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A relativistic approach to large-scale structure

David Wands

Institute of Cosmology and Gravitation, University of Portsmouth

with **Christian Fidler, Cornelius Rampf, Thomas Tram**,

Marco Bruni, Rob Crittenden, Kazuya Koyama

General relativistic corrections to N-body simulations, arXiv:1505.04756

The Intrinsic Matter Bispectrum in ΛCDM, arXiv:1602.05933

Relativistic interpretation of Newtonian simulations of structure formation, arXiv:1606.05588

motivation

cosmic structure provides a window onto the very early universe and the primordial density perturbation

can we trust standard Newtonian results for structure formation on the very largest scales?

- *coordinate invariance vs absolute space and time*
- *curved space vs flat space*
- *non-linear constraints vs linear Poisson equation*

Millenium simulation

questions

- Are Newtonian N-body simulations consistent with perturbative GR?
  - If so, what GR coordinate frame (gauge) do they work in?
  - What are the correct GR initial conditions for N-body simulations?
- What is the predicted GR galaxy distribution (observed along the past light cone, at given redshift and angle)?

Standard Newtonian+Gaussian initial fields

Gaussian primordial metric fluctuations *ζ(x)* from inflation + linear Einstein-Boltzmann code (e.g., CMBfast, CAMB, CLASS)

Gaussian initial Newtonian potential $\Phi = (3/5)\zeta$

Gaussian initial matter density using Poisson equation

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

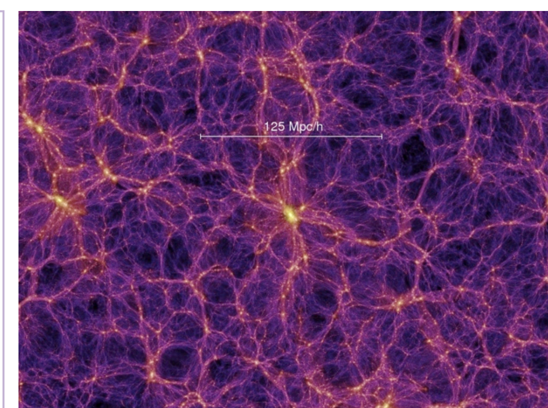
Gaussian initial displacement $\vec{\nabla}.\vec{\Psi} = -\delta$

Newtonian N-body simulations

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

$$\dot{\delta} + \vec{\nabla}.((1+\delta))\vec{v} = 0$$

$$\dot{\vec{v}} + \mathcal{H}\vec{v} + (\vec{v}.\vec{\nabla})\vec{v} = -\vec{\nabla}\Phi$$



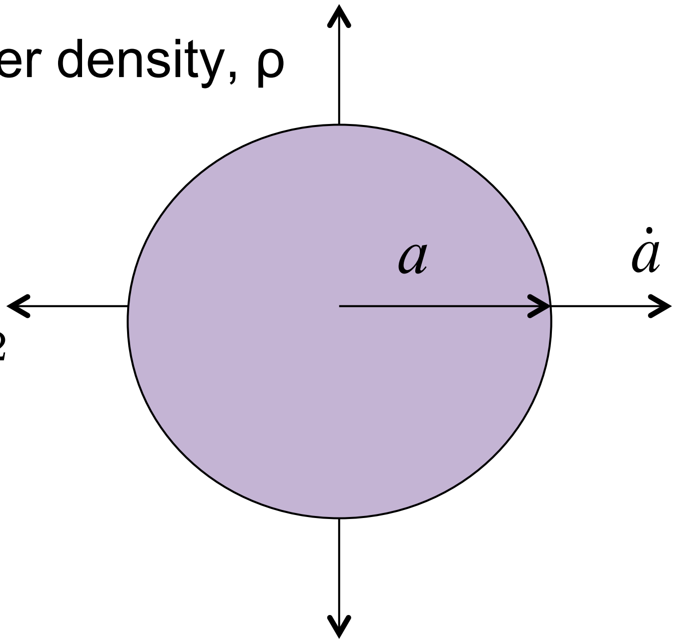
Newtonian ΛCDM cosmology

homogeneous+isotropic background (Milne 1930s)

same evolution+continuity equations for matter density, ρ

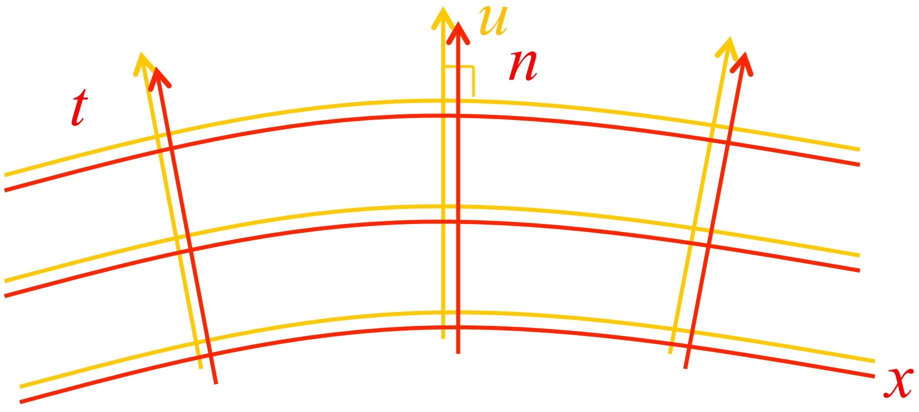
+ Friedmann (energy) constraint:

$$K = \frac{1}{2}\dot{a}^2 - \frac{G(4\pi\rho a^3/3)}{a} - \frac{\Lambda}{6}a^2$$



cosmological constant Λ = constant vacuum density

Newtonian energy = interpret in GR as *spatial curvature*

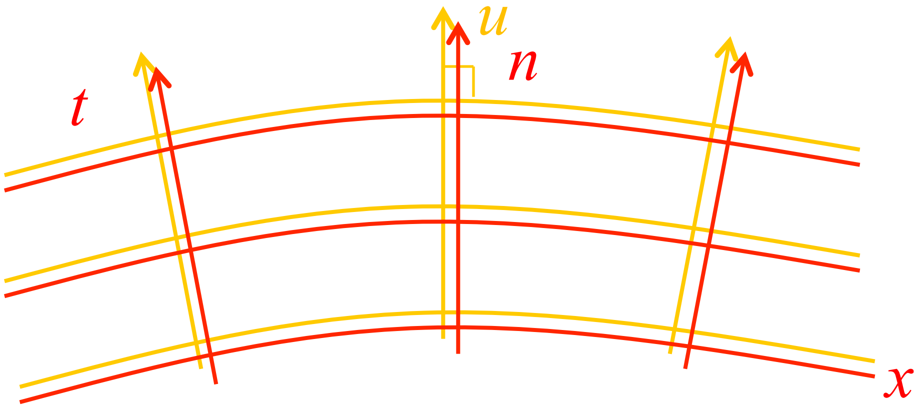


FRW cosmology

preferred coordinates for homogeneous and isotropic space

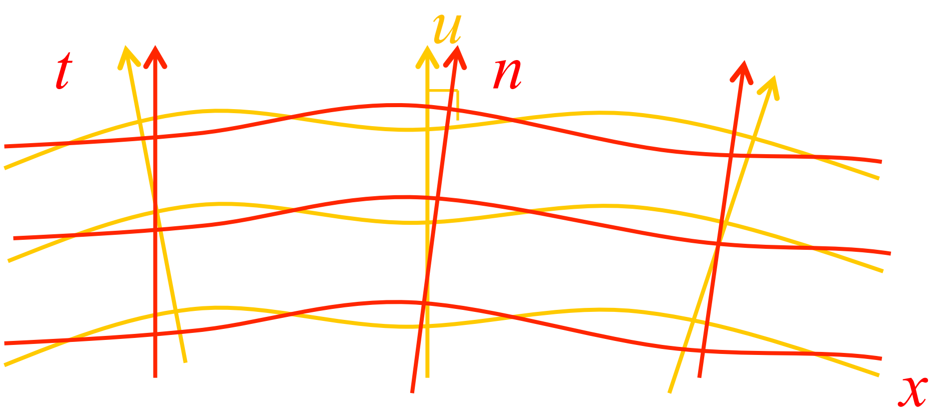
preferred space+time split in FRW cosmology breaks symmetry of Einstein's theory

