Environmetrics

Special Issue Paper

Received: 2 April 2014, Accepted: 7 April 2014, Published online in Wiley Online Library: 8 May 2014

(wileyonlinelibrary.com) DOI: 10.1002/env.2278

Lightning-caused forest fire risk in Northwestern Ontario, Canada, is increasing and associated with anomalies in fire weather

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Results from studies of climate model scenarios suggest that forest fire ignitions will increase in Canada in the future because of climate change. Yet, there have been few studies that monitor long-term trends in Canadian historical fire records. Although there are seasonal trends to historically reported fires within a fire season, there are also periods of zero-heavy behaviour as well as periods during which more fires are reported than usual. We develop a flexible mixture-modelling framework that permits the joint assessment of temporal trends in these dominant characteristics in terms of fire risk, defined as the daily probability that one or more fires are reported. The statistical power of such trend tests are also evaluated. We identify statistically significant increases in lightning-caused fire risk between 1963 and 2009 in the boreal forest regions of the Rainy River and Lake of the Woods ecoregions in Northwestern Ontario, Canada. These observed changes in lightning-caused fire risk were found to be associated with temperature and fire danger rating index anomalies. If such trends continue into the future, the duration of elevated periods of lightning-caused forest fire risk is forecasted to increase by over 50% by the middle of this century. Copyright © 2014 John Wiley & Sons, Ltd.

Keywords: climate change; ignition; logistic GAM; power; wildfire; wildland fire; zero heavy

1. INTRODUCTION

The seasonal nature of wildfire ignitions has been documented in many parts of the world where fire is a natural disturbance on the landscape, including regions of North and South America (Albert-Green *et al.*, 2013; Brillinger *et al.*, 2006; Di Bella *et al.*, 2006; Martell *et al.*, 1989; Preisler *et al.*, 2004; Woolford *et al.*, 2009, 2010), Australia (Craig *et al.*, 2002; Williams *et al.*, 1998), Asia (Seol *et al.*, 2012), Africa (Langaas, 1992) and Europe (Vilar *et al.*, 2010). How climate change may impact forest fire regime characteristics is of concern, as increasing temperatures could increase the frequency of severe fire weather, extend the fire season or lead to increases in the number of fire ignitions (Weber and Stocks, 1998). Some of these changes have been documented recently: increased frequency and duration of large fire activity have been associated with warmer temperatures and earlier snowmelts in the western United States (Westerling *et al.*, 2006), and the forest fire season has been lengthening in the provinces of Alberta and Ontario, Canada (Albert-Green *et al.*, 2013).

However, most forest fire research in this context has been prospective. Numerous studies have analysed data simulated by global climate models. Many of these studies forecast potential impacts on weather-based fire danger indices or characteristics of fire regimes under various scenarios. For Canada's boreal forests, such studies have suggested increased severity ratings (e.g. Flannigan and Van Wagner, 1991; Stocks *et al.*, 1998), area burned (e.g. Balshi, *et al.*, 2009; Flannigan *et al.*, 2005; Podur and Wotton, 2010), ignitions (Wotton *et al.*, 2003, 2010) and a lengthening of the fire season (Wotton and Flannigan, 1993).

Our focus here differs in that it is retrospective. We address the question of whether there is any evidence of trends across years in historical reports of lightning-caused forests fires. The analysis is complicated by the need to account for three dominant characteristics that are typically observed in such records: regular seasonal patterns and two sharp deviations from this pattern. To define these deviations from the general seasonal pattern, we refer to periods during which there are more fires reported than usual as *extreme* behaviour, while periods

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during which no fires are reported correspond to *zero-heavy* behaviour. Based on a large database that contains nearly a half century of fire reports (1963–2009) observed in a 3.4 million hectare portion region of the boreal forest region of Ontario, a province in central Canada, we investigate the evidence of possible temporal changes in reported patterns of natural (i.e. lightning-caused) forest fires. We also link the prevalence of extreme fire risk to temperature and other weather-related anomalies over this period. Our analysis is unique in that it focuses on the question of precisely what empirical evidence there is of trends over time for each of these dominant characteristics and with what statistical power such trends can be assessed. Previous exploratory analyses (Woolford *et al.*, 2010) found that the probability of observing lightning-caused fires had increased over time in this region, and extensions of that work designed to capture the main elements of the data have led to the consideration of the flexible, mixture-based modelling framework presented in this paper.

We employ a finite mixture of penalized-spline-based logistic generalized additive models (GAMs) with component-specific mixing probabilities that are allowed to vary over time. The structure of our mixture-model framework model accounts for the fire and non-fire events commonly reported in historical records.

It is important to note that the focus of our work is driven by the collaborative forestry context, which sets the questions for our investigation using statistical science (e.g. Nelder, 1999). Hence, this paper devotes considerable discussion to the forest science and management contexts, and their relationships to the statistical tools employed. Questions important to fire science and fire management are as follows: have rates of fire ignitions changed over time, are these changes related to a warming climate and what might happen in the future? Our modelling reveals a statistically significant shift towards an increased risk of lightning-caused fires during the fire season in our study area. This shift was also found to be associated with anomalies in temperature and forest floor moisture indices that are indicators of the receptivity of the forest floor to ignition by lightning. Should these trends continue, periods of peak lightning-fire risk are forecasted to increase by over 50% before the end of the current century.

In addition, we conduct a comparative power study of Wald, score and permutation-based hypothesis tests in the context of our modelling framework, focusing on determining how many years of data are required to be reasonably confident when concluding estimated trends are significant. This concern appears to be a key, yet commonly overlooked, point in many quantitative scientific investigations of trends that may be related to climate change.

The paper is organized as follows. Our modelling framework and the data we analyse are described in the next section. Then, in Section 3, we present the fitted model, assess its goodness of fit and make connections between the probability of extreme behaviour and observed anomalies in fire weather. Section 4 compares the power of three different hypothesis tests for testing for significant increasing trends in the probability of extreme component membership. We conclude with a discussion that touches on potential future impacts if the estimated trends were to continue and comments related to how this might impact fire management.

