[12]F. H. P. M. Habraken, O. L. J. Gijzeman, and G. A. Bootsma, Surf. Sci. **96**, 482 (1980).
[13]T. E. Furtak and J. Reyes, Surf. Sci. **93**, 351 (1980).
[14]S. L. Adler, Phys. Rev. **126**, 413 (1962); N. Wiser, Phys. Rev. **129**, 62 (1963).
[15]L. Lorenz, Ann. Phys. Chem. (Leipzig) **11**, 70 (1880).
[16]M. Omini, Physica (Utrecht) **83A**, 431 (1976); **84**, 129, 492 (1976); K. Vedam and P. Limsuwan, J. Chem. Phys. **69**, 4772 (1978).
[17]C. Grosse and J. L. Greffe, J. Chim. Phys. **76**, 305 (1979).
[18]J. C. M. Garnett, Philos. Trans. R. Soc. London **203**, 385 (1904); A **205**, 237 (1906).
[19]D. A. G. Bruggeman, Ann. Phys. (Leipzig) **24**, 636 (1935).
[20]D. E. Aspnes, J. B. Theeten, and F. Hottier, Phys. Rev. B **20**, 3292 (1979).
[21]G. B. Smith, J. Phys. D **10**, L39 (1977).
[22]G. A. Niklasson, C. G. Granqvist, and O. Hunderi, Appl. Opt. **20**, 26 (1981).
[23]C. G. Granqvist and O. Hunderi, Phys. Rev. B **18**, 2897 (1978).
[24]O. Wiener, Abh. Math. Phys. K1. Königl. Sächs. Ges. **32**, 509 (1912).
[25]Z. Hashin and S. Shtrikman, J. Appl. Phys. **33**, 3125 (1962).
[26]D. Bergmann, Phys. Rev. Lett. **44**, 1285 (1980).
[27]D. W. Milton, Appl. Phys. Lett. **37**, 300 (1980).

Nuclear spins in the Earth's magnetic field

P. T. Callaghan and M. Le Gros
Department of Chemistry, Biochemistry and Biophysics, Massey University, Palmerston North, New Zealand

(Received 24 March 1981; accepted for publication 11 November 1981)

Details are given of a simple apparatus for the observation of nuclear precession in the Earth's magnetic field. Spin echos are generated using a pulse of oscillatory magnetic field resonant at the Larmor frequency. Students can use the system to measure a nuclear magnetic moment, nuclear relaxation times, and the local strength and orientation of the Earth's magnetic field.

INTRODUCTION

The study of nuclear magnetism is one of those areas of physics which seems to endure beyond all expectations. Indeed few techniques have yielded so rich a harvest in physics, chemistry, and biology as the observations of nuclear spin dynamics. Nuclear spin manipulation is even responsible for the lowest laboratory temperature to date! This remarkable versatility of application results from the wide range of magnitudes manifest in the interaction of nuclear spin and environmental electromagnetic properties. An appreciation of nuclear magnetism is well within the grasp of an undergraduate physics student and an inexpensive apparatus which illustrates some of the fundamentals of the subject can be easily constructed. We describe here some simple equipment which can be used as the basis for a nuclear magnetism experiment in the teaching laboratory. Variations of this technique have been used in geomagnetic field measurements for many years but in our experiment the student not only measures precisely the magnitude of the Earth's local magnetic field but can also obtain the nuclear dipole moment of ^{19}F and the relaxation times of either ^{1}H and ^{19}F nuclei in different chemical environments. A novel aspect of our system is that it enables the observation of spin echos in the Earth's magnetic field. The spin echo is generated by a pulse of magnetic field oscillating at the Larmor frequency and is an audio frequency analog of the 180° rf pulse used in the traditional NMR spin echo sequence.

