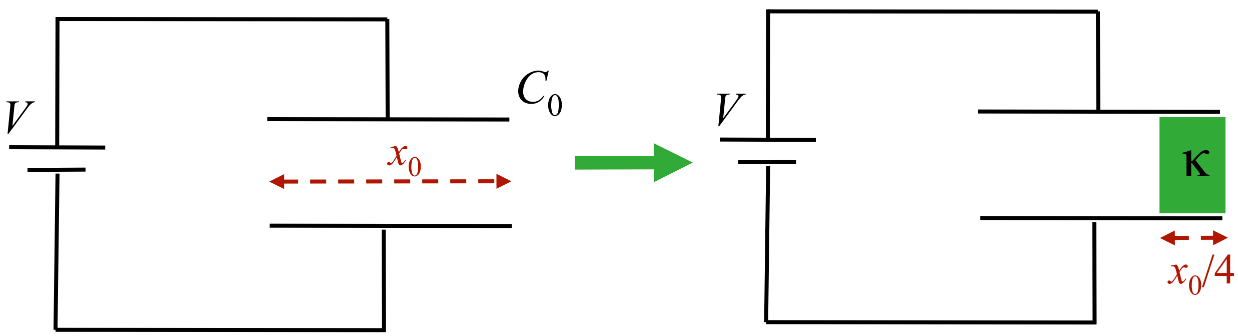
What is C_1 ?

A) $C_1 = C_0$ B) $C_1 = {}^3/_4 C_0$ C) $C_1 = {}^4/_3 C_0$ D) $C_1 = {}^1/_4 C_0$

In general. For parallel plate capacitor: $C = \varepsilon_0 A/d$

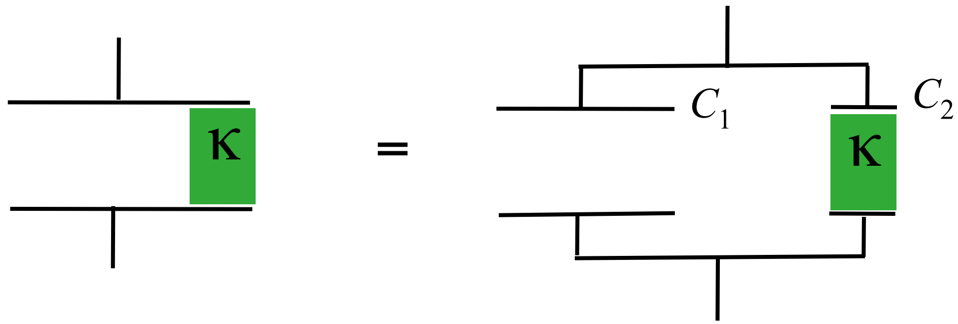
$$A = {}^3/_4 A_0, \quad d = d_0 \;\rightarrow\; C_1 = {}^3/_4 (\varepsilon_0 A_0/d_0) \;\rightarrow\; C_1 = {}^3/_4 C_0$$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.



What is Q_f, the final charge on the capacitor?

$C_1 = {}^3/_4 C_0$

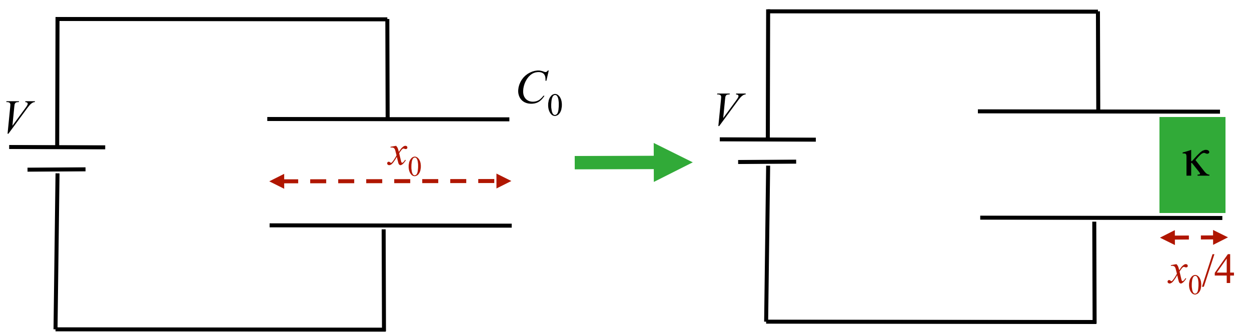
What is C_2 ?