2. TRENDS IN FIRE RISK

We use the terms ‘fire risk’ and ‘risk’ to refer to the probability of a lightning *fire day*, which we define to be a day during which one or more lightning-caused forest fires are reported. In Ontario, the risk of lightning-caused fires varies throughout the fire season, which normally begins on 1 April and ends on 31 October as stipulated in Ontario’s Forest Fires Prevention Act R.S.O. (1990). The seasonal nature of fire risk in Ontario is characterized by a curve that begins at zero risk in the early spring, increases to a period of peak risk in midsummer and then decreases back to zero at the end of the fire season (e.g. Albert-Green *et al.*, 2013; Woolford *et al.*, 2009). This seasonal behaviour is a trend that reflects how the proportion of days with fires within a narrow time window varies, on average, throughout the fire season.

As we noted in our introduction, apart from such regular seasonal behaviour, there are two other dominant patterns observed in the historical fire reports that we characterize as follows: (1) extreme behaviour, periods with far more fire days than expected at a given time of year, and (2) zero-heavy behaviour, periods during which there are days with no fires reported. Incorporating these three observed components into a model requires a framework that is reflective of such hidden clustering.

An additional issue that needs to be addressed is potential confounding effects. Woolford *et al.* (2010) noted that although historically observed lightning-fire risk appeared to be increasing in this study area, fire detection effectiveness may have improved over time. Hence, any association with a warming climate could be confounded with improvements in detection. Both changes in detection effectiveness and potential climate effects could lead to a reduction in zero-heavy behaviour as well as increase the frequency where extreme behaviour in the frequency of fire days would be observed.

Here, rather than modelling counts of fires, we model the probability or risk of a lightning-caused fire day. We do so because changes to detection effectiveness have less of an impact when modelling trends in fire days versus trends in counts of fire events: only the risk of presence versus absence of fires on the landscape is being quantified. Hence, when fires occur, changes to detection effectiveness will mainly affect the number of fires recorded more so than the presence/absence of fire on the landscape.

However, there is a loss in power when we move from the analysis of count data to working with a presence/absence response. Hence, the assessment of power will be an important consideration in this report.

We employ a finite-mixture-model framework (e.g. McLachlan and Peel, 2000) that consists of zero-heavy, regular seasonal and extreme seasonal fire risk components. Zero-heavy behaviour is used as our baseline, to which the relative changes in regular and extreme behaviour over time are assessed. The log-odds of membership in the regular component, and in the extreme component, relative to the zero-heavy component are modelled as a linear function of time, permitting us to test for significant trend shifts out of the zero-heavy behaviour towards either regular or extreme behaviour. Risk curves for the seasonal trends in regular and extreme behaviour are estimated as non-linear effects in component-specific logistic GAMs (e.g. Wood, 2006). These risk curves are estimated using penalized splines, which produce smooth curves, the shapes of which are driven by locally observed trends in the data. The zero-heavy component has a degenerate distribution with a point mass at 0: if the process is in the zero-heavy component, then no fires are observed. This could occur from a fire being ignited and then extinguishing naturally without being detected or a lack of lightning during a period when conditions are favourable for ignitions to occur.

2.1. Modelling framework

Define a lightning *fire day* as a day during which at least one lightning-caused fire was reported and let X_t be the number of fire days during period t, where $t = (w, y)$ with w denoting week and y denoting year. Let the zero-heavy, regular and extreme seasonal behaviour components be denoted by 0, R and E, respectively. We model the random variable X_t using the following mixed binomial distribution

$$X_t \sim \pi_0(y)\,\mathrm{Bin}(7, p_0 = 0) + \pi_R(y)\,\mathrm{Bin}(7, p_R(w)) + \pi_E(y)\,\mathrm{Bin}(7, p_E(w)) \tag{1}$$

where $p_R(w)$ and $p_E(w)$ are penalized spline functions used to model the component-specific ignition risk curves for the regular and extreme seasonal behaviours. The time-varying mixing probabilities of membership in the zero-heavy, regular and extreme seasonal components, $\pi_0(y), \pi_R(y)$ and $\pi_E(y)$, sum to 1 and are parametrized as

$$\mathrm{logit}\left(\frac{\pi_j(y)}{\pi_0(y)}\right) = \alpha_j + \beta_j y \ , \quad j = R, E \tag{2}$$

Hence, we model the number of fires at time t with a three-component mixture of logistic GAMs. The three components reflect the three dominant characteristics commonly observed in the historical fire records: the regular component accounts for 'normal' seasonal behaviour in terms of presence/absence of fires on the landscape; the zero-heavy component accounts for 'quiet periods' where no fire days are observed, even though fires may be expected; and the extreme component accounts for periods where far more fire days are observed than usual. The mixing proportions for these three components are modelled using a multinomial logistic regression model. Recall that the trend we are trying to isolate is one that could lead to either (or both) a reduction in the number of zeros or an increase in the number of extremes. Consequently, we use membership in the zero-heavy component as our baseline and consider the odds of regular to zero-heavy behaviour and the odds of extreme to zero-heavy behaviour because, if both the numerator and denominator terms change slowly, considering such ratios will lead to large joint effects, which should be easier to detect.

Parameters are jointly estimated through the maximization of the penalized complete likelihood function using a variant of the expectation maximization (EM) algorithm (Dempster *et al.*, 1977). Wood (2006) provides a thorough overview of GAMs parameterized as penalized splines and their estimation. The EM algorithm for a finite mixture of generalized linear models is discussed by McLachlan and Peel (2000). That estimation routine extends to our finite mixture of GAMs. Standard errors for these curves are estimated via a parametric bootstrap (e.g. Davison and Hinkley, 1997). Computations were performed in R (R Core Team, 2012).

