A Faraday rotation experiment for the undergraduate physics laboratory

Frank J. Loeffler
Department of Physics, Purdue University, Lafayette, Indiana 47907

(Received 12 July 1982; accepted for publication 8 September 1982)

I. INTRODUCTION

Traditional college courses in general physics cover the subject in two semesters, with the second devoted to electromagnetics, optics, and "modern" physics. It has always seemed difficult to me to convince the student that the transition from the study of electricity and magnetism to optics is a logical and smooth one as far as physical principles are concerned, despite the use of analogies and a lot of hand waving. Perhaps the performance by the student in the laboratory of the experiment discussed below will not completely achieve this goal either, but it at least does bring together fundamental elements of both electromagnetism and optics in the examination of a fundamental physical phenomenon—the rotation of plane polarized light by magnetic fields in transparent liquids.

This was first observed by Michael Faraday[1] in 1845 and is usually called the Faraday effect. The rotation is caused by circular birefringence and the relationship of the external magnetic field to this birefringence can be discussed in terms of the Zeeman effect. The phenomenon has been discussed in classical terms by several authors,[2] and in recent years several more modern treatments have been given.[3–5] It has been used some, but not widely as the subject for an experiment in the undergraduate laboratory.[6,7]

A schematic view of the experimental setup is shown in Fig. 1. Light from a lamp is collimated with a simple lens and aperture and directed through a color filter to make it approximately monochromatic. Next it passes through a polaroid and thus enters the liquid medium as plane polarized light. When there is current in the magnetic solenoid, a longitudinal field **B** exists in the liquid and the plane of polarization is rotated by an angle θ_R. This angle is given by

$$\theta_R = V\int_0^l \mathbf{B}\cdot d\mathbf{l} = V\,\overline{Bl},$$

where V is the Verdet rotation constant for the liquid being used and l is the path length traveled by the light in the liquid. V is a function of the wavelength and changes appreciable over the visible range (see Fig. 2). It is therefore appropriate to use the color filter.

After passing through the solenoid and having its polarization direction changed by θ_R, the beam traverses a second polaroid (analyzer) and is then focused onto a photodiode which acts as an intensity monitor. By adjusting the analyzer angle for maximum intensity both with the magnetic field on and off, the rotation angle θ_R can be determined. Thus either V or $\overline{Bl}$ can be measured if one or the other is already known. Using the modest apparatus described below and convenient liquids such as water or xylene, rotations of several degrees can be obtained.

II. APPARATUS

The apparatus described schematically in Fig. 1 is mounted on a simple optical bench which is 42 in. long. Let me describe the components shown in Fig. 1 by progressing from left to right.

A standard projection lamp (BEH-150 W) is used as the source. A 25-Ω, 50-W resistor is connected in series with the lamp to prolong its lifetime.

In order to provide a collimated beam of light a simple converging lens ($f = 10$ cm; diam = 51 mm) is used in conjunction with an aperture plate (3/8 in. diam. hole in a round aluminum plate). This provides a beam that is somewhat smaller in diameter than the liquid medium cell.

The color filter is a square green glass filter (5×5 cm; 2 mm thick) with a transmission peak at 5400 Å. One example of a suitable filter is number A26-3350 distributed by the Ealing Corporation.

The polarizer and analyzer are identical and consist of polaroid sheet sandwiched in round pieces of glass (Edmund Scientific, 40209) and mounted so that they can be rotated about an axis parallel to the light beam. Relative angles are measured with a protractor attached to the mounting.

The magnetic solenoid is wound on an aluminum spool with a bore of 5/8 in., an overall length of $7\frac{1}{2}$ in. and an outside diameter of 5 in. It consists of 1000 turns of #10 magnet wire and requires about 12 V to produce a current of 15 A, which is the normal operating condition. The average magnetic field along the bore is about 1100 G at 15 A, and $\overline{Bl}$ is typically 22 000 G cm (0.022 T m).

The power supply has an output voltmeter and ammeter on the front panel. Current values can be maintained within a range of $\pm$ 1% or better. We do not use the absolute value of the current for any numerical calculations of experimental results, but we do need to maintain the magnetic field at a reasonably constant level.

