Muon Experiment Physics 431 (Jan 2007)

Suggested procedure:

Follow the suggested outline in the Teachspin manual "student exercises". There are some errors in the TEachspin discussion of atmospheric physics. Make sure your read the corrected analysis file (muon_corrected_annalysis.pdf)
Start by taking an overnight scan of ~24 hours

Do a preliminary analysis. See whether the data are sufficient to achieve the objectives of the write-up (polarization ratios, etc)

Perform at least one long scan spanning several days. While you are accumulating this data, continue to work on the analysis of the preliminary data.

Estimate your "false detection rate" for muon decays. Compare with the observed muon decay background.

From your SFU data (elevation 340m), estimate the expected stopping rates at another elevation assuming some of the corrections discussed in the Teachspin manual. Try Mount Seymour parking lot (~1200m). Neglect any changes in muon energy spectrum with elevation for your rough estimate.

In order to measure the stopping rate at a different elevation, how long would you have to do the measurement assuming a count rate of roughly 1 decay per 2 minutes? Assume 2% uncertainty in the count rate. (This is of practical importance if we drive the apparatus up to Mount Seymour).

Experimental points:

Why do you see no signal from the PMT on the scope unless you first add a 50 Ohm resistor across the scope terminals?

Appendix:

Time Dilation Effect (rewritten by S. Watkins, Jan 2007)

(This section explains in a bit more detail and fixes some errors in the time dilation analysis in the TeachSpin notes)

The energy lost by a muon as it travels through a vertical distance dx of density ρ is empirically given by $dE = C_0 \rho dx$ where ρ is the density of the medium in g/cm^{-3} and C_0 is around 2MeV g^{-1} cm^2.

The density of the earth's atmosphere varies with altitude. As a crude approximation, one can assume that the atmosphere is at constant temperature at all altitudes (e.g. see http://en.wikipedia.org/wiki/Atmospheric_models). This assumption together with the ideal gas law can be shown (see Carrington) to give the following pressure dependence with elevation *h*: $P = P_0 e^{-h/h_0}$ where h_0 = 7.99 km and typically P_0=1.01×10^5 Pa .

The density (g/cm^{-3}) at a height *h* in a fluid can easily be shown to be $\rho = -\frac{1}{g}\frac{dP}{dh}$ (note that one cannot use the special case $P = \rho g h$ because the system is a compressible fluid. i.e. air).

Consider a layer of air between two elevations, H_U and H_L where the former is say Mount Seymour and the lower is say SFU.

The energy loss for high energy muons travelling through this region is given by given by

$$\Delta E = \int_{H_L}^{H_U} C_0 \rho dx = -C_0 \int_{H_L}^{H_U} \frac{1}{g}\frac{dP}{dh} dh = -C_0 \int_{P_L}^{P_U} \frac{1}{g} dP = \frac{C_0 P_0}{g}\left[e^{-H_L/h_0} - e^{-H_U/h_0}\right]$$

where C_0 is 2MeV g^{-1}cm^2 and P_0/g works out to around 1010 g/cm^2 (assuming atmospheric pressure at the surface of the earth is 1.01×10^5 Pa.

For the case of Mount Seymour parking lot, H_U ~1200m = 1.2×10^5 cm.
For SFU, H_L=340m = 3.4×10^4 cm

In the muon rest frame the transit time for the muon to travel from some initial elevation H_U down to some lower elevation H_0 is given by

$$t' = \int_{H_U}^{H_L} \frac{dh}{c\beta(h)\gamma(h)}$$

where H_L = SFU and H_U = Mount Seymour

Using simple estimate for energy loss, this becomes: (derive this)

$$t' = \frac{mc^2}{c\rho C_0} \int_{\gamma_2}^{\gamma_1} \frac{d\gamma}{\sqrt{\gamma^2 - 1}}$$

(note that the numerator is written so as to emphasize rest mass/energy of muon)

Note that density here is average density per unit volume between the upper and lower elevations (this is an approximation which makes the integration easier, but probably small compared to all the rest!)

The energy at the upper elevation (corresponding to γ_1) is given by 160 MeV (the energy at the lower elevation) plus the energy loss estimate from above.

Hence, E_U=160MeV + ΔE
From this we can calculate γ_1 and then evaluate the above integral numerically. (http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/integral/integral.html)

As discussed in the muon manual, γ_2~1.5 (this corresponds to a muon energy of 160MeV, which is the approximate energy that muons must have on average to be stopped inside the detector. Any higher than this and they pass through.

The rest lifetime of the muon is 2.197 μs.

The predicted ratio of the stopping rates (SFU/Mount Seymour) is therefore given by

$$R = e^{-t'/\tau}$$

This neglects the effect of changes in the muon flux with energy.

In practice, the muon energy spectum peaks is an increasing function of energy in the range of interest. This means that muons that are measured at the lower detector started out as higher energy muons in the upper detector. Because the muon energy spectrum peaks at higher energy, this means that the number of muons at lower energy will appear to be higher than it would if we had a flat energy distribution.

The paper by Coan et al. gives some suggestions for how to correct for this effect. Their method involves measuring the stopping rates between two reference elevations and then obtaining an empirical correction factor that can be used to predict the stopping rate at a third elevation.

In principle one can also use the data in the figure below to estimate the correction factor by making a rough estimate of the extra flux which 160MeV muons initially had at the higher energy. This is a simple scalar correction factor.

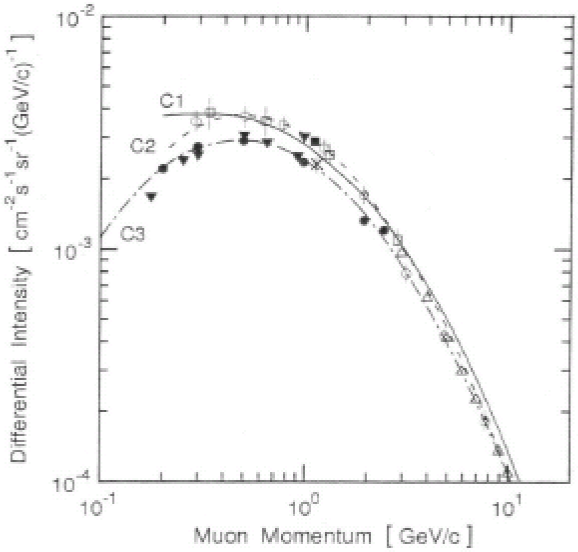

Figure 4 from the TeachSpin muon manual

Momentum energy relationship:

$$E^2 - (cp)^2 = (mc^2)^2$$

References

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