As a brief aside to conclude the description of our modelling framework, we note that a single logistic GAM that models the log-odds of fire risk as a smoother of week and year may not be adequate for quantifying historical trends. When experimenting with such a model, diagnostic plots had strong runs of positive residuals as well as runs of negative residuals at various periods within each year, suggesting the use of the mixed binomial distribution we employ. In our concluding discussion, we provide some further commentary on the merits of using a single versus mixed binomial framework. In particular, we find that the aims of the analysis appear to dictate which framework should be used. Our mixture-modelling framework is motivated by the need to monitor trends in historical fire risk, while single-binomial models are

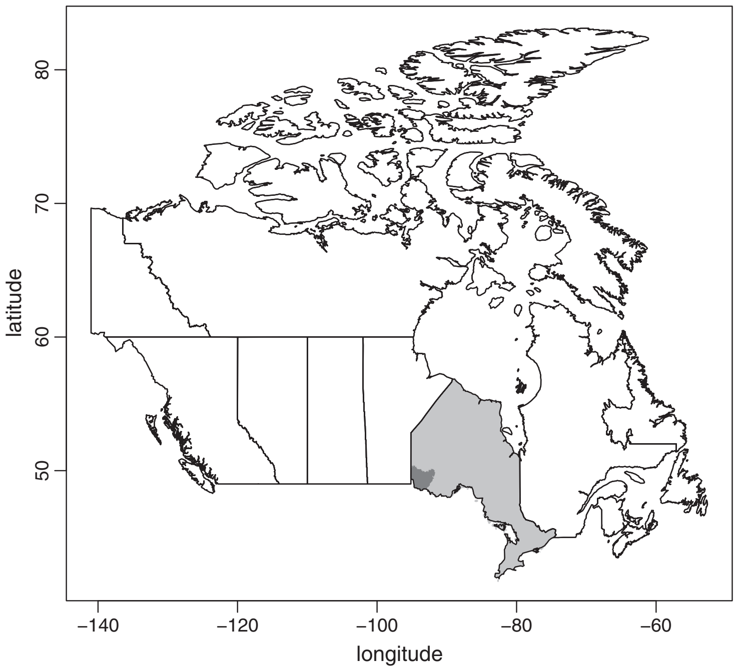

Figure 1. A map of Canada illustrating the location of our study area (shaded dark grey) within the province of Ontario (shaded light grey)

useful when the focus is the development of a decision support tool that forecasts fire risk as a function of other locally observed covariates for use by fire management agencies.

2.2. The data and study area

Figure 1 illustrates the location of our study area, a 3 371 186 ha region of boreal forest in Northwestern Ontario, Canada. The study area consists of the Rainy River and Ontario portion of the Lake of the Woods ecoregions. Ecoregions are areas that are reasonably homogeneous with respect to their ecological characteristics, such as climate, vegetation and geography (Ecological Stratification Working Group. 1996). Historically, the Ontario Ministry of Natural Resources (OMNR) has actively detected and suppressed most forest fires in this region since the early 1900s. Consequently, the study region can be assumed to be relatively homogeneous with respect to both fire management and ecological characteristics.

The data were provided by the Aviation, Forest Fire and Emergency Services of the OMNR. These consist of reports of all lightning-caused forest fires in the study region that were reported to the OMNR during Ontario's official fire season (1 April through 31 October of each year) for the years 1963–2009. There were 1986 days where there was at least one lightning-caused forest fire reported in the study region during this period. We use the date a fire was reported to the OMNR as the time of the fire, because it is more reliable than the date of ignition recorded, which in many cases is estimated for lightning-caused fires.

3. RESULTS

Prior to presenting the fitted model, we note that during the first week of the 1977 fire season, there were seven consecutive fire days. This was an extremely rare event that was related to extreme drought conditions. Stocks (1979) noted 'substantial precipitation deficits had developed over much of the province' leading up to April of 1977. By the start of the April 1977 fire season, the 2-year precipitation deficit as measured at the Kenora, Ontario, weather station (which is located near the centre of our study area) was 404.6 mm below normal (Stocks, 1979, Table 1).

Those seven consecutive fire days observed during the first week of April in 1977 turned out to be a highly influential outlier. There are usually no fires during the first week of April each year. That point was influential, because estimation of the component-specific ignition curves involves weighted maximization and these weights are the posterior probabilities of membership in each of the components. The first week of 1977 was the only observation at the start of any of the fire seasons that was assigned a non-zero posterior probability of membership in the extreme component. Consequently, it led to unreasonably larger values in the extreme fire risk curve at the start of the fire season from the forestry perspective. Because that single observation was such an influential outlier, we excluded it from our analysis presented herein.

3.1. The fitted fire risk curves

Figure 2 displays the estimated seasonal fire risk curves for regular and extreme behaviours along with 95% confidence intervals. Regular seasonal behaviour is as expected: historically, fire risk increased in the spring, peaked in the summer and then decreased in the fall. This pattern reflects the seasonal nature of Ontario's climate, which gets warmer through the months of June, July and August, in combination with the seasonal patterns in cloud-to-ground lightning strikes, the majority of which occur during these summer months (Wotton and Martell, 2005, Figure 3a; Burrows and Kochtubajda, 2010, Figure 2b). The seasonal variation in the risk curve for the extreme component is essentially two values: low at the start and end of the fire season and high otherwise.

