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## S1 Additional Simulation Results

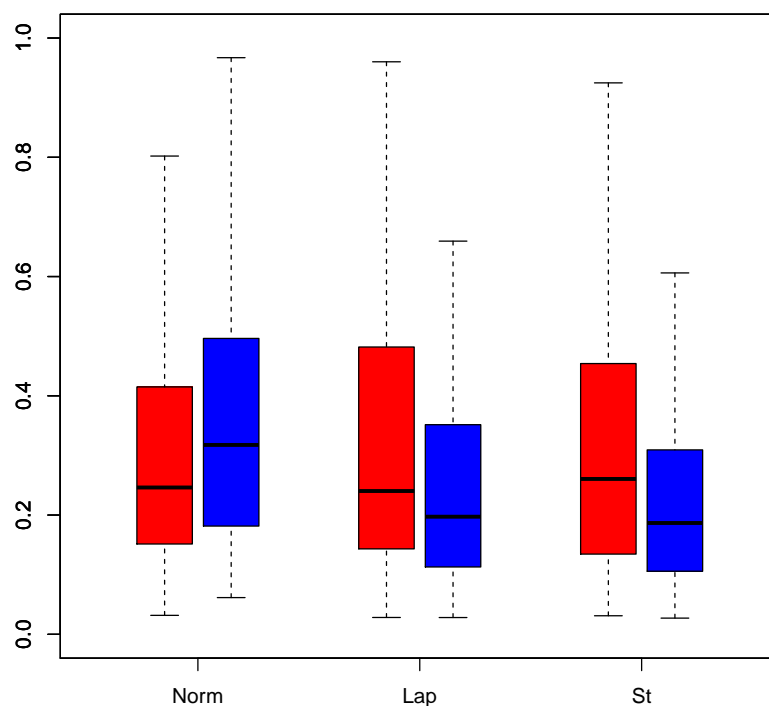


Figure S1: Boxplots of the mean squared prediction errors (MSE) across the 300 replicated simulations for all three designs and for  $n = 100$ . “Norm”, “Lap” and “St” are abbreviations of normal, Laplace and Student t, respectively. Blue and red blocks denote errors obtained from the proposed functional single index quantile regression and the functional single index mean regression by ?, respectively.

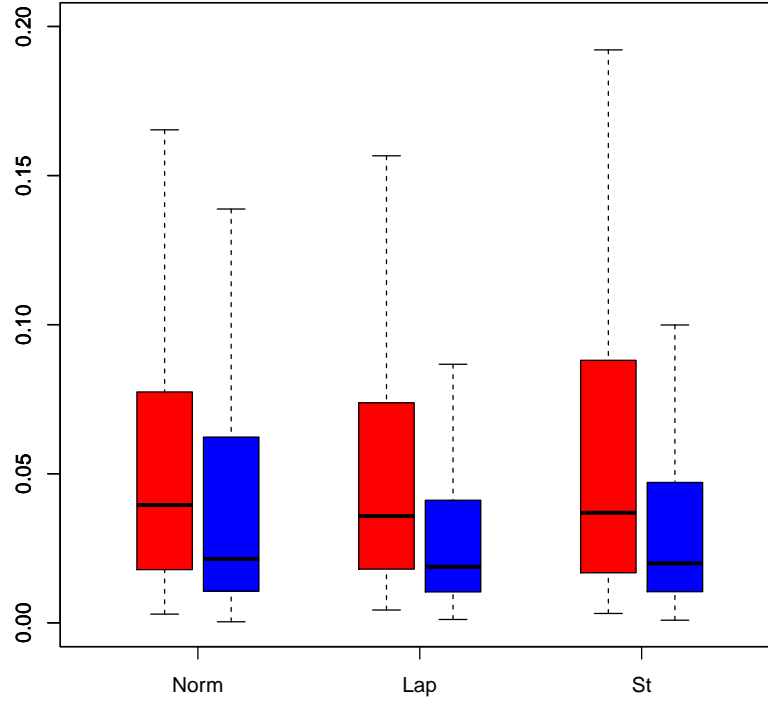


Figure S2: Boxplots of the integrated squared errors (ISE) for estimation of the index function across the 300 replicated simulations for all three designs and for  $n = 100$ . “Norm”, “Lap” and “St” are abbreviations of normal, Laplace and Student t, respectively. Blue and red blocks denote errors obtained from the proposed functional single index quantile regression and the functional single index mean regression by ?, respectively.