A) $C_2 = \kappa C_0$ B) $C_2 = {}^3/_4\, \kappa C_0$ C) $C_2 = {}^4/_3\, \kappa C_0$ **D) $C_2 = {}^1/_4\, \kappa C_0$**

In general. For parallel plate capacitor filled with dielectric: $C = \kappa\varepsilon_0 A/d$

$A = {}^1/_4 A_0$, $d = d_0$ → $C = ¼(\kappa\varepsilon_0 A_0/d_0)$ → $C_2 = {}^1/_4\, \kappa C_0$

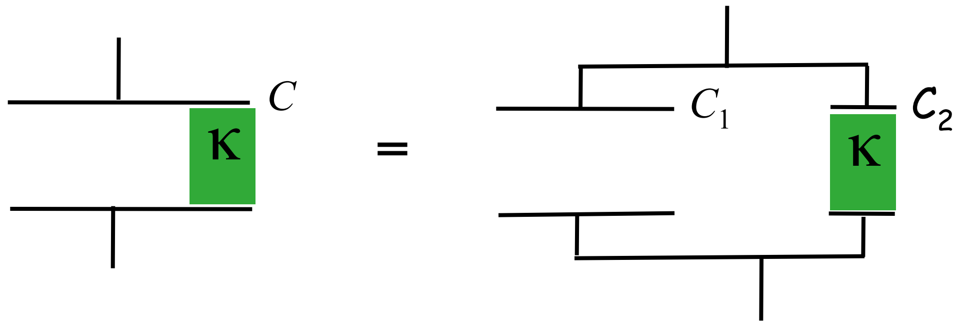
Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?



$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

What is C?

A) $C = C_1 + C_2$

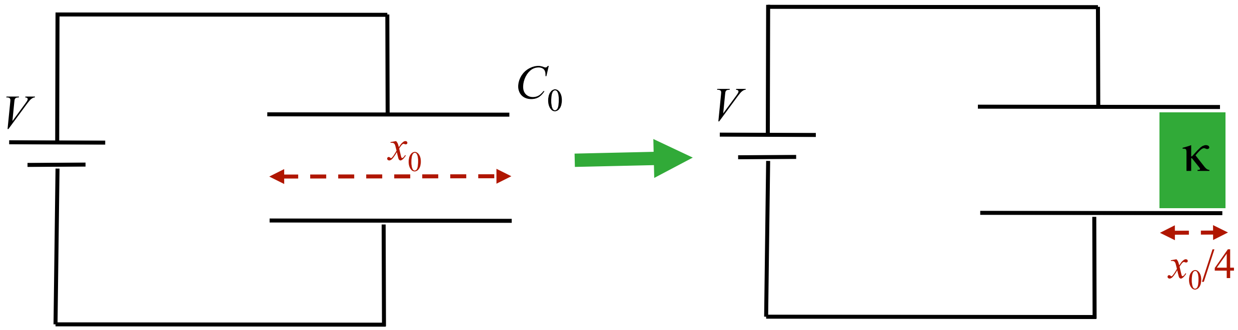
B) $C = C_1 + \kappa C_2$

C) $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1}$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

$$\rightarrow \quad C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$$

Calculation



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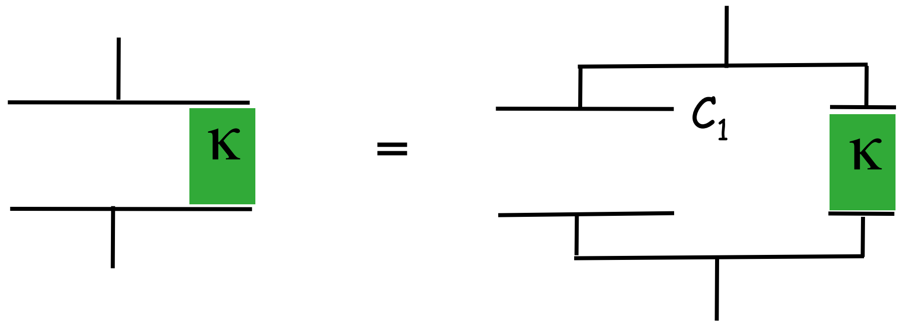


What is Q_f, the final charge on the capacitor?