DETECTING NUCLEAR MAGNETISM

Whenever the polarization axis of a nuclear dipole ensemble is momentarily displaced from the polarizing magnetic field direction an oscillatory emf can be subsequently detected in a receiver coil oriented perpendicular to the field.[1] This decaying emf, known as the free induction decay (FID), results from the Larmor precession about the static magnetic field of the nuclear spin angular momentum vectors. The decay rate is governed by the nuclear spin–spin interaction which cause a relentless loss of phase coherence in the nuclear ensemble. For an ensemble of similar spin-$\frac{1}{2}$ nuclei, the FID signal is accurately described by[2]

$$S(t) = S_0 \exp(-t/T_2)\cos(\gamma B_0 t + \alpha), \quad (1)$$

where T_2 is the transverse or spin–spin relaxation time and γB_0 is the Larmor precession frequency, proportional both to the static environmental magnetic field B_0 and to the nuclear gyromagnetic ratio γ. γ is a constant relating the nuclear magnetic dipole moment μ to the spin angular momentum quantum number j via the equation

$$\mu = \gamma \hbar j. \quad (2)$$

Where the static magnetic field in which the nuclei are to precess is the local field of the Earth, a simple means of reorienting the nuclear spin magnetization is to momentarily apply a much larger laboratory magnetic field in a transverse direction. This method was first suggested by Packard and Varian[3] who demonstrated that a simple but precise magnetometer could be constructed using the coil configuration shown in Fig. 1 with a water sample abundant in ^{1}H spins. The transverse polarizing field B_p, if applied for longer than a few spin-lattice relaxation times T_1, induces a larger transverse proton magnetization which subsequently precesses about the Earth's magnetic field on removing the polarizing field. B_p thus had a dual purpose

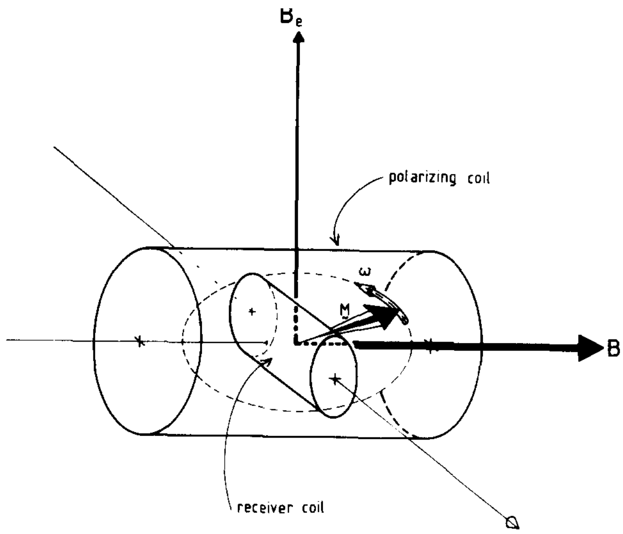

Fig. 1. Coil configuration for the free-precession experiment.

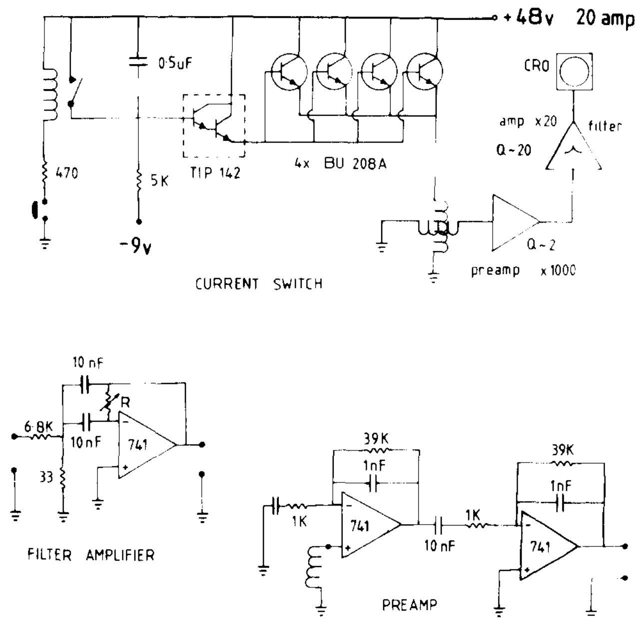

Fig. 2. Details of circuitry used in the static reorientation–free-precession experiment. The BU 208A are 3-kV 5-A silicon power transistors similar to those used in auto ignition systems. R is a series combination of a 500-kΩ ten-turn variable resistor and a 680-kΩ fixed resistor.

in that it both realigns the equilibrium proton magnetization and significantly increases its magnitude.