Newtonian description sufficient



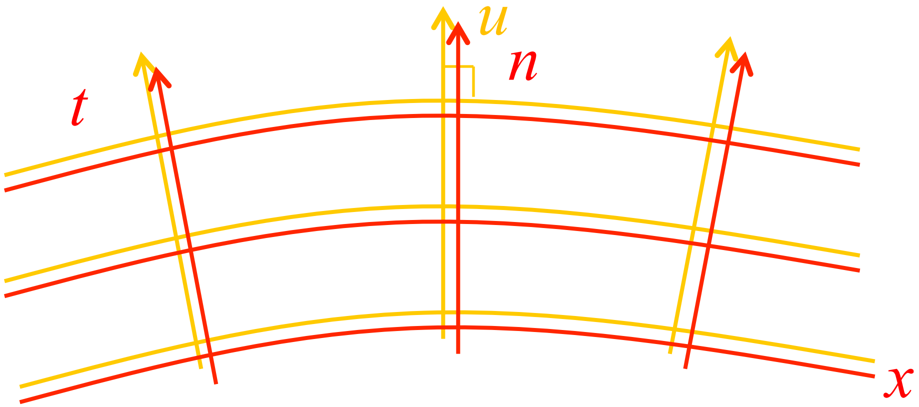
FRW cosmology

no unique choice of time (slicing) and space coordinates (threading) in an inhomogeneous universe



FRW cosmology
+ perturbations

arbitrary gauge *(t,x)*



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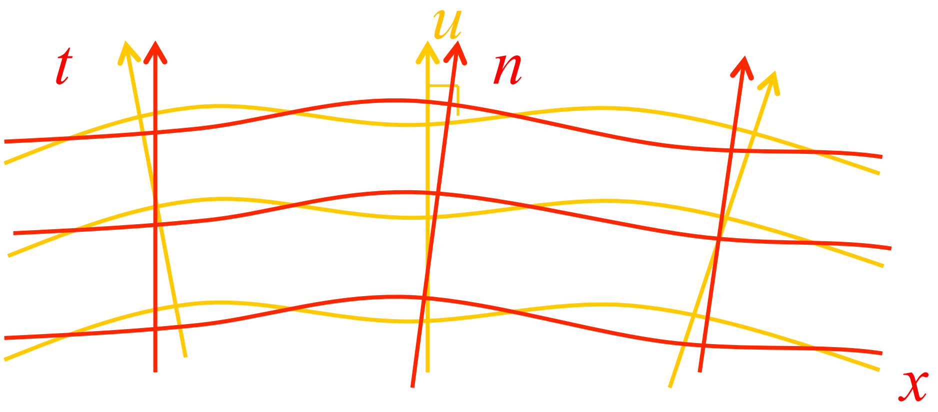
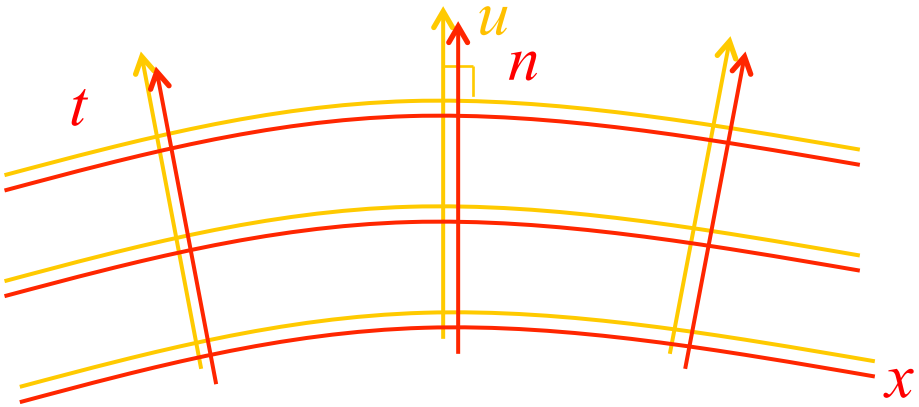


FRW cosmology
+ perturbations

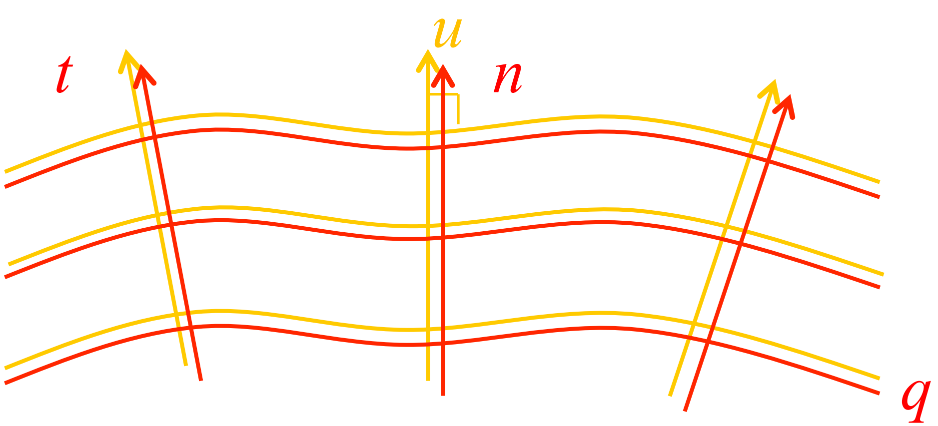
arbitrary gauge *(t,x)*

gauge problem: see different GR perturbations in different gauges
what gauge are N-body simulations using?



FRW cosmology

synchronous+comoving with pressureless cold dark matter
time-slicing orthogonal to comoving worldlines

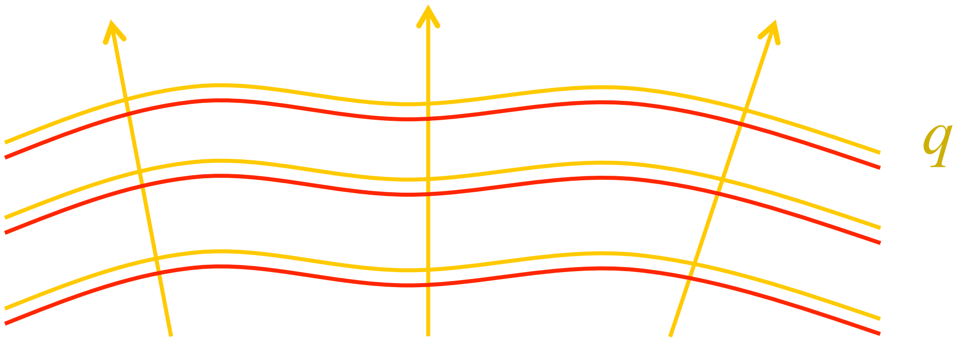


FRW cosmology
+ perturbations

comoving-Lagrangian coordinates *(t,q)*

Newtonian vs relativistic Lagrangian frame

- Newtonian (absolute) time, t
  - ➔ comoving-orthogonal time-slicing - *unique*

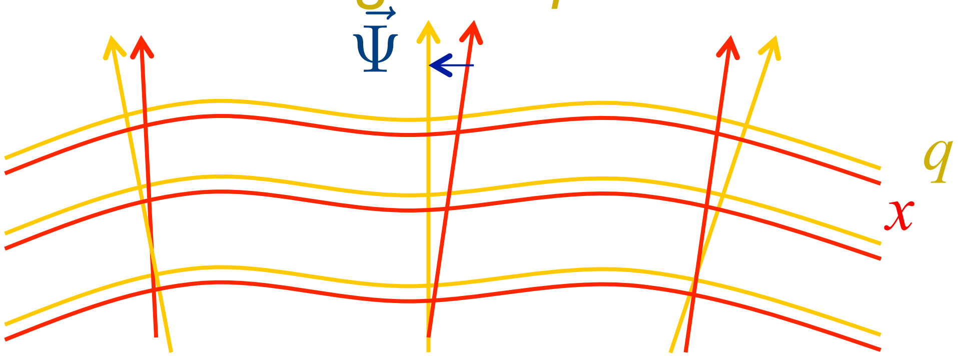


- Lagrangian coordinates, q
  - ➔ comoving spatial coordinates – *unique*

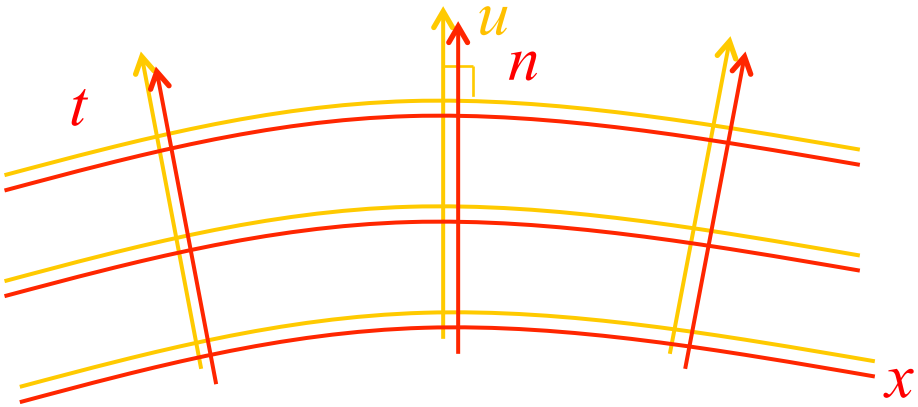
Newtonian vs relativistic Eulerian frames