The optical rotation occurs in the cell containing the

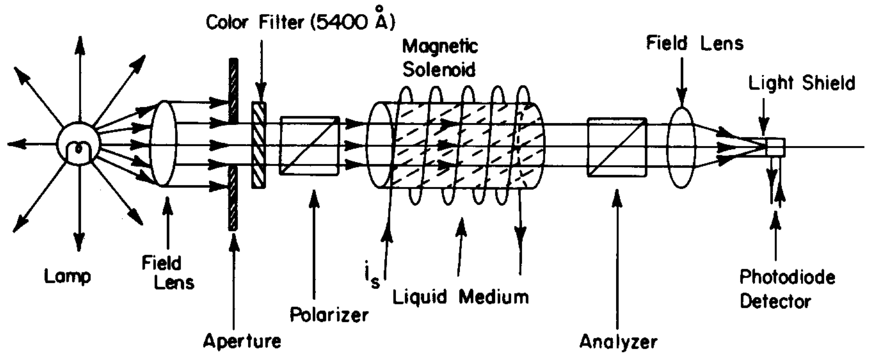

Fig. 1. Schematic diagram of the Faraday rotation apparatus.

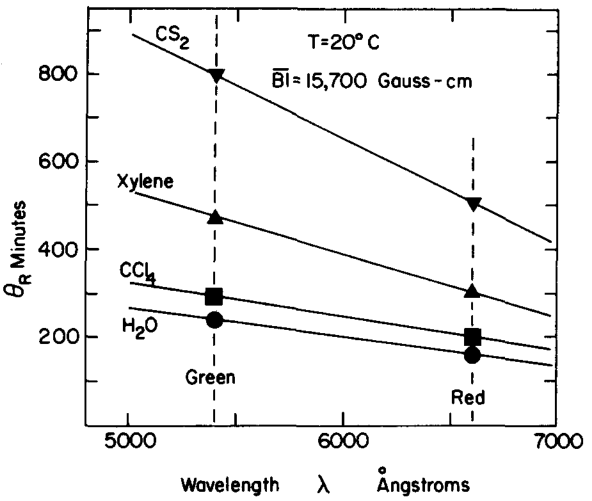

Fig. 2. Some values of the Faraday rotation angle, θ_R as measured by the author for four liquids.

active liquid medium. It consists of a bakelite tube 8 in. long, with an inner diameter of $\frac{1}{2}$ in. and outer diameter of $\frac{5}{8}$ in. The ends of the tube are sealed with $\frac{1}{2}$ in. diam. glass plugs cut from $\frac{1}{4}$ in. thick optical glass. These are glued into the tube with an epoxy resin. The tube is filled with liquid through a $1\frac{1}{2}$ in. long, $\frac{1}{4}$ in. diam. aluminum filler–expansion port which is glued into a hole drilled through the bakelite at one end of the tube.

The choice of active optical liquids was dictated by two considerations—fairly large Verdet constants and relatively nonhazardous materials. Figure 2 shows eight measurements of rotation that I made with four different liquids—water, CCl_4, *o*-xylene, and CS_2. These were done at two different wavelengths of light, 5400 Å and 6600 Å. The straight lines drawn on the graph are merely to guide the eye and are not intended to predict values of rotation at other wavelengths.

We use water and xylene in the laboratory. Xylene is sufficiently different from water in its optical properties that it is easy to distinguish one from the other in laboratory measurements.

The anlayzer is identical to the polarizer. It of course is used to determine the amount of rotation produced by the magnetic field.

To monitor the intensity of the light transmitted by the apparatus, the beam leaving the analyzer is focussed by the last field lens ($f = 10$ cm; diam. = 51 mm) on to a photodiode. The photodiode is a VACTEC VT701. This detector, is connected in series with a 9-V battery, a 2-kΩ fixed resistor, a 10-kΩ potentiometer, and a 0–1 mA panel meter. The potentiometer allows the experimenter to set a convenient reading level on the panel meter.

III. EXPERIMENTAL TECHNIQUE

Alignment of the optical elements in the apparatus is fairly easy and is done by laboratory teaching assistants before each laboratory period. The students are encouraged to understand the alignment by making minor changes in the positions of the various elements and observing the effects.

The object of the experiment is quite simple. First the rotation of the polarization plane produced by a water sample is measured. Knowing the Verdet constant V for water one is able to determine the quantity $\overline{Bl}$ for the apparatus. Next an unknown liquid sample (xylene) is measured. The students are asked to identify the unknown liquid by using a table of values for V.

Table I. Typical rotation data for several symmetric intensity values (θ_1 and θ_2) which give measures of the angle for maximum intensity θ_m. One set of data corresponds to zero magnetic field ($i_s = 0$ A) and the other to full magnetic field ($i_s = 15$ A). In this example the angle of magnetic rotation, θ_R is found to be 5.40 deg. The optically active liquid is CCl_4.