Figure 2 also shows the changes over the study period of the component-specific mixing probabilities as defined by Equation (2). There has been a strong decline in the probability that the system is in the zero-heavy component, 38% in 1963 to 26% in 2009 (more than a 25% reduction). This decrease in zero-heavy behaviour is dominated by a shift towards a higher likelihood of regular seasonal behaviour (46.6% in 1963 to 57.9% in 2009). However, the remaining small portion of the decrease in zero-heavy behaviour is attributed to a shift towards a

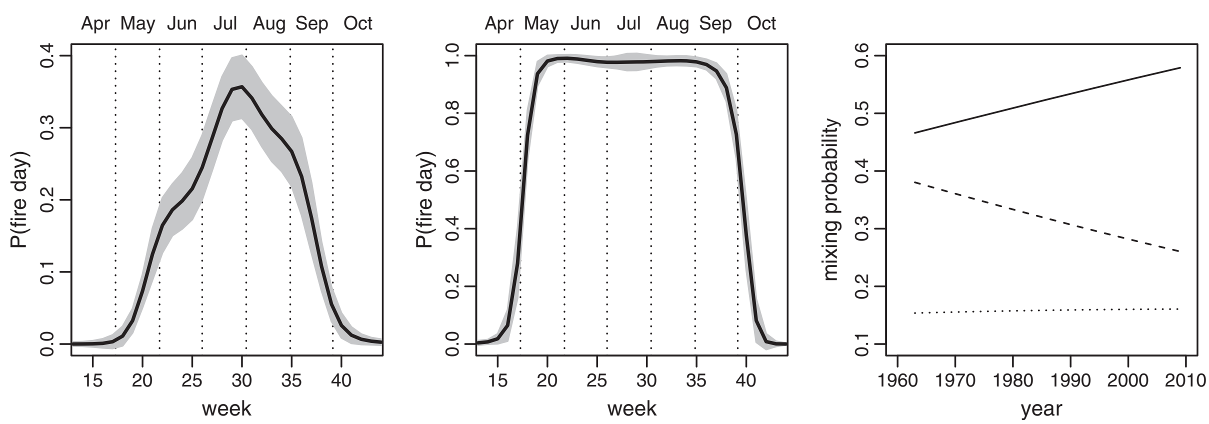

Figure 2. Left and middle panels: the estimated component-specific seasonal fire risk curves and pointwise 95% confidence intervals for the regular (left) and extreme (middle) components. Right panel: the estimated mixing probabilities for the regular (solid), zero-heavy (dashed) and extreme (dotted) components

Table 1. Estimates along with corresponding standard errors, Wald-based test statistics and p-values corresponding to the parameters for trends in the mixing probabilities as defined by Equation (2)

Parameter	Estimate	Standard error	Wald statistic	p-value
α_R	0.1904	0.1881	1.0118	0.1558
β_R	0.0129	0.0075	1.7329	0.0416
α_E	−0.9153	0.1858	−4.9261	≈0
β_E	0.0092	0.0072	1.2820	0.0999

The p-values for the slope parameters, β_R and β_E correspond to testing the null hypothesis of no slope on the log-odds scale versus a one-sided alternative that the slope is positive.

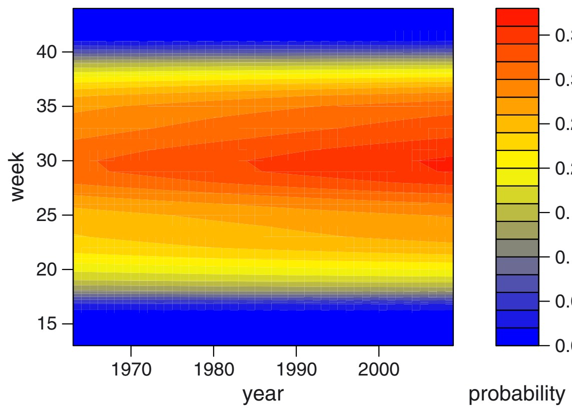

Figure 3. Contour plot of the estimated temporal changes in historical fire risk in the study area over the period 1963–2009. Here, fire risk is defined as the probability of a lightning-fire day, a day during which one or more lightning-caused fires are reported

higher likelihood of extreme behaviour, which experienced a 4.5% relative increase over these 47 years. The estimated probability of extreme behaviour in 2009 was 16.1%. Table 1 presents the parameter estimates, their standard errors and the results of standard (i.e. Wald type) hypothesis tests for significance. The shift towards increased regular seasonal behaviour was significant (p − value $= 0.0416$). The shift towards increased extreme seasonal behaviour was marginally significant (p-value $= 0.0999$).

The overall temporal trends (both within and across years) in historical fire risk are visualized by combining all of the jointly estimated components to produce the risk contours presented in Figure 3. There, it is clear that the risk of lightning-caused forest fires has been increasing over this period, especially peak historical risk, which increased from 31.7% in 1963 to 36.4% in 2009—a 15% relative increase in peak risk over the 47 years in our study.

3.2. Goodness of fit

The model fits the data reasonably well. Figure 4 examines the goodness of fit by comparing observed counts versus those expected under the model for each of the three components. Observed trends in the weekly proportions of fire days, weighted by their respective probabilities of membership in the regular and extreme components, are overlain on the component-specific risk curves in the first two panels of Figure 4; these follow the individual risk curves very closely. In terms of the zero-heavy behaviour, the trends in predicted and observed counts of excess zeros are also in agreement: the right-most panel in Figure 4 visually compares the expected number of zeros from the zero-heavy component and the empirical number of excess zeros on an annual basis. There, the empirical number of excess zeros is the number of observed zeros minus the number of zeros expected to arise from both the regular and extreme seasonal components.

3.3. Associations with anomalies in temperature and fire-weather indices

Fire ignition risk is widely understood to be influenced by the number of potential ignition sources (in this case, the number of lightning strikes) as well as the moisture content of forest fuels. Weather or weather-based fire danger rating variables were not included as possible predictors in our model because our focus was on quantifying temporal changes in historical fire risk using a modelling structure that was representative of patterns commonly observed is historical fire reports.