- Newtonian (absolute) time, t
  - ➔ comoving-orthogonal time-slicing - *unique*

$$\vec{x} = \vec{q} + \vec{\Psi}(\vec{q}, t)$$



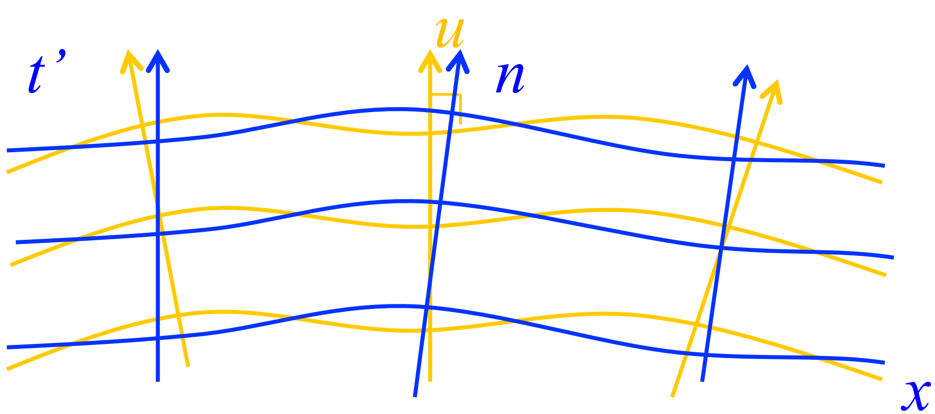
- Lagrangian coordinates, q
  - ➔ comoving spatial coordinates – *unique*
- Eulerian coordinates, x (only "comoving" in background)
  - *not unique (see, e.g., Rampf, arXiv:1307.1725)*



FRW cosmology

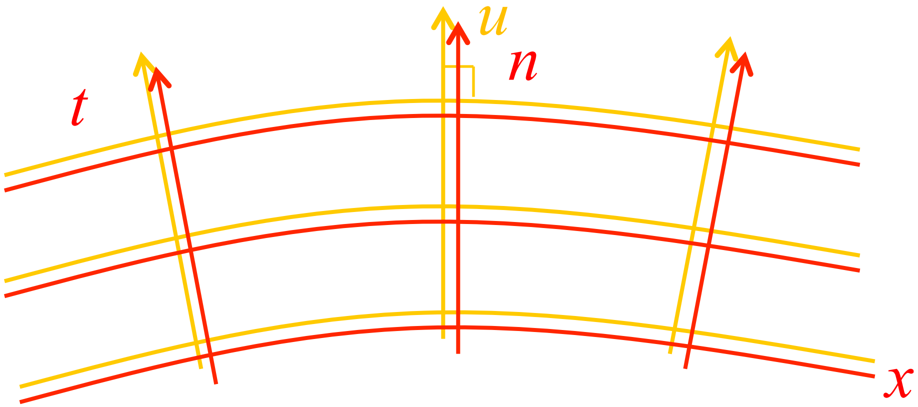
Poisson = conformal Newtonian = longitudinal gauge

hypersurface-orthogonal 4-vector field ***n*** *is shear-free*



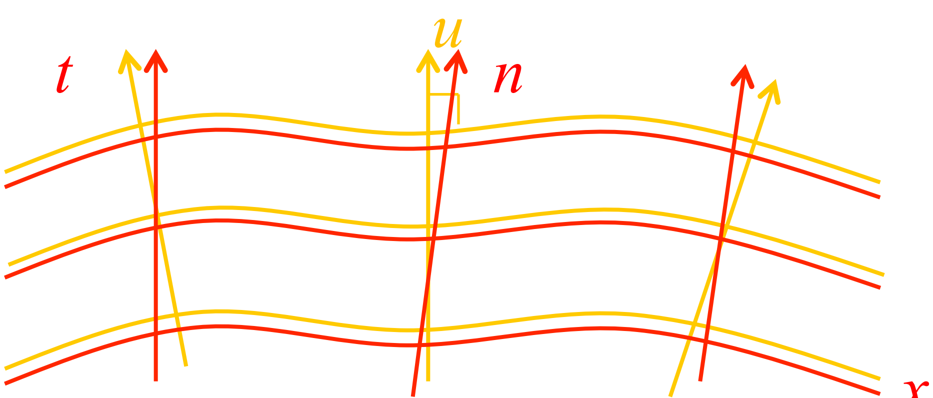
FRW cosmology
+ perturbations

Poisson gauge coordinates *(t',x)*



FRW cosmology

time-slicing orthogonal to comoving worldlines
spatial threading is same as Poisson gauge (Eulerian, not Lagrangian)



FRW cosmology
+ perturbations

total-matter coordinates *(t,x)*

Linear matter perturbations in total-matter gauge

- Energy and momentum conservation
  - comoving density contrast:

$$\dot{\delta} = -\vec{\nabla}.\vec{v} - 3\dot{\mathcal{R}}$$

  - total-matter velocity:

$$\dot{\vec{v}} + H\vec{v} = \vec{\nabla}\Phi$$

- Energy constraint:
  - Conformal Newtonian potential

$$\nabla^2\Phi = -4\pi G\bar{\rho}a^2\delta$$

- Momentum constraint:
  - Comoving curvature perturbation (for zero pressure):

$$\dot{\mathcal{R}} = 0$$

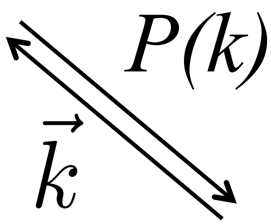
First-order density in Fourier space

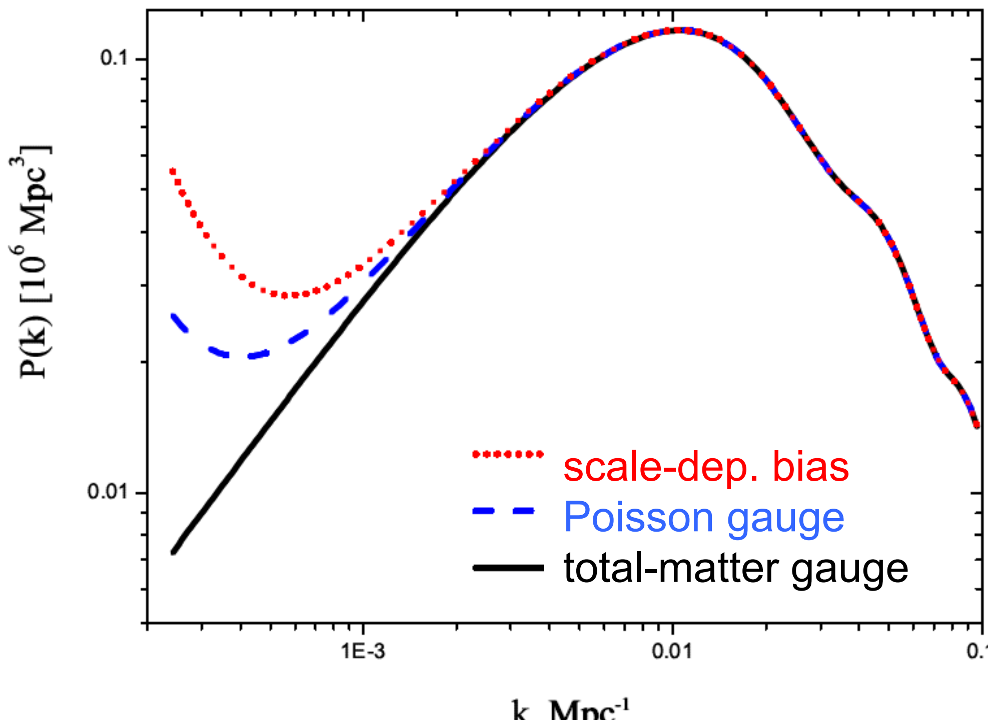
$$\delta_{\vec{k}}(z) = \int d\vec{x}\, e^{i\vec{k}.\vec{x}} C_1(\vec{x}) D_+(z)$$

linear transfer function through radiation era from Einstein-Boltzmann code

$$\delta_{\vec{k}}(z) = T_{\vec{k}}(z)\Phi_{\vec{k},\mathrm{ini}} = T_{\vec{k}}(z)\frac{3}{5}\zeta_{\vec{k}}$$

