Supplementary Document for the Manuscript entitled "Estimating Time-varying Directed Neural Networks"

1 Derivatives

The derivatives of the penalized log-likelihood function for each intensity trajectory have analytic forms as follows. The first derivative of $H(\boldsymbol{c}_g)$ with respect to the coefficients is defined as

$$\frac{dH(\boldsymbol{c}_g)}{d\boldsymbol{c}_g} = \sum_{j=1}^{m_g} \boldsymbol{\phi}_g(s_{gj}) - \int_0^T \exp[\boldsymbol{\phi}_g^T(s)\boldsymbol{c}_g]\boldsymbol{\phi}_g^T(s)ds - 2\lambda_g \boldsymbol{R}\boldsymbol{c}_g \tag{1}$$

The second derivative of $H(\boldsymbol{c}_g)$ takes the following form

$$\frac{d^2H(\boldsymbol{c}_g)}{d\boldsymbol{c}_g^T\boldsymbol{c}_g} = -\int_0^T \exp[\boldsymbol{\phi}_g^T(s)\boldsymbol{c}_g]\boldsymbol{\phi}_g(s)\boldsymbol{\phi}_g^T(s)ds - 2\lambda_g \boldsymbol{R} \tag{2}$$

2 Additional Real Application results

This section provides additional real data application results for all 12 neurons. Estimated regulation functions for 12 differential equations are displayed in the following figures. The directed connection from regulator g-th neuron to receiver l-th neuron is determined to exist only if the corresponding regulation function is non-zeo at some t in the observation period.