For a receiver coil of area A_r and N_r turns, the induced emf is

$$\mathscr{E} = N_r \frac{d\phi}{dt} = N_r A_r \omega \mu_0 M \cos \gamma B_0 t. \quad (3)$$

A spin-$\frac{1}{2}$ ensemble such as the protons of a water sample has a thermal equilibrium magnetization M in the polarizing magnetic field of

$$M = N \langle \bar{\mu}_z \rangle \simeq N \frac{\gamma \hbar}{2} \frac{\gamma \hbar B_p}{2 k_B T}, \quad (4)$$

where k_B is Boltzmann's constant and N is the number of nuclear spins per unit volume.

The initial signal magnitude S_0, defined by Eq. (1), is proportional to the magnetization acquired by the time the polarizing field is removed. This magnetization M depends on the polarization time t_p during which spin-lattice relaxation occurs. Accordingly, one may write

$$S_0 = S_{0m} [1 - \exp(-t_p/T_1)], \quad (5)$$

where S_{0m} is the maximum signal corresponding to the full thermal equilibrium magnetization of Eq. (4).

STATIC REORIENTATION: THE FREE-PRECESSION EXPERIMENT

A compact magnetometer using the nuclear free-precession signal can be built using a common polarization and receiver coil.[4] We have chosen a separate coil system for the laboratory apparatus since it enables independent axis orientation, a useful feature if the direction of the Earth's field is to be investigated. Separate coils also facilitate independent optimization of the polarizing and receiver coil design. For example, the externally arranged polarization coil can be wound with a thickness much greater than the af skin depth in copper (about 1.5 mm) without impairing the nuclear precession signal. Details of the coils and associated electronics are given in Fig. 2 and Table I. 12-V automotive batteries were used to provide the polarization current ($\simeq$20 A) and a manually triggered Darlington switch was used to provide the current pulse. An important feature of the current switching circuit is that while the turn-off rate must be controlled to avoid back emf damage to the output transistors, it must be sufficiently rapid that the spins are "left behind" along the initial polarization direction as the current drops to zero and the net magnetic field swings back to the remaining Earth's field axis. This corresponds to the so-called nonadiabatic condition.

The receiver coil is tuned ($Q \sim 4$) to the approximate Larmor frequency and is connected locally to a tuned ($Q \sim 2$) broadband, high-gain, preamplifier employing "741" operational amplifiers. The signal is then taken via a tunable filter amplifier to a Hewlett Packard HP 1223A storage oscilloscope, which is internally triggered by the large emf induced when the polarizing field is removed (or applied).

The magnitudes of the signals obtained are consistent with the predictions of Eqs.(3) and (4), making due allowance for electronic amplification in the receiver circuit.

Figure 3(a) shows the free-precession signal obtained from the protons of a 200-ml water sample after adjusting the center frequency of the filter amplifier for maximum signal. This center frequency is then the Larmor frequency, $\gamma B_0/2\pi$, which in our experiment is 2.424 $\pm$ 0.005 kHz giving a value for the Earth's magnetic field in Palmerston

Table I. Details of coils used in nuclear precession apparatus.

	Polarizing coil	Receiver coil
Number of turns	560	2700
Area (cm^2)	200	38
Length (cm)	16	10
Resistance (Ω)	2.7	200
Wire diameter (mm)	2.0	0.28