- Power spectrum





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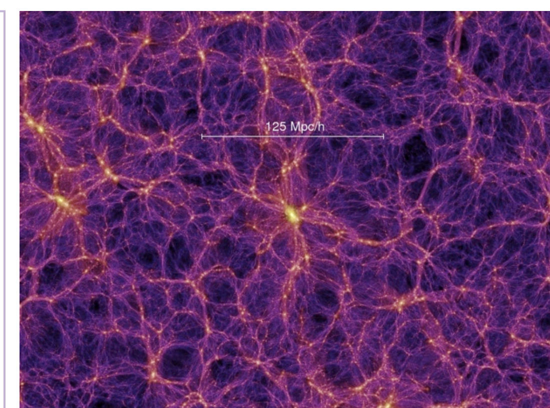
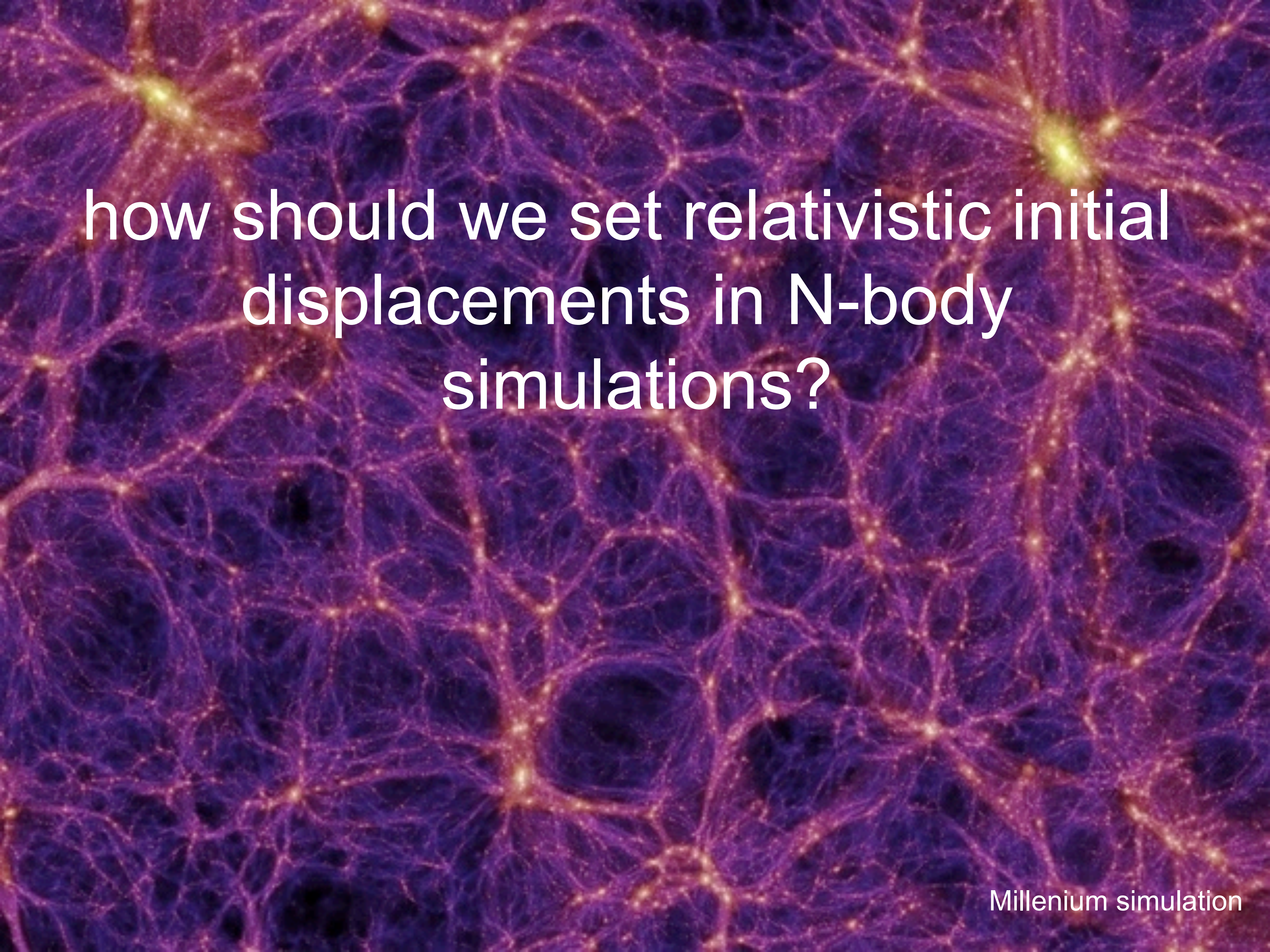


how should we set relativistic initial displacements in N-body simulations?



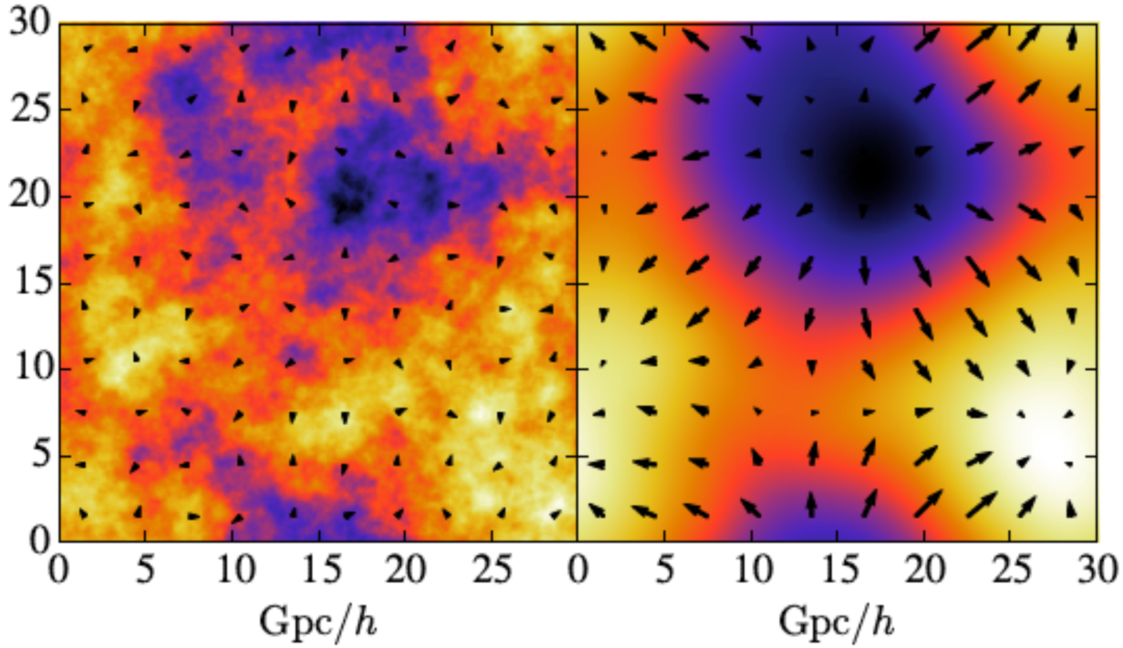
Millenium simulation

First-order displacement in total-matter gauge

Fidler, Rampf, Tram, Crittenden, Koyama & Wands, arXiv:1505.04756

$$\dot{\delta} = -\vec{\nabla}.\vec{v} - 3\dot{\mathcal{R}} \quad \Rightarrow \quad \vec{\nabla}.\vec{\Psi} = -\delta - 3\mathcal{R}$$