I/I_0	$i_s = 0$ A			$i_s = 15$ A		
	θ_1	θ_2	θ_m	θ'_1	θ'_2	θ'_m
0.3	171.5	7.5	89.5	157	10	83.5
0.4	159.5	19.5	89.5	152.5	15.5	84
0.5	149.5	28	88.75	147	20.5	83.75
0.6	140.5	38	89.25	141	27	84
0.7	129.5	49	89.25	134	34	84
		$\bar{\theta}_m = 89.25$			$\bar{\theta}'_m = 83.85$	
		$\theta_R = \bar{\theta}_m - \bar{\theta}'_m = 5.40$ deg				

The technique we use for determining θ_R is somewhat detailed. Merely rotating the analyzer until a maximum intensity is observed with the detector is not a very sensitive method since the observed intensity I is related to the maximum intensity I_0 and the angle of the analyzer θ by the equation

$$I = I_0 \cos^2 \theta.$$

Thus when θ is near the angle θ_m where maximum intensity occurs one is working close to the nearly flat top of the $\cos^2 \theta$ function.

Therefore we use the following method. First the analyzer is rotated until the observed intensity is a maximum and the potentiometer in the detector is adjusted until the meter reads full scale. Then we measure the angle θ_1 and θ_2 on the analyzer protractor which give rise to equal intensity readings corresponding to symmetric points on the sides of the $\cos^2 \theta$ curve. We do this for five points corresponding to $I = (0.3, 0.4, 0.5, 0.6, 0.7)I_0$. The mean value of each set of θ_1, θ_2 measurements gives a measure of θ_m. Table I shows a typical set of data taken with the magnetic solenoid off and then energized (corresponding to magnetic solenoid current i_s values of 0 and 15 A). These data were taken by a student and it is clear from the dispersion of the measurements that one can typically determine the value of θ_R to better than 10%.

IV. SUMMARY

I am convinced that this experimental apparatus and the associated measuring techniques are well suited to the undergraduate laboratory in general physics. The kinds of measurements suggested above lend themselves to straightforward error analyses which are a very important part of undergraduate laboratory training.

One criticism that can be made of this laboratory experiment is that it is not feasible to give a detailed theoretical treatment of the Faraday rotation phenomenon within the scope of most courses in general physics. I don't feel that this is a serious shortcoming. Acquainting the student with the phenomena of nature is an important part of our job even though we are not prepared in all cases to give thorough explanations of these phenomena. For example, we present the rough ideas of the quark–lepton model of matter but hardly develop its theoretical foundations in beginning physics courses.

ACKNOWLEDGMENTS

I want to thank E. C. Beard, E. F. Durr, and J. Meyer for help in constructing the equipment.

[1] Entry 7504, 13 September 1845 in Faraday's Diary (Bell, London, 1933), Vol. IV.
[2] R. A. Houston, *A Treatise on Light* (Longmans, Green, London, 1945), pp. 475–478.
[3] P. M. Rinard and J. W. Calvert, Am. J. Phys. **39**, 736 (1971).
[4] R. M. Peterson, Am. J. Phys. **43**, 969 (1975).
[5] W. G. Harter and N. DosSantos, Am. J. Phys. **46**, 264 (1978).
[6] K. J. Gordon, Am. J. Phys. **46**, 530 (1978).
[7] B. K. Tanner, Am. J. Phys. **48**, 59 (1980).

One-dimensional hydrogen atom and hydrogen molecule ion in momentum space

I. Richard Lapidus
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030

(Received 28 October 1981; accepted for publication 28 July 1982)

The exact computation of the momentum-space wave functions for bound states of quantum-mechanical systems is usually not possible, and one must obtain the momentum representation in terms of an integral equation.

A system of particular interest is the hydrogen atom. For this system the momentum-space wave functions for the bound states in three dimensions have been obtained.[1] However, they are quite complicated. In this note the bound-state momentum-space wave function of the one-dimensional hydrogen atom with δ-function interaction is obtained. The exact momentum representation wave function for the one-dimensional hydrogen molecule ion with δ-function interations is also obtained. Because these calculations can be carried out quite simply, this problem may be useful in discussing the momentum representation in a quantum mechanics course.