We did, however, examine annual trends in two main environmental variables that may have a physical link to increased ignitions: average seasonal air temperature and organic layer moisture. Air temperature has been found to be correlated with area burned (Flannigan *et al.*, 2005) which can, in the case of lightning fire, be physically attributed to the influence of air temperature on both drying forest fuels and, on a

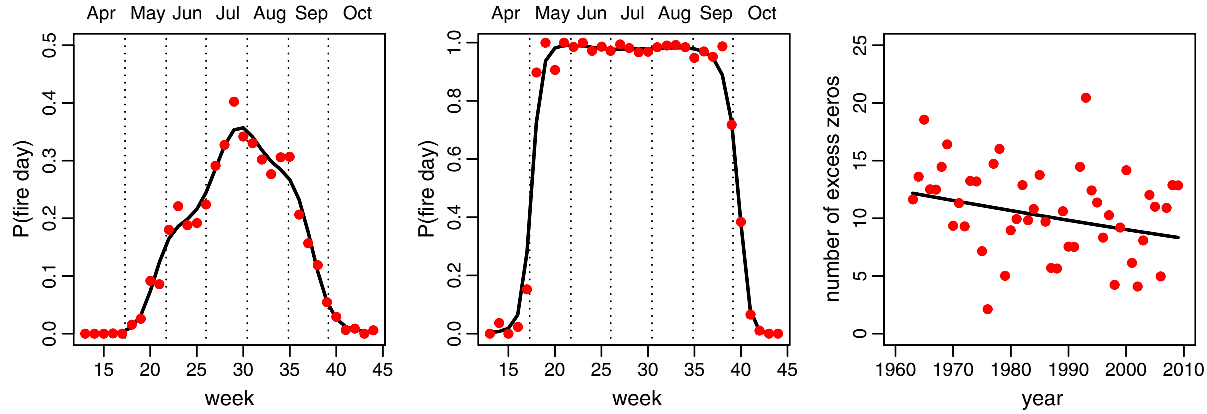

Figure 4. The left and middle panels plot the estimated component-specific fire risk curves for the regular (left) and extreme (middle) components. Overlaid on each of these curves are the observed empirical weighted proportion of the number of fire days per week over all years, where the observed data were weighted by the posterior probabilities of membership for the corresponding regular or extreme component. The right panel compares observed and expected frequencies of excess zeros. The solid line is the expected number of zeros from the zero-heavy component plotted versus year. The points are the empirical number of excess zeros: the number of observed zeros minus the number of zeros expected to arise from both the regular and extreme seasonal components

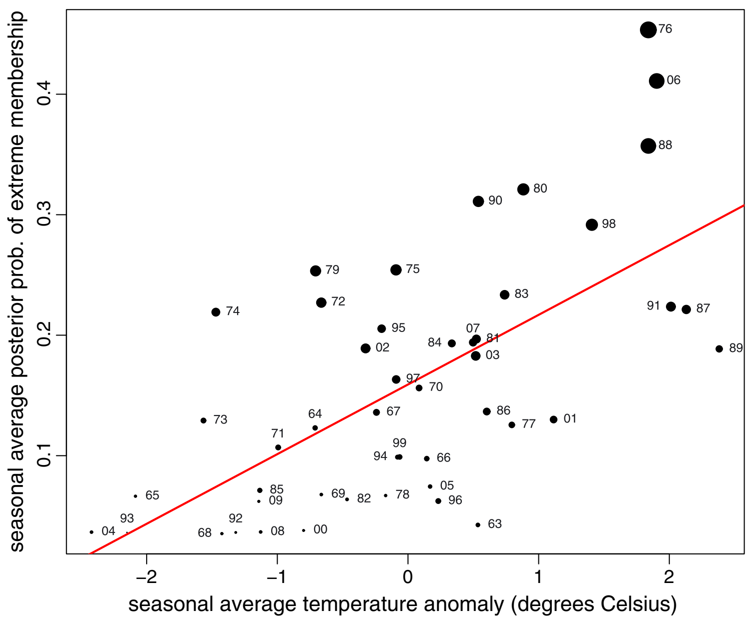

Figure 5. Seasonal averages of the posterior probability of extreme component membership versus seasonal average temperature anomalies. Each point is labelled to indicate the year, and the size of each point is proportional to the number of lightning-caused fires reported in the study area that year. The trend is highlighted by the simple linear regression line overlaid on the scatterplot

larger scale, the lightning activity. To explore these variables, we analysed daily fire-weather streams from the Kenora, Ontario, OMNR fire-weather station. This location was used because it is situated near the centre of our study area and it is a long-established OMNR weather station, containing records that spanned the entire study period.

In Figure 5, we explore the relationship between results from our model and temperature using a second-stage analysis that utilizes posterior probabilities from the previous fit. The average seasonal (i.e. within the fire season) posterior probability that the process is in the extreme component is plotted against the seasonal average temperature anomalies for each year. Here, a temperature anomaly is the observed difference between a day's recorded temperature and the historical average for that given day over the study period (i.e. 47 years). Each point is labelled to indicate the year, and the size of each point is proportional to the number of lightning-fire days observed that year. The probability of extreme behaviour, which contributes to increases in historical fire risk, is positively correlated with anomalous temperatures, and the association is highly significant: during years where it is warmer than average, the fire signal is more likely to be in the extreme component. Moreover, as highlighted by the size of each point in this figure, during years when it is warmer than average, the presence of fire ignitions on the landscape increases. For example, in 1976, which was the worst year on record, there were 105 days during the fire season where at least one new fire was reported. Observations for that year have the highest average posterior probability of membership