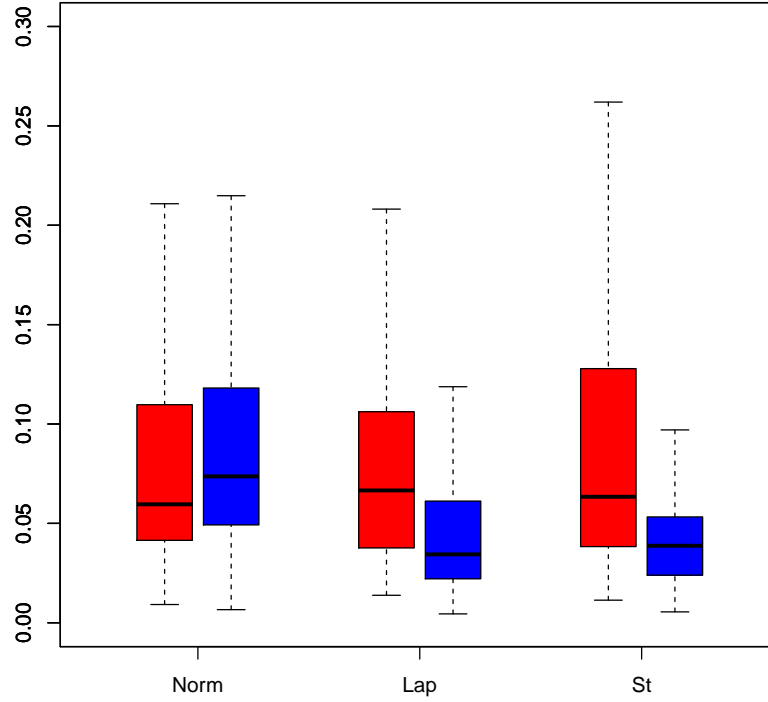


Figure S3: Boxplots of the mean squared prediction errors (MSE) across the 300 replicated simulations for all three designs and for  $n = 500$ . “Norm”, “Lap” and “St” are abbreviations of normal, Laplace and Student t, respectively. Blue and red blocks denote errors obtained from the proposed functional single index quantile regression and the functional single index mean regression by ?, respectively.

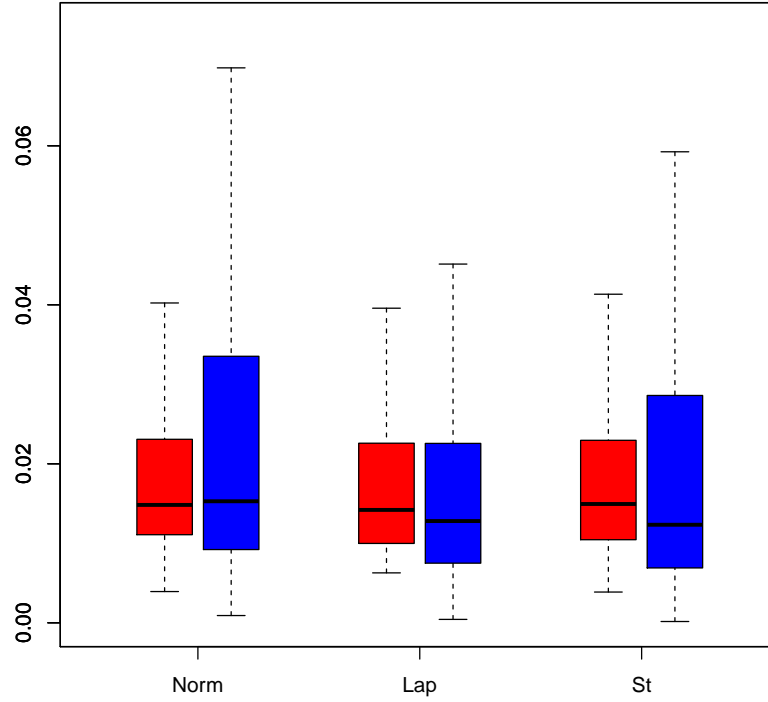


Figure S4: Boxplots of the integrated squared errors (ISE) for estimation of the index function across the 300 replicated simulations for all three designs and for  $n = 500$ . “Norm”, “Lap” and “St” are abbreviations of normal, Laplace and Student t, respectively. Blue and red blocks denote errors obtained from the proposed functional single index quantile regression and the functional single index mean regression by ?, respectively.

$n = 100$					
Distribution	MSE			ISE	
	FSiQ	FSiM	FKnn	FSiQ	FSiM
Laplace	.151(.148)	.256 (.184)	.556 (.208)	.064 (.090)	.106 (.112)
Normal	.257(.214)	.246 (.187)	.553 (.156)	.057 (.062)	.097 (.093)
Student $t$	.165 (.145)	.255 (.195)	.550 (.185)	.078(.120)	.110 (.126)
Chi-square	.227 (.178)	.244 (.182)	.553 (.163)	.075(.097)	.110 (.106)

$n = 500$					
Distribution	MSE			ISE	
	FSiQ	FSiM	FKnn	FSiQ	FSiM
Laplace	.029 (.032)	.040 (.020)	.437 (.154)	.038 (.046)	.035 (.034)
Normal	.062 (.042)	.040 (.026)	.470 (.152)	.041 (.042)	.043 (.043)
Student $t$	.026 (.018)	.040 (.021)	.442 (.169)	.043 (.038)	.032 (.037)
Chi-square	.031 (.025)	.043 (.028)	.454 (.148)	.043 (.043)	.041 (.052)

Table S1: Summary of the averages and the standard errors (in brackets) of MSEs and ISEs across the 300 replicates in all four designs for training data of size  $n = 100$  and 500, when the functional principal component scores are exponentially distributed.