Newtonian displacement **_Relativistic displacement in total matter_**



GR volume distortion, $\mathcal{R}$, absent in Newtonian N-body density

$$\rho_{\rm N-body} = a^{-3} \sum_{\rm particles} m_p \delta^{(3)}(\vec{x} - \vec{x}_p) = \rho(1 + 3\mathcal{R})$$

first-order solution: **N-body gauge** *R=0*

Fluid equations for density contrast and velocity:

- Newton vs Einstein*

Newton:

$$\dot{\delta}_m + \vec{\nabla}.\vec{v}_m = 0$$

$$\dot{\vec{v}}_m + \mathcal{H}\vec{v}_m = -\vec{\nabla}\Phi$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta_m$$

Einstein*:

$$\dot{\delta}_m + \vec{\nabla}.\vec{v}_m = 0$$

$$\dot{\vec{v}}_m + \mathcal{H}\vec{v}_m = -\vec{\nabla}\Phi + \vec{\nabla}(\ddot{H}_T + \mathcal{H}\dot{H}_T)$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta_m$$

$$\nabla^2\dot{H}_T = -3\mathcal{H}\frac{c_s^2}{1+w}\delta_m$$

*in N-body gauge (comoving-orthogonal slicing, zero volume distortion)

$$\rho \equiv \rho_{\mathrm{N-body}} = a^{-3}\sum m_p \delta^{(3)}(\vec{x} - \vec{x}_p)$$

- **no volume distortion**
- Bardeen potential $\Phi = (1/3)\nabla^2 H_T + \mathcal{H}(\dot{H}_T - v)$

first-order solution: **N-body gauge** *R=0*

Fluid equations for density contrast and velocity:

- Newton vs Einstein* zero pressure

$$\dot{\delta}_m + \vec{\nabla}.\vec{v}_m = 0 \qquad \dot{\delta}_m + \vec{\nabla}.\vec{v}_m = 0$$

$$\dot{\vec{v}}_m + \mathcal{H}\vec{v}_m = -\vec{\nabla}\Phi \qquad \dot{\vec{v}}_m + \mathcal{H}\vec{v}_m = -\vec{\nabla}\Phi$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta_m \qquad \nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta_m$$

$$\nabla^2 \dot{H}_T = 0$$

*in N-body gauge (comoving-orthogonal slicing, zero volume distortion)

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beyond zero-pressure, linear matter?

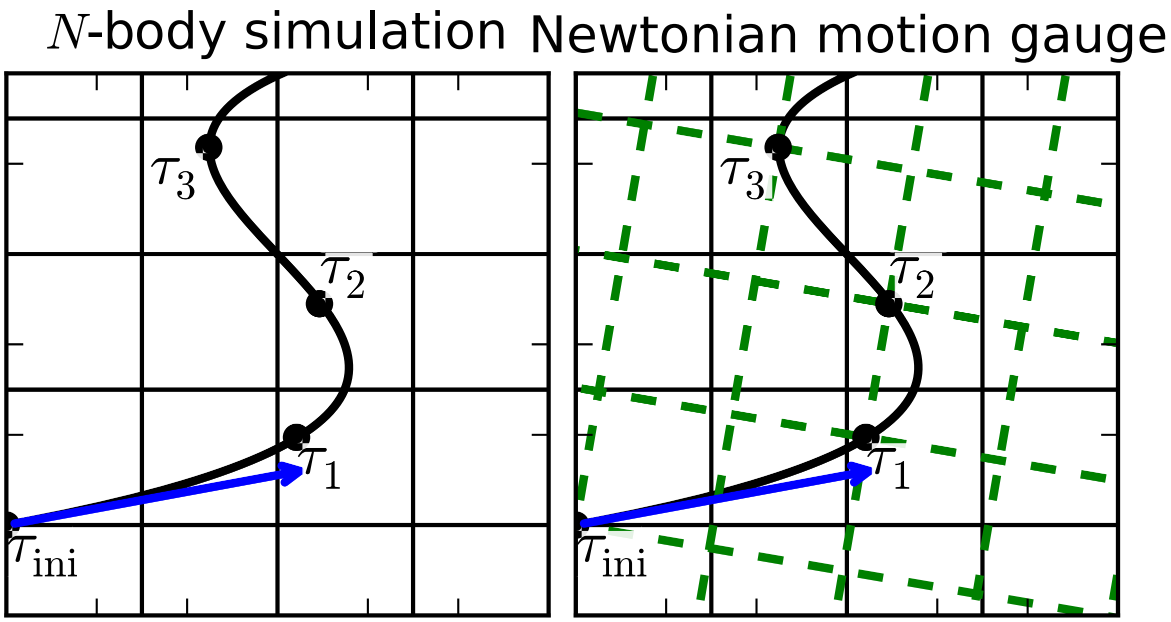
- *including radiation*

Newtonian motion gauges

Fidler, Rampf, Tram, Crittenden, Koyama & Wands, arXi:1606.05588

- in GR coordinates are arbitrary
- coordinates+metric define the physical spacetime
  - so… construct gauge such that
    Newtonian displacement = GR displacement

$$\Psi^{N}(t,\vec{q}) \equiv \Psi^{GR}(t,\vec{q})$$



Newtonian motion gauges

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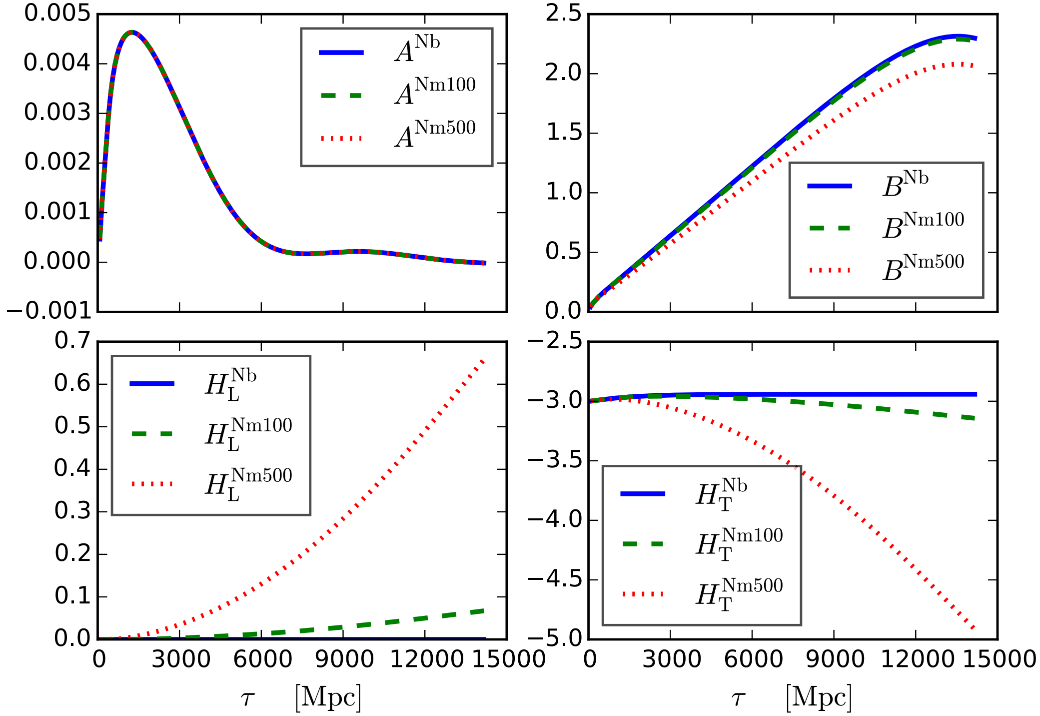
- **N-body simulations** solve Newtonian non-linear collapse for matter
- **Einstein-Boltzmann code** solves relativistic perturbation equations for metric potentials and radiation in Newtonian motion gauge



$$g_{00} = -a^2(1+2A)\,,$$

$$g_{0i} = -a^2 B_i\,,$$

$$g_{ij} = a^2\left[\delta_{ij}\left(1+2H_{\mathrm{L}}\right) - 2H_{\mathrm{T}\,ij}\right]$$

beyond zero-pressure, linear matter?

- *including radiation*
- *massive neutrinos**
- *dark energy**
- *second-order perturbations**

** left as an exercise for the reader*

beyond first order?

so, does Newtonian = GR for matter?