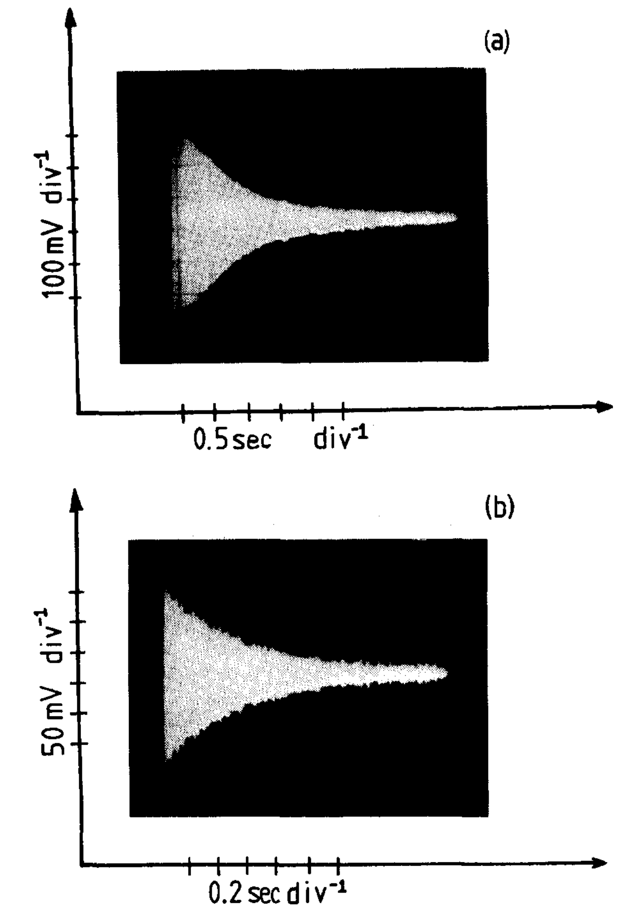

Fig. 3. (a) ^{1}H FID from a 200-ml sample of tap water. (b) ^{19}F FID from a 200-ml sample of perfluorodecalin. These signals correspond to the form of Eq. (1) where $\gamma B_0/2\pi$ is (a) 2.424 ± 0.005 kHz and (b) 2.258 ± 0.008 kHz.

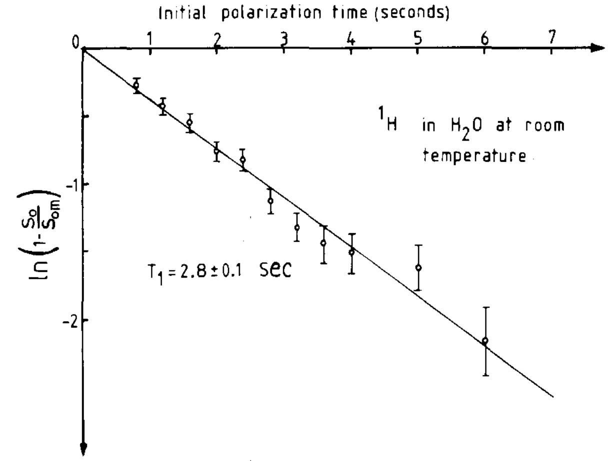

Fig. 4. ^{1}H spin-lattice relaxation for tap water at 20 °C as determined by the time taken for the spin ensemble to come to thermal equilibrium in the polarizing field. S_0 is the FID amplitude, immediately following the removal of the polarizing field and has the maximum value S_{0m} when the polarizing time is long.

North of $56.9 \pm 0.1\,\mu$T. This agrees with the known value[5] within experimental error. The direction of the Earth's field may be investigated by reorienting the receiver coil (and sample) to obtain maximum signal consequent on a rectangular alignment of the receiver coil, polarizing coil, and terrestrial field axes. We obtain a local inclination angle of $64° \pm 3°$ in good agreement with that found by more precise methods. With the magnitude of the Earth's field well established the nuclear magnetic moment of other nuclei may be examined. Figure 3(b) shows the ^{19}F signal obtained from 200 ml of perfluorodecalin. The filter frequency for maximum signal was 2.258 ± 0.008 kHz yielding