$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

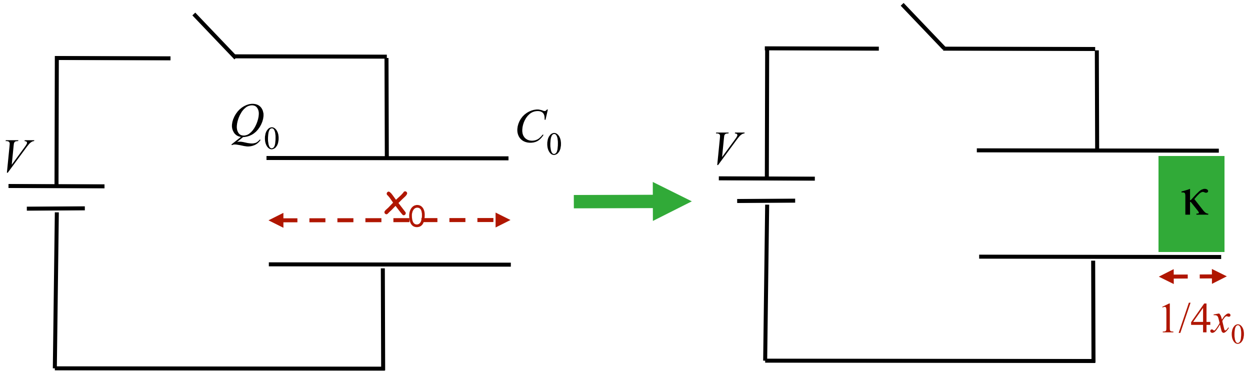
$$\rightarrow \quad C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$$

What is Q?

$$C \equiv \frac{Q}{V} \quad \rightarrow \quad Q = VC$$

$$Q = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$

Different Problem



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V and then battery is disconnected.

A dielectric (κ) of width $1/4x_0$ is inserted into the gap as shown.

What is V_f, the final voltage on the capacitor?

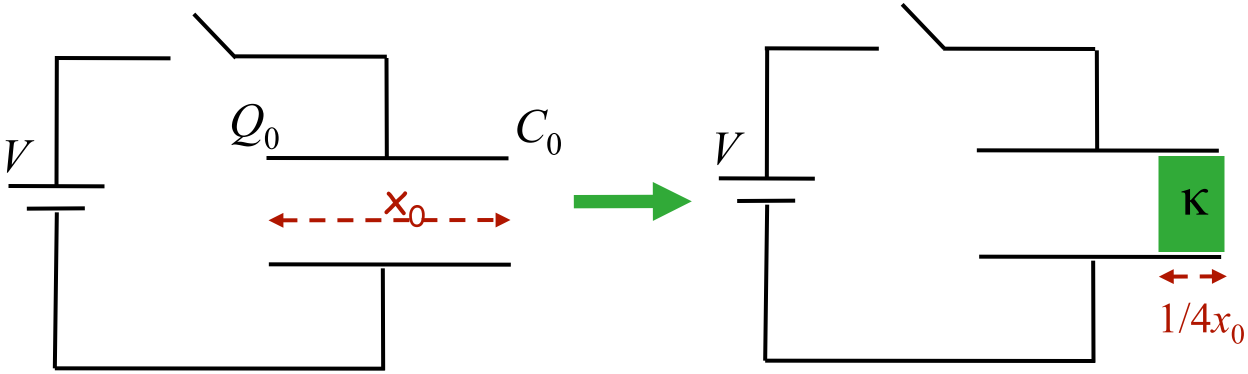
A) $V_f < V$ B) $V_f = V$ C) $V_f > V$

Q stays same: no way to add or subtract → $Q = Q_0 = C_0 V$

We know C: (property of capacitor) → $C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$

→ $V_f = Q/C = V/({}^3/_4 + {}^1/_4\,\kappa)$

Different Problem



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V and then battery is disconnected.

A dielectric (κ) of width $1/4x_0$ is inserted into the gap as shown.

What is V_f, the final voltage on the capacitor?

$$V_f = Q/C = V/({}^3/_4 + {}^1/_4\,\kappa)$$

How did energy stored in capacitor change when dielectric inserted?

A) U increased B) U stayed same C) U decreased

$U = ½\, Q^2/C$
Q remained same
C increased

→ U decreased