- **"even to the second order perturbations, equations for the relativistic irrotational flow… coincide exactly with the previously known Newtonian equations"**

  Hwang & Noh gr-qc/0412128

  - fluid flow evolution equations are the same in comoving-orthogonal time-slicing (e.g., total-matter gauge)
  - ***but*** there are ***non-linear constraints in GR***
    - there is no linear Poisson equation relating density to metric perturbations beyond first order
    - there are GR corrections to linearly growing mode at second- and higher-order

Second-order solutions in GR

Tomita (1975)… Bartolo, Matarrese and Riotto (2005); Bruni, Hidalgo, Meures & Wands (2014) …

Matter density contrast, δ, obeys second-order differential equation:

- first-order linearly growing mode:

$$L\{\delta^{(1)}\} = 0$$

$$\Rightarrow \delta^{(1)} = \underline{C_1(\vec{x})D_+(t)}$$

$$\text{constraint} : C_1 \sim \nabla^2\zeta_1$$

- second-order:

$$L\{\delta^{(2)}\} = Q\{(\delta^{(1)})^2\}$$

$$\Rightarrow \delta^{(2)} = \underline{C_2(\vec{x})D_+(t)} + \underline{P_2(\vec{x})D_{2+}(t,\vec{x})}$$

$$\text{constraint} : C_2 \sim \nabla^2\zeta_2 + \zeta_1\nabla^2\zeta_1 + (\nabla\zeta_1)^2$$

- *usual "Newtonian" solution is the particular solution: $P_2(x) \sim (C_1(x))^2$*
- *intrinsic non-linear GR constraint leads to non-trivial homogeneous solution: $C_2(x)$ (similar to primordial non-Gaussianity from inflation)*

Second-order solutions

Bernardeau et at (2001), …

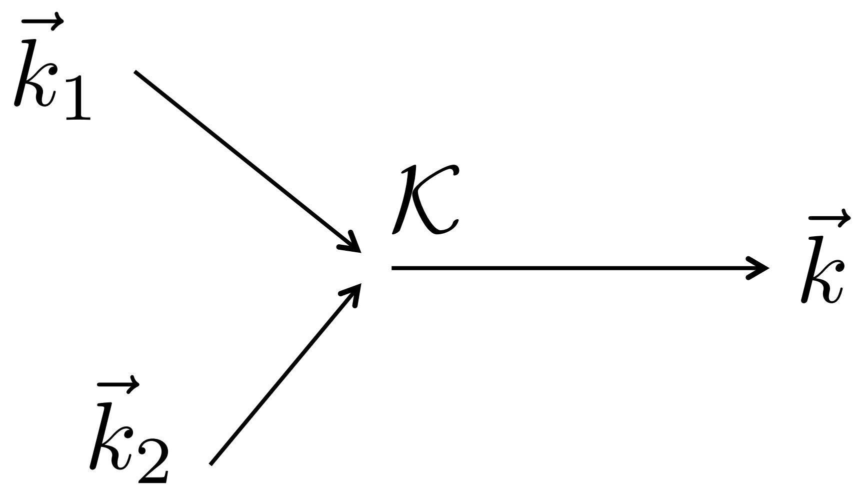
Fourier space

$$\delta_{\vec{k}} = \int d\vec{x}\, e^{i\vec{k}.\vec{x}} \delta(\vec{x})$$

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- kernel



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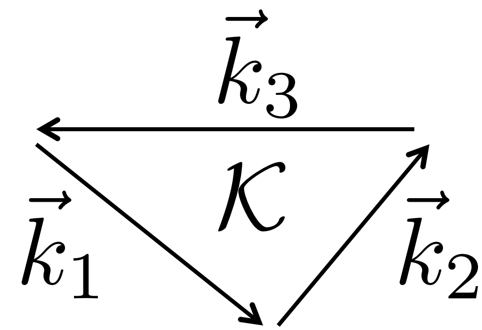
- reduced bispectrum



$$B(k_1, k_2, k_3) = P(k_1)P(k_2)\mathcal{K}(k_1, k_2, k_3) + 2 \text{ permutations}$$

Second-order *Newtonian* matter solution

Peebles (1980); Bernardeau et at (2002)

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- Newtonian kernel

$$\mathcal{K}_N(k, k_1, k_2) \equiv (\beta_N - \alpha_N) + \frac{\beta_N}{2} \frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2} \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right) + \alpha_N \left(\frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2} \right)^2 ,$$

  - Constant coefficients for matter-dominated (EdS) cosmology

$$\alpha_N = \frac{2}{7} , \quad \beta_N = 1$$

  - weak time-dependence at late times for ΛCDM cosmology
  - Newtonian kernel vanishes in squeezed limit, $k_2 << k_1$

Second-order *GR* matter solution

Bartolo, Matarrese & Riotto (2005); Bruni, Hidalgo, Meures & DW (2014); Uggla & Wainwright

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- GR kernel in total-matter gauge

$$\mathcal{K}(k, k_1, k_2) \equiv (\beta - \alpha) + \frac{\beta}{2}\frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) + \alpha\left(\frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2}\right)^2 + \gamma\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)^2$$

  - coefficients for matter-dominated cosmology

$$\alpha = \frac{2}{7} + \frac{5}{2}\frac{\mathcal{H}^2}{k^2}\,, \quad \beta = 1 - \frac{15}{2}\frac{\mathcal{H}^2}{k^2}\,, \quad \gamma = -\frac{5}{2}\frac{\mathcal{H}^2}{k^2}$$

GR corrections dominate at early times / large scales

Second-order *GR* matter solution

Bartolo, Matarrese & Riotto (2005); Bruni, Hidalgo, Meures & DW (2014); Uggla & Wainwright

Second-order convolution

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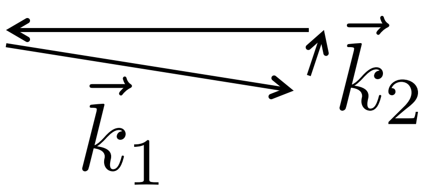
GR corrections non-vanishing in squeezed limit



$$k_2/k_1 \to 0\,, \quad \mathcal{K}(k, k_1, k_2) \to \gamma\left(\frac{k_1}{k_2}\right)^2 = -\frac{5}{2}\frac{\mathcal{H}^2}{k_2^2}$$

Second-order *GR* matter solution

Bartolo, Matarrese & Riotto (2005); Villa & Rampf (2015)

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- GR kernel *in Poisson gauge (adapted from Villa & Rampf (2015))*

$$\mathcal{K}(k, k_1, k_2) \equiv (\beta - \alpha) + \frac{\beta}{2}\frac{\vec{k}_1.\vec{k}_2}{k_1 k_2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) + \alpha\left(\frac{\vec{k}_1.\vec{k}_2}{k_1 k_2}\right)^2 + \gamma\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)^2$$

  - coefficients for matter-dominated cosmology

$$\alpha_{\rm P} = \frac{2}{7} + \frac{59\mathcal{H}^2}{14k^2} + \frac{45\mathcal{H}^4}{2k^4}, \quad \beta_{\rm P} = 1 - \frac{\mathcal{H}^2}{2k^2} + \frac{54\mathcal{H}^4}{k^4}, \quad \gamma_{\rm P} = -\frac{3\mathcal{H}^2}{2k^2} + \frac{9\mathcal{H}^4}{2k^4}$$