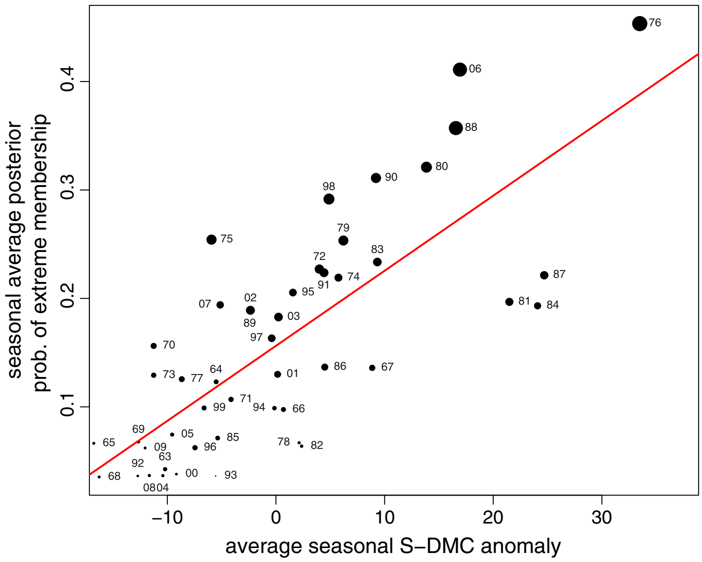

Figure 6. Seasonal averages of the posterior probability of extreme component membership versus seasonal anomalies in the Sheltered Duff Moisture Code (SDMC). Each point is labelled to indicate the year, and the size of each point is proportional to the number of lightning-caused fires reported in the study area that year. The trend is highlighted by the simple linear regression line overlaid on the scatterplot

in the extreme component, and daily temperatures were 1.84 °C warmer on average. It is widely accepted that climate change is leading to warmer temperatures. A simple linear model was fitted to the daily temperature anomaly data, resulting in a positive slope estimate (0.0118) that was significant (p-value $= 0.0050$). This result suggests that temperatures during the fire season have also been increasing over the course of our study period in our study area.

In terms of the receptivity of forest fuels to ignitions, increases in temperature can be offset by increases in rainfall. Thus, examining trends in moisture content of key forest fuels over the study period seemed useful. Fire management agencies throughout Canada have for several decades used the Duff Moisture Code (DMC) element of the Canadian Forest Fire Weather Index System (Van Wagner, 1987) to estimate the potential for lightning-fire ignition after lightning storms (Wotton, 2009). Wotton *et al.* (2005) recently modified that moisture code to improve its ability to track day-to-day changes in moisture in the key locations of the forest floor where lightning fires ignite and holdover, namely in the loosely compacted organic material near the trunks of dominant overstorey trees in a given stand of forest. Wotton and Martell (2005) found that the log-odds of lightning ignition risk are strongly associated with this new code, the Sheltered DMC (SDMC), and noted that it was a better predictor of lightning-fire risk in Ontario than the standard DMC. Figure 6 presents an analysis of the relationship between the posterior probability of extreme component membership and anomalies in the SDMC. Similar to the results for the temperature anomalies, there is a strong positive association, with more lightning fires occurring when the SDMC is anomalously high, on average. It is important to note that higher SDMC values indicate a drier forest floor. The scale of the moisture code value is the inverse of actual moisture.

4. POWER STUDY

The estimated temporal trends, as visualized in Figures 2 and 3, suggest that there have been changes to patterns in historical fire risk in this area over the course of our study period, and the results in Table 1 suggest that these trends are statistically significant. Here, simulation is used to assess how many years of data are required to be reasonably confident that our model would correctly detect such an increasing trend in the probability of membership in extreme component. Power curves, as a function of the length of the study period, for three different hypothesis tests related to the slope parameter, β_E, are estimated and compared.

The p-values in Table 1 were calculated using Wald-based hypothesis tests, which assume that the underlying sampling distributions approach a normal distribution as the size of the data set increases. However, different methods of testing for statistical significance in the trends for the proportion of time the system spent in these states can be employed. Besides the Wald test, score and permutation tests were also considered when testing for a shift towards increased extreme component behaviour. The score test statistic is the ratio of the score function (the log-likelihood) to Fisher's information (its standard deviation), evaluated under the null hypothesis. This statistic is also asymptotically normally distributed under the null hypothesis. In contrast, the permutation test is a non-parametric method. Here, the response variables (or equivalently their covariates) are randomly rearranged, and the model is fitted to these pseudo-data. The permutation and fitting steps are repeated a large number of times, and the distribution of the resulting set of parameter estimates numerically approximates the sampling distribution of the statistic of interest under the null hypothesis. Our null hypothesis is that there are no changes across years. Consequently, we employ a block resampling approach where the order of observations within each fire season is kept intact. For more details on permutation tests and block resampling, see Davison and Hinkley (1997), and for another application of this approach in the context of forest fires, see Albert-Green *et al.* (2013).

Table 2. A comparison of p-values from Wald-based, permutation-based and score-based hypothesis tests of $H_0 : \beta_E = 0$ versus $H_A : \beta_E > 0$, where β_E is the slope parameter defining the trend in the log-odds of being in the extreme component, relative to the zero-heavy component, as described in Equation (2)

Test	p-value
Wald	0.0999
Permutation	0.2500
Score	0.3285

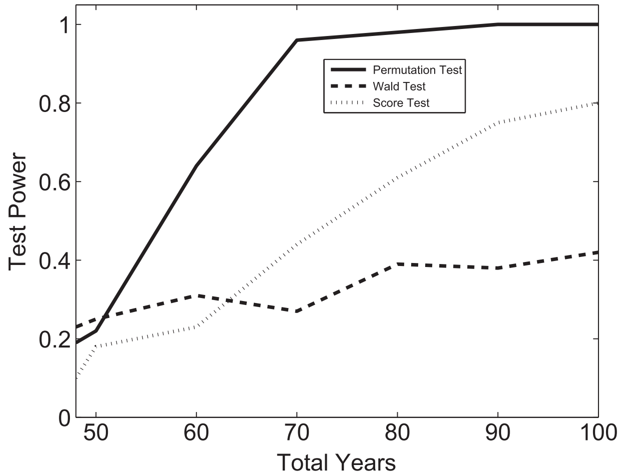

Figure 7. Power curves, plotted as functions of the total number of years of data, of permutation-based (solid), Wald-based (dashed) and score-based (dotted) hypothesis tests for a significant increasing trend in the log-odds of membership in the extreme component membership relative to the zero-heavy component

Table 2 compares the p-values of these three different methods when testing for an increasing trend in the log-odds of extreme component membership, relative to the zero-heavy component as specified in the multinomial logistic regression model for changes to the mixing proportions across time. The p-values for the score and permutation are above the standard cut-offs for declaring significance (permutation test p-value $= 0.25$, score test p-value $= 0.33$).