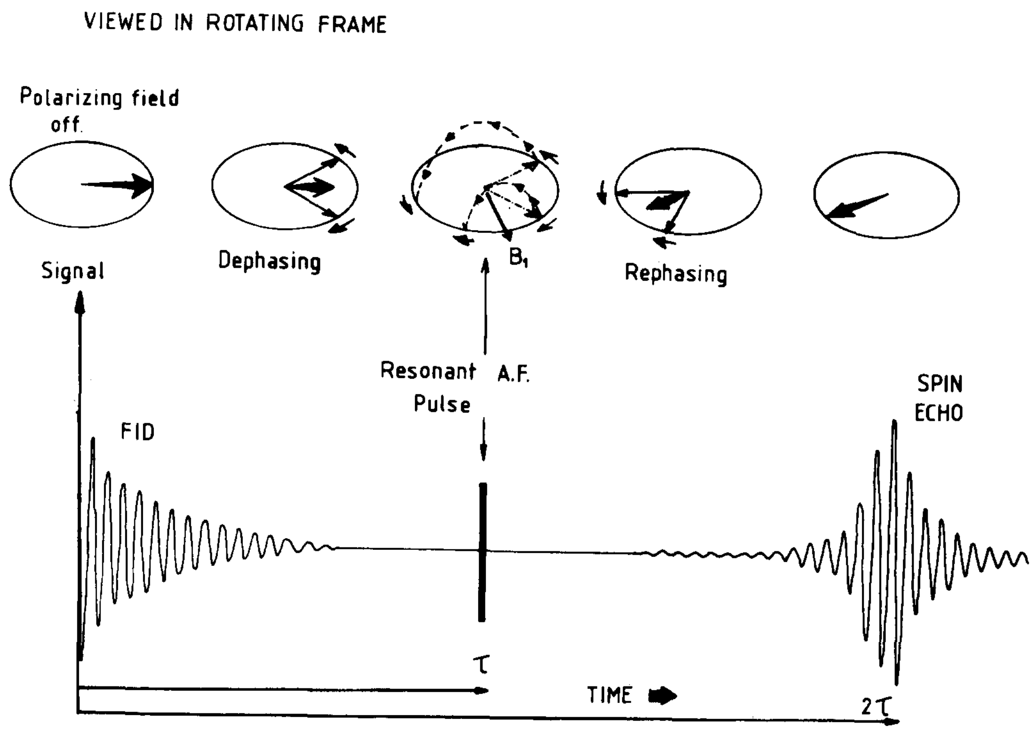

Fig. 5. The formation of a spin echo due to the refocussing effect of a pulse of af magnetic field resonant at the Larmor frequency. The pulse magnitude and duration obey the "180° condition" while the af phase is arbitrary.

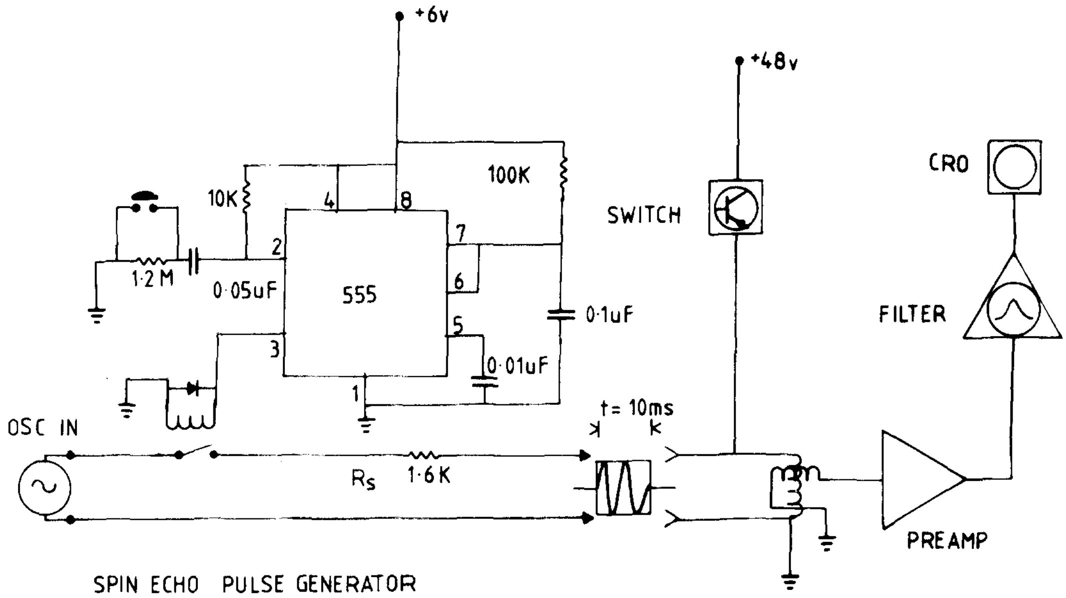

Fig. 6. Circuit diagram of af pulse generator showing its connection to the static reorientation apparatus. R_s may be adjusted to give the required field for a 180° pulse.