gauge transformation dominates at early times/large scales

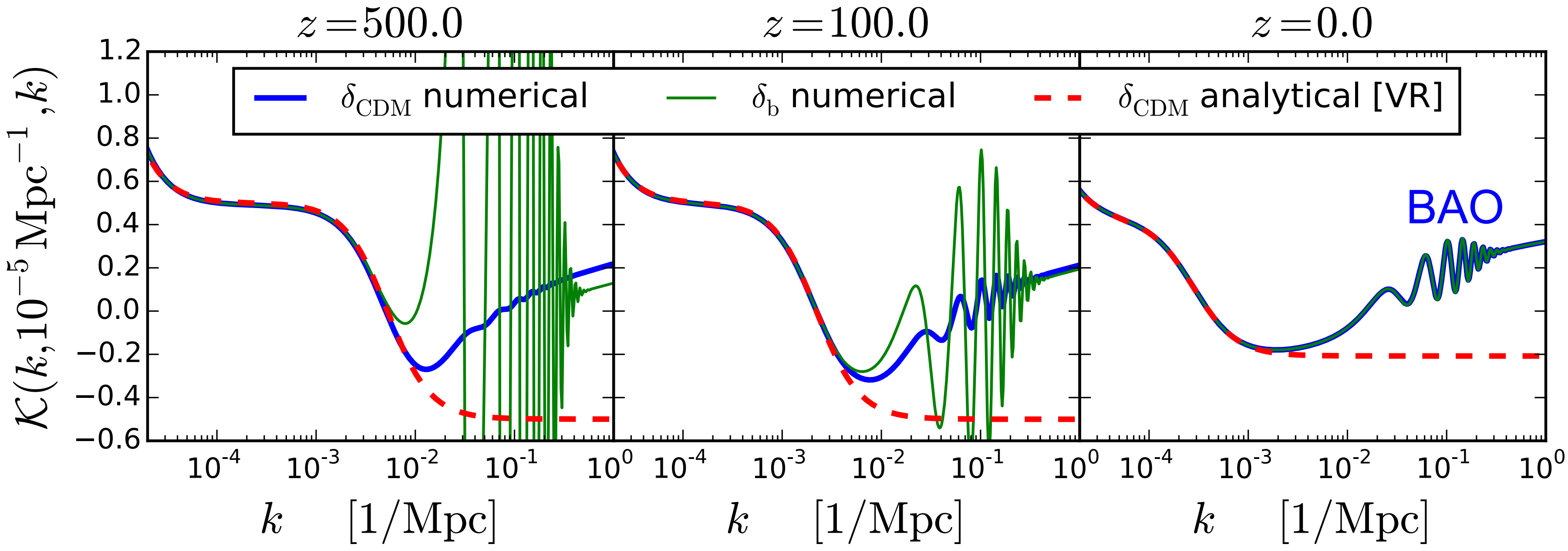
Second-order *Einstein-Boltzmann* solution

Tram, Fidler, Crittenden, Koyama, Pettinari & DW (2016)

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- Full numerical GR kernel in *Poisson gauge* using SONG*



**photon scattering simplest in Poission / conformal Newtonian gauge*

Second-order *GR* matter solution

Tram, Fidler, Crittenden, Koyama, Pettinari & DW (2016)

Second-order convolution

$$\delta^{(2)}_{\vec{k}} = \int \frac{d\vec{k}_1\, d\vec{k}_2}{(2\pi)^3}\, \delta^{(1)}_{\vec{k}_1}\, \delta^{(1)}_{\vec{k}_2}\, \mathcal{K}(k_1, k_2, k)\, \delta^D(\vec{k}_1 + \vec{k}_2 - \vec{k})$$

- GR kernel *in Poisson gauge*

$$\mathcal{K}(k, k_1, k_2) \equiv (\beta - \alpha) + \frac{\beta}{2}\frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) + \alpha\left(\frac{\vec{k}_1 . \vec{k}_2}{k_1 k_2}\right)^2 + \gamma\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)^2$$

  - coefficients for matter-dominated cosmology

$$\alpha_{\rm P} = \frac{2}{7} + \frac{59\mathcal{H}^2}{14k^2} + \frac{45\mathcal{H}^4}{2k^4}\,, \quad \beta_{\rm P} = 1 - \frac{\mathcal{H}^2}{2k^2} + \frac{54\mathcal{H}^4}{k^4}\,,$$

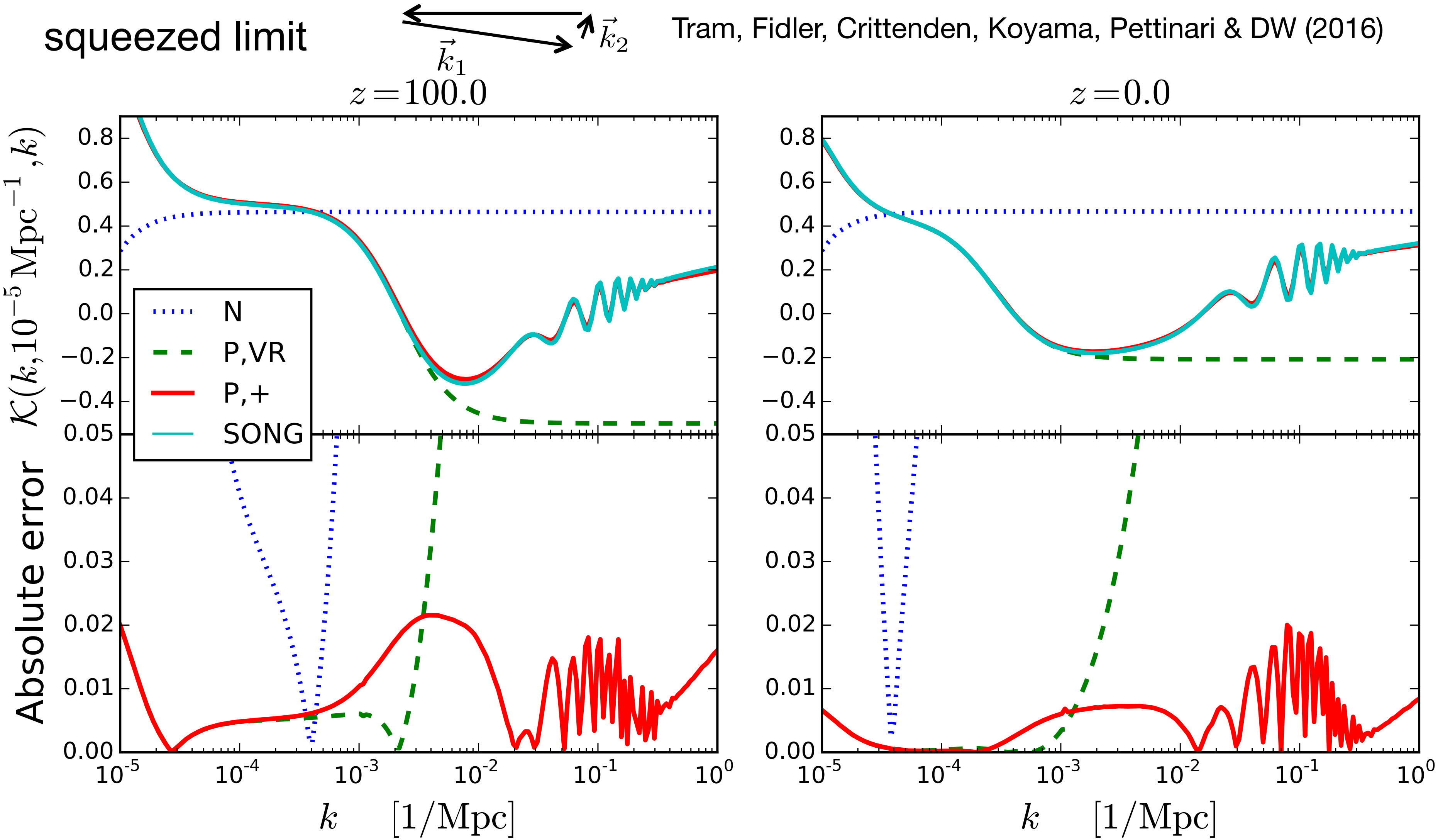
  - \+ squeezed limit from separate universe / peak-background split

$$\gamma_{P,\rm sq} = -\frac{3}{2}\frac{\mathcal{H}^2}{k^2} + \frac{9}{2}\frac{\mathcal{H}^4}{k^4} - \frac{5}{4}\left[\frac{\mathcal{H}^2}{k^2} + 3\frac{\mathcal{H}^4}{k^4}\right]\frac{\partial \ln T}{\partial \ln k}$$

Second-order *Einstein-Boltzmann* solution

squeezed limit

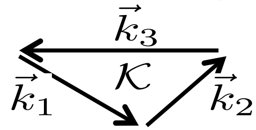
Tram, Fidler, Crittenden, Koyama, Pettinari & DW (2016)