## S2 Additional Results in Real Data Application

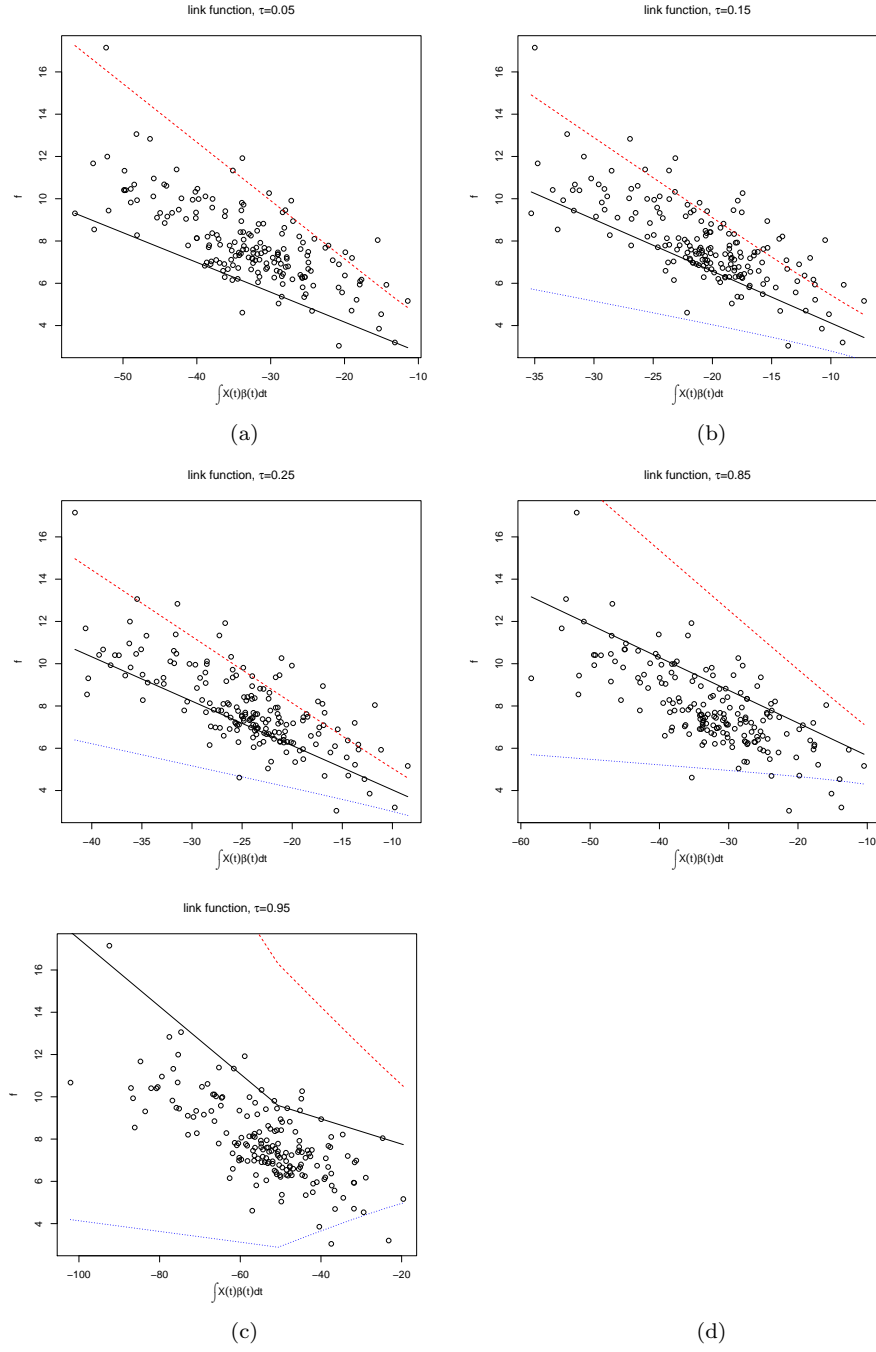


Figure S5: The estimated link functions for the quantile  $\tau = 0.05, 0.15, 0.25, 0.85$ , and  $0.95$ , and their 95% pointwise confidence intervals.