The power of these three different methods that test for increases in extreme behaviour relative to zero-heavy behaviour was also explored through simulation. Analogous to the concept of 'what sample size do I need', we focused on identifying the following: *How many years of data would be required to correctly detect a significant trend with high probability?* The results from our simulation study (Figure 7) suggest that the permutation test has the highest power. For this test, no assumptions are made about the underlying sampling distribution. However, all three hypothesis tests exhibit very similar levels of power for the size of our data set. For our study, which is based on 47 years of fire reports, we have weak confidence (power $\approx$ 20%) in correctly concluding that shifts in the proportion of time that the system spends in the extreme state are significant.

5. DISCUSSION

We conducted the power study, in part, to investigate what level of confidence we have in our results, as the concept of power is a key point that appears to be commonly overlooked in studies of fire risk. As illustrated, the power of the permutation test rapidly increases with the size of the data set. In about 15 years from now, the power of the permutation test will exceed 80%. This threshold may be preferred, as our modelling framework would be four times more likely to produce correct conclusions than make a type I error. And, in about 25 years, the power of the permutation test will exceed 90%.

Although our current power is weak and, therefore, we are unable to make strong conclusions about whether or not there is a significant trend towards increased extreme component behaviour, the overall estimated historical changes in risk are quite dramatic (Figure 3). In particular, consider the duration of periods where fire risk is highest each year, which we refer to as periods of 'peak risk'. Hence, how we view our estimated p-values must account for the potential future impacts that may be incurred should the estimated trends continue. Consider the changes to peak risk and, in particular, the duration of the peak towards the end of our study period. A risk threshold of 33% roughly corresponds to the two top fire risk categories in the fitted model presented in Figure 3. In 2009, fire risk above this threshold lasted for slightly over a month (about 37 days). Should current trends continue, our model forecasts this period to increase by just under 60% in less than 50 years. By 2050, the amount of time that lightning-fire risk will exceed a threshold of 33% is forecasted to last for approximately 59 days, or nearly 2 months (with a standard error of 33.1 days, calculated using a parametric bootstrap with 1000 replications).

Recall that we are only modelling log-odds of a fire day. The actual counts could increase at a different rate. Regardless, any increases to fire (either risk or counts) would likely result in increased fire management costs and impacts. Consider for example, the potential impact of an increase in the number of fire days each year. Fire managers would have to respond by placing detection, initial attack transport helicopters and firefighters on duty at elevated alert levels on more days, which would increase 'readiness' costs. Increases in the number of fires would, in addition, result in increased suppression expenditures and potentially increases in the detrimental impacts of fires that threaten public safety, property and forest resources.

While the environmental anomaly analysis of fire season air temperature and duff moisture was not meant to be definitive, those comparisons do seem to provide the basis of a sound physical rationale for the increase in lightning ignition with time. A general decrease in organic layer moisture (indicated by an increase in SDMC) would, in a typical lightning year, be associated with an increased occurrence of lightning fires because the drier forest floor fuels are more receptive to ignition. We lack any reliable historical indication of change in lightning strike activity for most of the period of analysis in this study; however, numerous recent studies have shown positive associations between lightning and surface air temperature (e.g. Williams, 2005), and thus, the use of air temperature for the region seems a reasonable surrogate for lightning strike observation itself. It should be considered, however, that increased surface air temperature would also tend to increase the drying rate of the forest floor organic material and, in the absence of increased rainfall associated with this increased temperature, could also lead to a drier forest floor overall. Thus, the association between air temperature anomaly and probability of lightning fire may be the result of a complex interaction of key factors driving the potential for lightning-fire occurrence. However, the moisture content anomalies studied do provide a more clear physical linkage between increased dryness and increased lightning-fire activity.

Wildfire risk is most commonly quantified using generalized linear (or additive) models, which model the probability of fire occurrence as a function of other variables, such a weather, fuel moisture and other locally observed covariates (e.g. Preisler *et al.*, 2004; Vilar *et al.*, 2010; Wotton and Martell, 2005). Our mixture-modelling framework avoided including covariates other than temporal variables because we were motivated by the need for a retrospective study of long-term trends in Canadian historical fire records. However, since the trends in historical risk observed in the fitted mixture model were found to be associated with air temperature and duff moisture anomalies, we also fit the following logistic GAM:

$$\text{logit}[p(\mathbf{x})] = f(w) + g(y) + h(DMC) + k(\textit{temperature}) \quad (3)$$

where p denotes the probability of a fire day, $\mathbf{x}$ is a vector of covariates and f, g, h and k are penalized spline smoothers.

The fitted smoothers for the partial effects of week, year, duff moisture and temperature appear in Figure 8. All of these additive effects were highly significant (p-values ≈ 0). A seasonal pattern within each fire season is again evident, but it appears to be a blend of the regular and extreme seasonal curves from Figure 2, as one might expect. The smoother of year is quite variable. When building this model, we found that increasing the number of knots in the yearly smoother continued to lead to improvements in fit as measured by the deviance explained