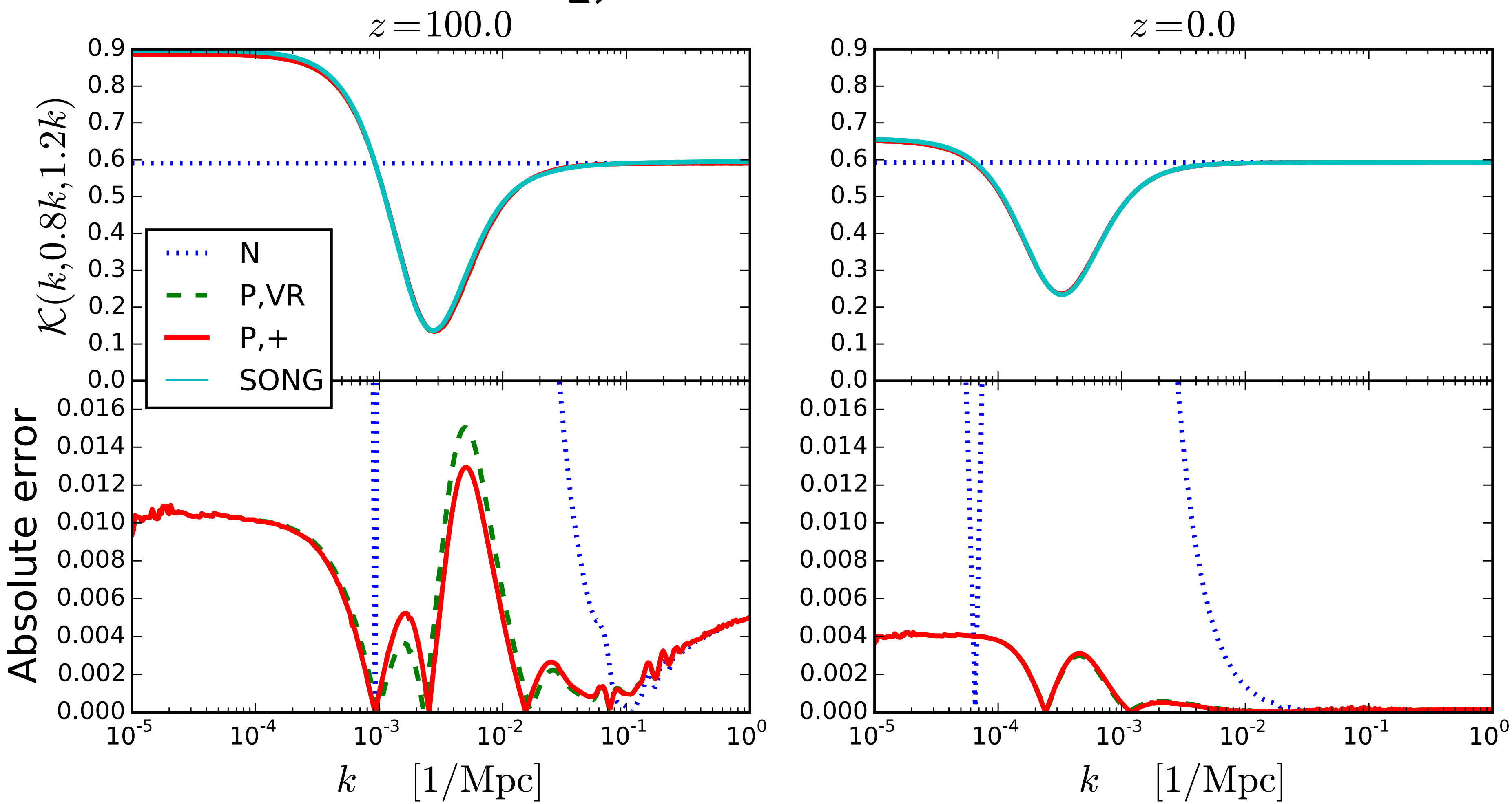
Newtonian (N), GR matter (P, VR) and improved GR (P,+) approximations

Second-order *Einstein-Boltzmann* solution

equilateral



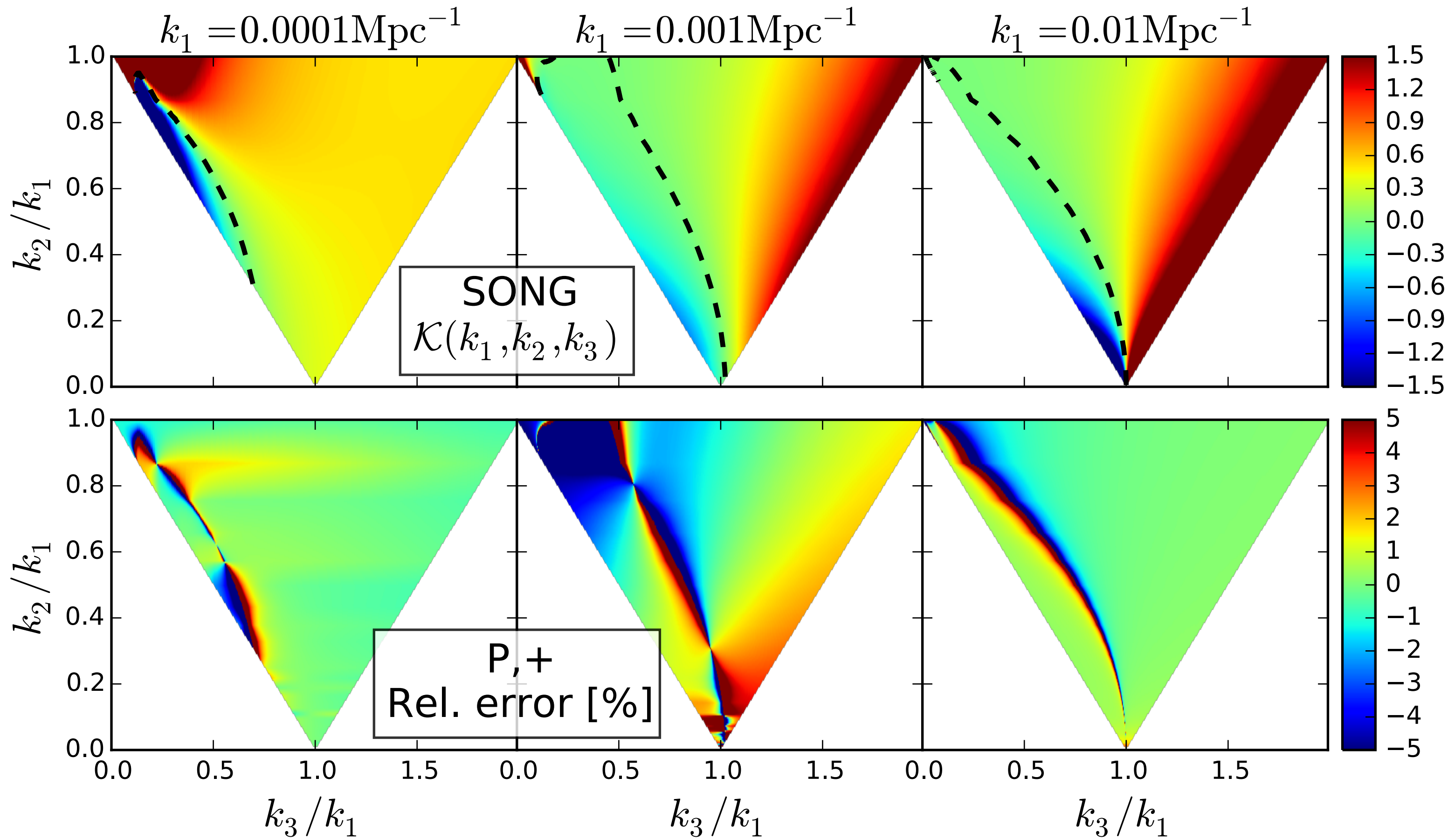
Tram, Fidler, Crittenden, Koyama, Pettinari & DW (2016)



Newtonian (N), GR matter (P, VR) and improved GR (P,+) approximations

Second-order *Einstein-Boltzmann* solution

Tram, Fidler, Crittenden, Koyama, Pettinari & DW (2016)



work in progress…

- **second-order initial conditions for N-body simulations**
  - modern simulations use “2LPT” (Newtonian second-order Lagrangian perturbation theory)
  - “GR2LPT” now exists in Poisson gauge
  - but need consistent second-order GR interpretation of N-body simulations in order to set GR initial conditions
- **other approaches**
  - incorporate Post-Newtonian corrections to modified N-body simulations in Poisson gauge (Adamek, Durrer, et al)
    - still linear (weak-field) on large scales
    - see also Millilo, Bruni, et al or non-linear formalism
- **must also include relativistic corrections along line-of-sight for observed angular power spectra…**

Conclusions

- **Newtonian cosmology works well**
  - needs consistent interpretation within GR

- **Einstein gravity** imprinted **in initial conditions**
  - **Gaussian metric perturbations** from inflation, $\zeta(x)$, generate **non-Gaussian matter distribution (bispectrum)**
    - **Newtonian non-linearity dominates at small scale / late times**
    - **GR non-linear corrections at large scale / early times**
  - need consistent **GR interpretation of N-body simulations**
  - **observations** also introduce significant non-linearities
    - need angular bispectrum of galaxies, shear, etc, in redshift space
  - GR effects could provide a target for future surveys