I. HYDROGEN ATOM

The Schrödinger equation for the one-dimensional hydrogen atom with δ-function interactions is given by[2–4]

$$-(\hbar^2/2m)\psi'' - e^2\delta(x)\psi = E\psi. \tag{1}$$

Equation (1) has a bound-state solution:

$$\psi(x) = a_0^{-1/2}\exp(-|x|/a_0), \tag{2}$$

where $a_0 = \hbar^2/me^2$. The energy eigenvalue for this state is

$$E = E_0 = -me^4/2\hbar^2. \tag{3}$$

The momentum-space wave function may be obtained most easily by Fourier transforming Eq. (1). One obtains

$$\begin{aligned}(p^2/2m - E)\phi(p) &= e^2(2\pi\hbar)^{-1/2}\\ &\quad\times\int_{-\infty}^{\infty}\exp(-ipx/\hbar)\delta(x)\psi(x)dx\\ &= e^2(2\pi\hbar a_0)^{-1/2}.\end{aligned} \tag{4}$$

Thus

$$\phi(p) = (2a_0/\pi\hbar)^{1/2}/[1 + (pa_0/\hbar)^2]. \tag{5}$$

This result was obtained by Lieber[5] using a different method.

Plots of $|\psi(x)|^2$ as a function of x and $|\phi(p)|^2$ as a function of p are shown in Fig. 1.

The widths of the distributions for x and p yield the uncertainty relation. Using Eq. (2),

$$(\Delta x)^2 = \langle x^2\rangle = \int_{-\infty}^{\infty} x^2\psi^2(x)dx = \frac{a_0^2}{2}. \tag{6}$$

Using Eq. (1),

$$(\Delta p)^2 = \langle p^2\rangle = 2m\langle E\rangle + 2me^2\langle\delta(x)\rangle = (\hbar/a_0)^2. \tag{7}$$

Thus

$$\Delta x\,\Delta p = (\hbar/2)\sqrt{2}, \tag{8}$$

which is greater than the minimum value possible (Gaussian wave function) by a factor of $\sqrt{2}$.

II. HYDROGEN MOLECULE ION

The Schrödinger equation for the one-dimensional hydrogen molecule ion with δ-function interactions is given by[4,6]

$$-(\hbar^2/2m)\psi'' - e^2\delta(x-a)\psi - e^2\delta(x+a)\psi = E\psi, \tag{9}$$

where the nuclei are located at the positions $x = \pm a$.

Equation (9) has one symmetric bound-state solution[4,6]

$$\psi(x) = A\cosh(\gamma x/a) \quad |x| < a \tag{10a}$$

$$= B\exp(-\gamma|x|/a) \quad |x| > a, \tag{10b}$$

with $A = (2/a_0)^{1/2}(1 + 2\gamma - \beta)^{-1/2}\exp(-\gamma)$ and $B = (2/a_0)^{1/2}(1 + 2\gamma - \beta)^{-1/2}\cosh(\gamma)$, where $\beta = 2a/\alpha_0 = \gamma[1 + \tanh(\gamma)]$.

The energy eigenvalue for this state is

$$R = E_0(2\gamma/\beta)^2. \tag{11}$$

The momentum-space wave function is obtained most easily by Fourier transforming Eq. (9). One obtains

$$\begin{aligned}(p^2/2m - E)\phi(p) &= e^2(2\pi\hbar)^{-1/2}\int_{-\infty}^{\infty}\exp(-ipx/\hbar)\\ &\quad\times[\delta(x-a) + \delta(x+a)]\psi(x)dx\end{aligned}$$

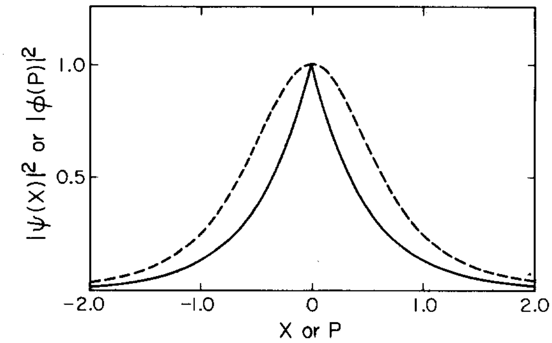

Fig. 1. Plots of the coordinate-space probability density $|\psi(X)/\psi(0)|^2$ vs $X = x/a_0$ (——), where $\psi(0) = a_0^{-1/2}$, and the momentum-space probability density $|\phi(\mathbf{p})/\phi(0)|^2$ *vs* $P = pa_0/\hbar$ (- - -), where $\phi(0) = (2a_0/\pi\hbar)^{1/2}$, for the one-dimensional hydrogen atom.