$2.60 \pm 0.02\,\mu_N$ for the ^{19}F magnetic moment in comparison with the tabulated value of $2.628\,\mu_N$.[6]

The nuclear free-precession apparatus can easily be used to investigate nuclear relaxation times. The spin-lattice relaxation time of the water protons was determined by observing the change in the initial FID height S_0, as the polarization time is varied, this time being simply determined by measuring the delay on the storage oscilloscope which was triggered at current turn-on. Figure 4 shows the results obtained for tap water at 20 °C in the spin-lattice relaxation experiment. The value of T_1 (here 2.8 ± 0.1 sec) is a function of the dissolved oxygen in the water. In a perfectly homogeneous magnetic field the decay rate of the free-precession signal is governed by the transverse relaxation time T_2. If the apparatus is constructed of entirely nonmagnetic materials this ideal can be very nearly realized, although ferrous metal even some meters distant can upset the homogeneity sufficiently that the FID is more rapidly attenuated than the rate predicted by relaxation alone. The classical means by which field imperfections may be compensated is to generate a spin echo.[7]

RESONANT REORIENTATION: THE SPIN ECHO EXPERIMENT

Magnetic field variation over the sample space causes dephasing of the nuclear transverse magnetization as the Larmor frequencies differ slightly for spins at different locations. In the spin echo experiment[2] the phase displacements are all inverted by rotating the nuclear spins about an axis in the transverse plane as shown in Fig. 5. Proton spin echos in the terrestrial magnetic field were reported by Bené in 1971 who used a static field inversion technique.[8] In high-field NMR experiments the required rotation is achieved by applying a finite duration rf magnetic field resonant at the Larmor frequency. An equivalent experiment is possible in the Earth's field and we demonstrate the results here. What is required is an oscillating transverse field of magnitude $2B_1$ (resolved as two counter rotating com-

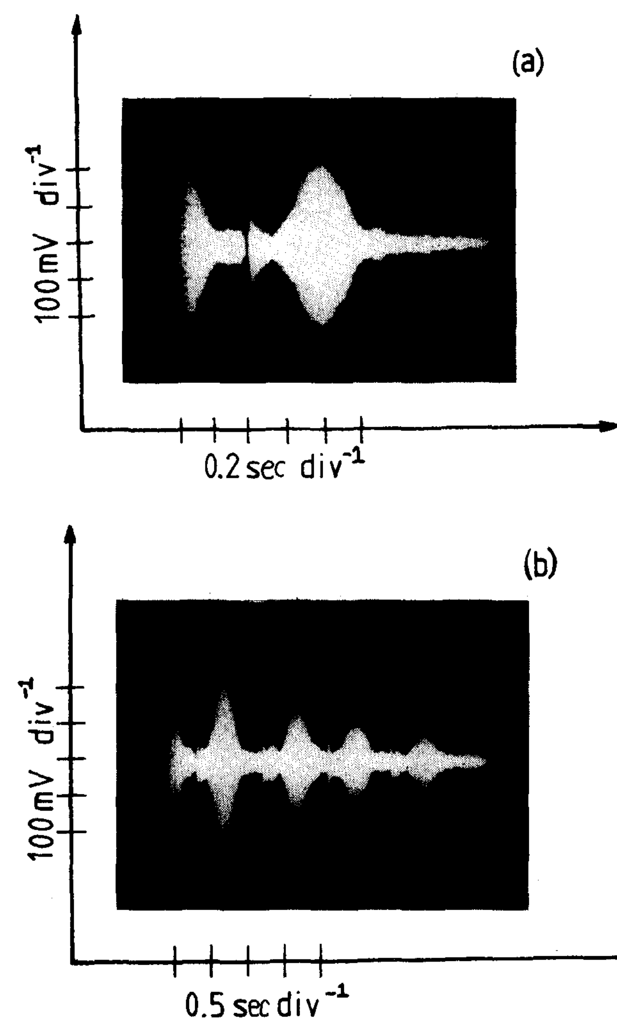

Fig. 7. (a) Oscilloscope trace of the signal due to a spin echo produced under conditions of enhanced magnetic field inhomogeneity. (b) Oscilloscope trace of the echo signal produced by a Carr–Purcell sequence of repeated 180° af pulses.

ponents of magnitude B_1) applied as a pulse of duration t, such that

$$\gamma B_1 t = \pi. \tag{6}$$

The phase of the af field relative to that of the preceding FID will determine the signal phase at the echo maximum but is otherwise unimportant (see Fig. 5). We have used a standard Laboratory oscillator (Philips PM5106, max output 10 V rms) to provide af to a manually triggered 10-msec gate. The gate circuit, shown in Fig. 6, employs a 555 timer and DIL reed relay. The pulse of af is applied to the polarizing coil.