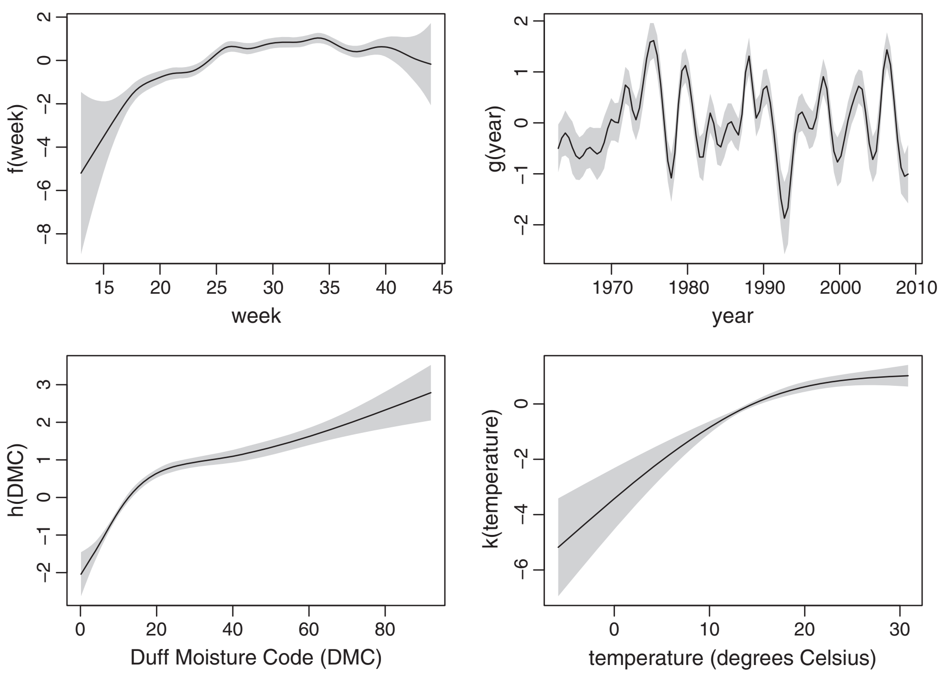

Figure 8. Estimated partial effects along with pointwise 95% confidence intervals of a logistic generalized additive model that models the log-odds of the risk of a lightning-caused fire day as a function of week (top left), year (top right), Duff Moisture Code (bottom left) and temperature (bottom right)

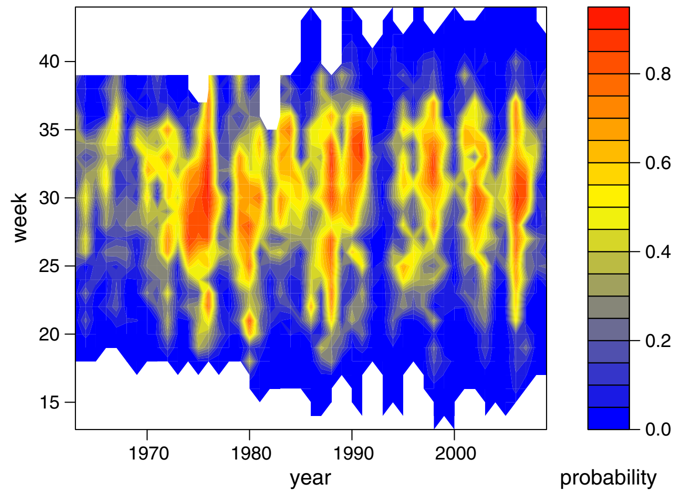

Figure 9. Contour plot of the estimated temporal changes in historical fire risk in the study area over the period 1963–2009 using the logistic generalized additive model specified in Equation (3). White areas are periods where no fire-weather data were available

by the model. The final smoother ended up using 47 knots (one for each year). We note that using this many knots is very similar to using a model with fixed effects for each year, rather than a smooth partial effect. However, we prefer the use of the smoother because our focus was on trends. Although this may seem to be overfitting the data, this is consistent with the notion that lightning-fire risk should reset itself each year because of a wet fall, a snowy winter and a wet spring. Using too few knots would oversmooth the data and be implying that fire risk in neighbouring years influences a given year, which is an extremely rare event, such as in the extreme drought conditions of 1976 that persisted into the start of the 1977 fire season as described by Stocks (1979). The variability of the smoother for year also corroborates the findings of Brillinger *et al.* (2006) who concluded that a random effect for year was needed when assessing wildfire fire risk in California as a function of space and time on the log-odds scale. In particular, our model supports the notion that a random effect for year would likely be required should the focus be forecasting. The relationship with the fuel moisture is consistent with our exploratory modelling. The log-odds of a lightning-caused fire day is positively associated with the level of dryness in the duff layer as measured by the DMC (increasing values of the DMC indicate a drier organic layer), as well as temperature. Note that we used DMC instead of the SDMC because the former was found to lead to a better fit as measured by common model selection criteria (e.g. greater deviance explained and a smaller Akaike information criterion).

Figure 9 can be used to contrast our mixture model to this simpler logistic GAM. It visualizes the fitted probabilities from the logistic GAM model specified in Equation (3) as a function of time. White areas are where there are no fire-weather data available, and these occur at the start and end of the fire season (i.e. early April and late October) when the fire-weather station was not in operation. In recent years, the weather is recorded for longer periods. Figures 3 and 9 show similar patterns. Fire risk is essentially nil in the first few weeks at the start and end of the fire season, and there are periods towards the middle of the fire season where the fire risk is higher. Two key differences between the plots are the variability and the scale of the response. The contours from the mixture modelling are much smoother, which is consistent with our goal of monitoring trends. The contours from the logistic GAM are very variable. This is the effect of fuel moisture entering the model; moisture in the upper duff layer can rise substantially from one day to another because of recent rain. The hotspots where fire risk is high are periods where the DMC is high, signifying low fuel moisture and because the logistic GAM is not smoothing out trends over years; the risk probabilities can peak to a much higher value. It is our opinion that the single logistic GAM modelling framework would definitely be preferred if the focus was on short-term predictive modelling for fire management, whereas our modelling framework has advantages if the focus is on assessing trends over time or if one wanted to simulate realistic counts of fire days within and across fire seasons without using weather data.

Acknowledgements

We thank the OMNR for providing the data used in this study and the associate editor and referee for their helpful comments on an earlier version of this manuscript. This research was supported by grants from Geomatics for Informed Decisions and the Natural Sciences and Engineering Research Council of Canada.

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