The duration of the af pulse is dictated by the af bandwith desired to enable simple tuning of the oscillator. However, the pulse bandwidth is limited by the available af field since $B_1 \propto (1/t)$. The 10-msec pulse chosen here gives a suitable compromise. In order to enhance the echo appearance we magnify the field inhomogeneity by placing a crescent spanner 1 m from the sample. The oscillator is set to the Larmor frequency and the output adjusted until a near-perfect 180° pulse is achieved for a constant 10-msec pulse time. The resulting echo is shown in Fig. 7(a). By varying the delay time at which the af pulse is applied and measuring the resultant echo amplitude the transverse relaxation time can be deduced using Eq. (1) where $S(t)$ is the echo height at time t following the initial polarization. Repeated application of 180° pulses [Fig. 7(b)] produces a sequence of echos in a single sweep akin to the Carr–Purcell technique.[9] In such a sweep the student is able to simultaneously view the irreversible decay due to relaxation (echo envelope) and the reversible decay caused by field inhomogeneity (echo width). A special feature of the Carr–Purcell sequence is that it inhibits additional attenuation of the echo envelope due to diffusion of the spins in the sample. If a single af pulse applied at time τ produces a single spin echo at time 2τ the echo attenuation due to diffusion is[9]

$$A(G)/A(0) = \exp(-\tfrac{2}{3}\gamma^2 G^2 D\tau^3), \tag{7}$$

where G is the gradient in magnetic field over the sample and may be determined from the echo width. $A(0)$ is given by the magnitude of the echo envelope at 2τ obtained using a closely spaced Carr–Purcell sequence for which the attenuation due to diffusion is inhibited.

CONCLUSION

The experiment and apparatus described here form a useful basis for the illustration of fundamental principles of nuclear magnetism. The simplicity of the equipment makes it possible for the student to participate in its design and construction. The use of the Earth's magnetic field affords a degree of homogeneity which would be regarded as luxurious in a laboratory magnet and has the added advantage of locating the Larmor frequencies in the audio region. This advantage can be put to spectacular effect by connecting the output of the experiment to an audio amplifier and loudspeaker. The 2.5-kHz chirp from 10^{25} protons rephasing their spin precession is music indeed!

[1] A. Abragam, *The Principles of Nuclear Magnetism* (Clarendon, Oxford, 1961).

[2] For a simple description of the nuclear free induction decay and other NMR phenomena, see T. C. Farrar and E. D. Becker, *Pulse and Fourier Transform NMR* (Academic, New York, 1971).

[3] M. Packard and R. Varian, Phys. Rev. **93**, 941 (1954).

[4] G. S. Waters and P. D. Francis, J. Phys. E **35**, 88 (1958).

[5] G. A. Eibey and W. I. Reilly, in *New Zealand Atlas*, edited by I. McL. Wards (Shearer, Wellington, 1976).

[6] I. Lindgren, *Table of Nuclear Spins and Moments, in Alpha, Beta and Gamma Ray Spectroscopy Appendix 4*, edited by K. Siegbahn (North-Holland, Amsterdam, 1964).

[7] E. Hahn, Phys. Rev. **80**, 580 (1950).

[8] G. J. Bené, Abstracts of 4th Int. Symposium on Magnetic Resonance, Rehovot, Israel 25–27 Aug. 1971, Weizmann Inst. Science, 1971 (unpublished).

[9] H. Y. Carr and E. M. Purcell, Phys. Rev. **94**, 630 (1954).

Science and society test VII: Energy and environment

David W. Hafemeister

Physics Department, California Polytechnic State University, San Luis Obispo, California 93407

(Received 2 July 1981; accepted for publication 5 October 1981)

Approximate numerical estimates are developed in order to quantify a variety of environmental effects that result from energy production. The results of these calculations are consistent with either direct observations or with more complex calculations. This paper will cover some of the possible environmental effects of the following: (1) the greenhouse effect caused by increased CO_2 in the atmosphere; (2) loss of coolant accidents in nuclear reactors; (3) increased radon concentrations in buildings with very low air infiltration rates; (4) acid rain from the combustion of fossil fuels; (5) expolosions of liquified natural gas (LNG); and (6) ozone in the stratosphere.

I. INTRODUCTION

Man has always had fear and fascination for cataclysmic environmental effects from before the early days of the concept of the battle of Armageddon in Revelations up to our present time. Nowdays we can contemplate some of these events: the collapse of the Teton Dam, the accident at Three Mile Island, acid rains with a pH of 2.4 (equivalent to