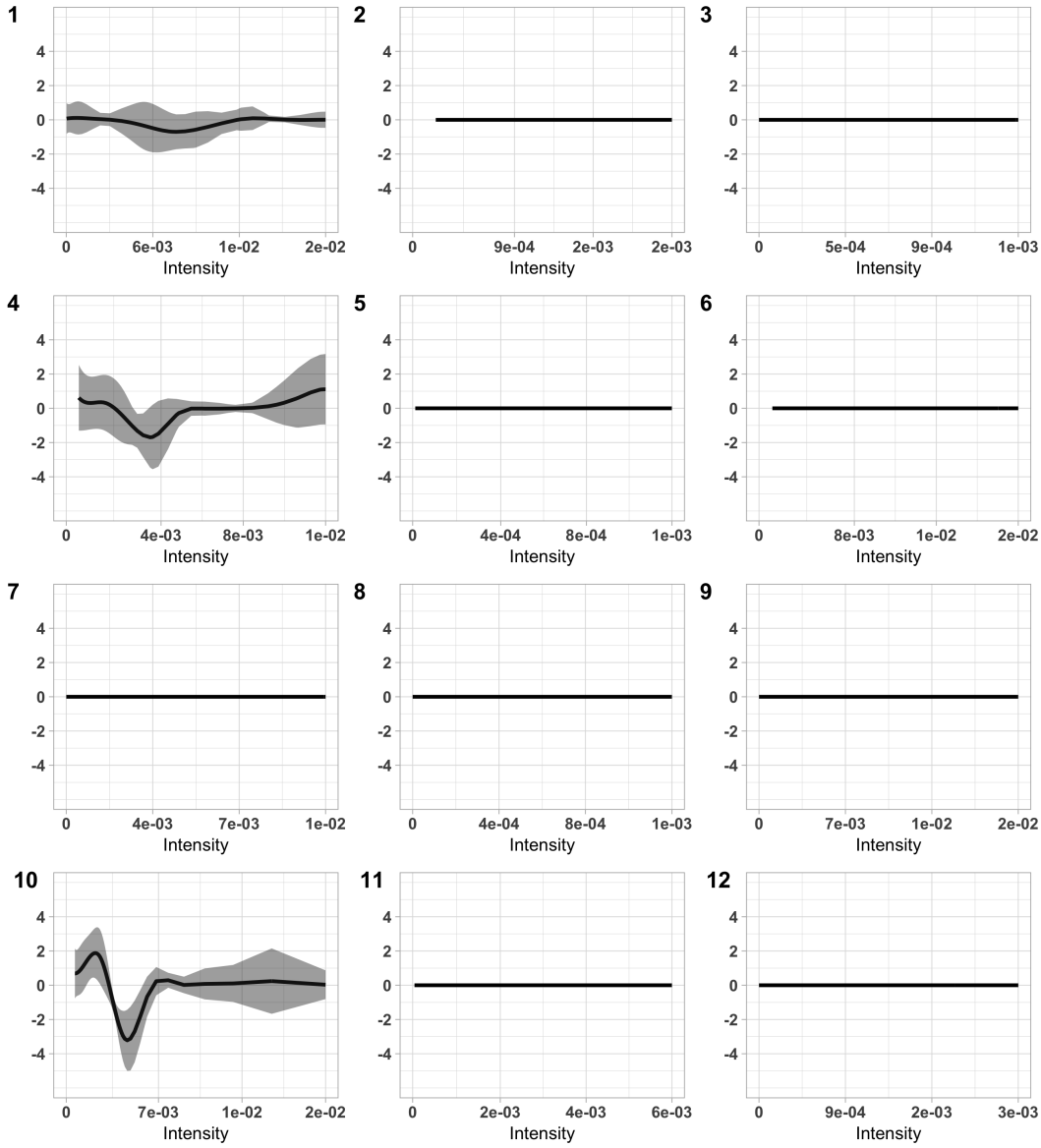

Figure 1: Regulation functions for the target neuron 1. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

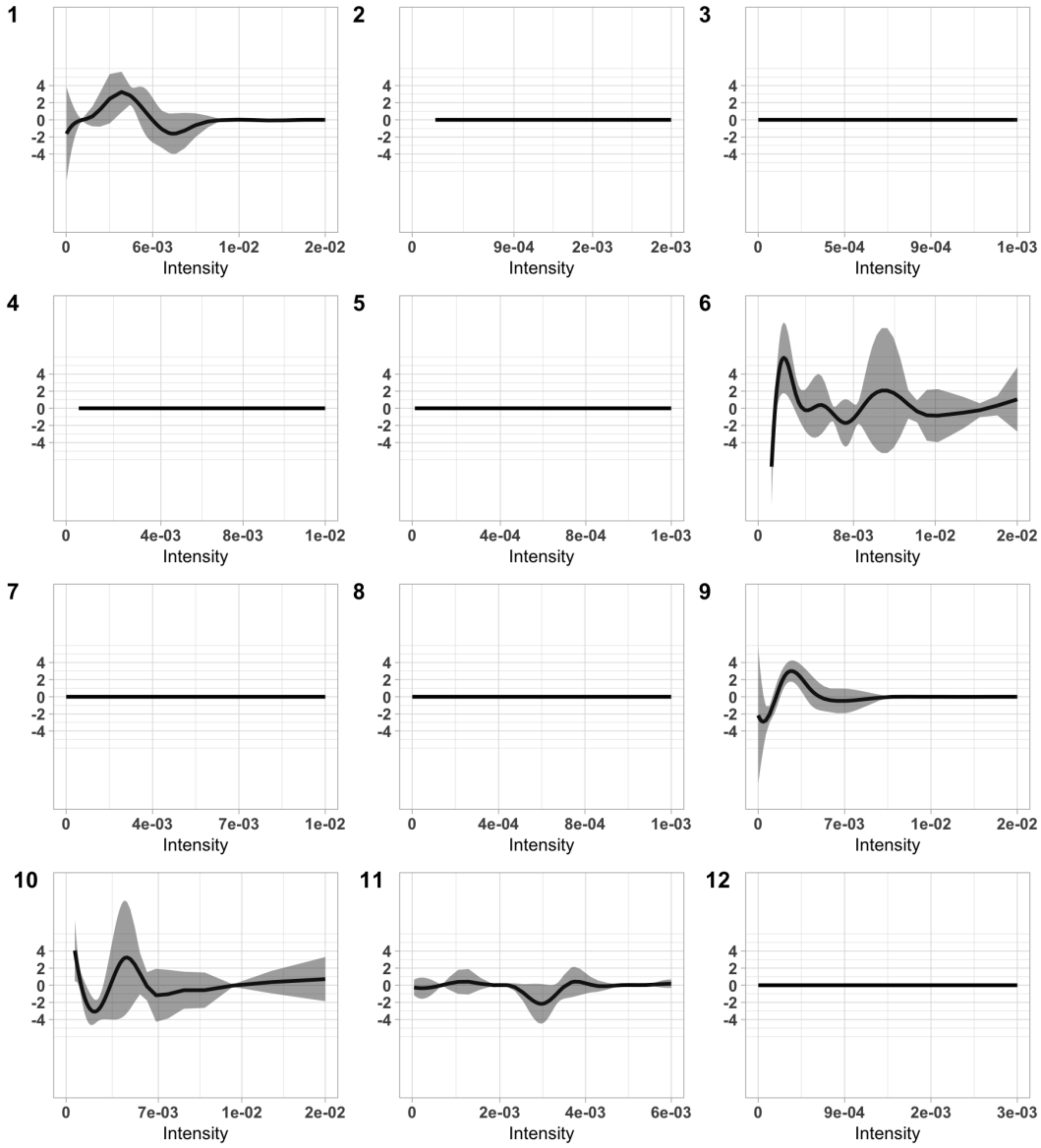

Figure 2: Regulation functions for the target neuron 2. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

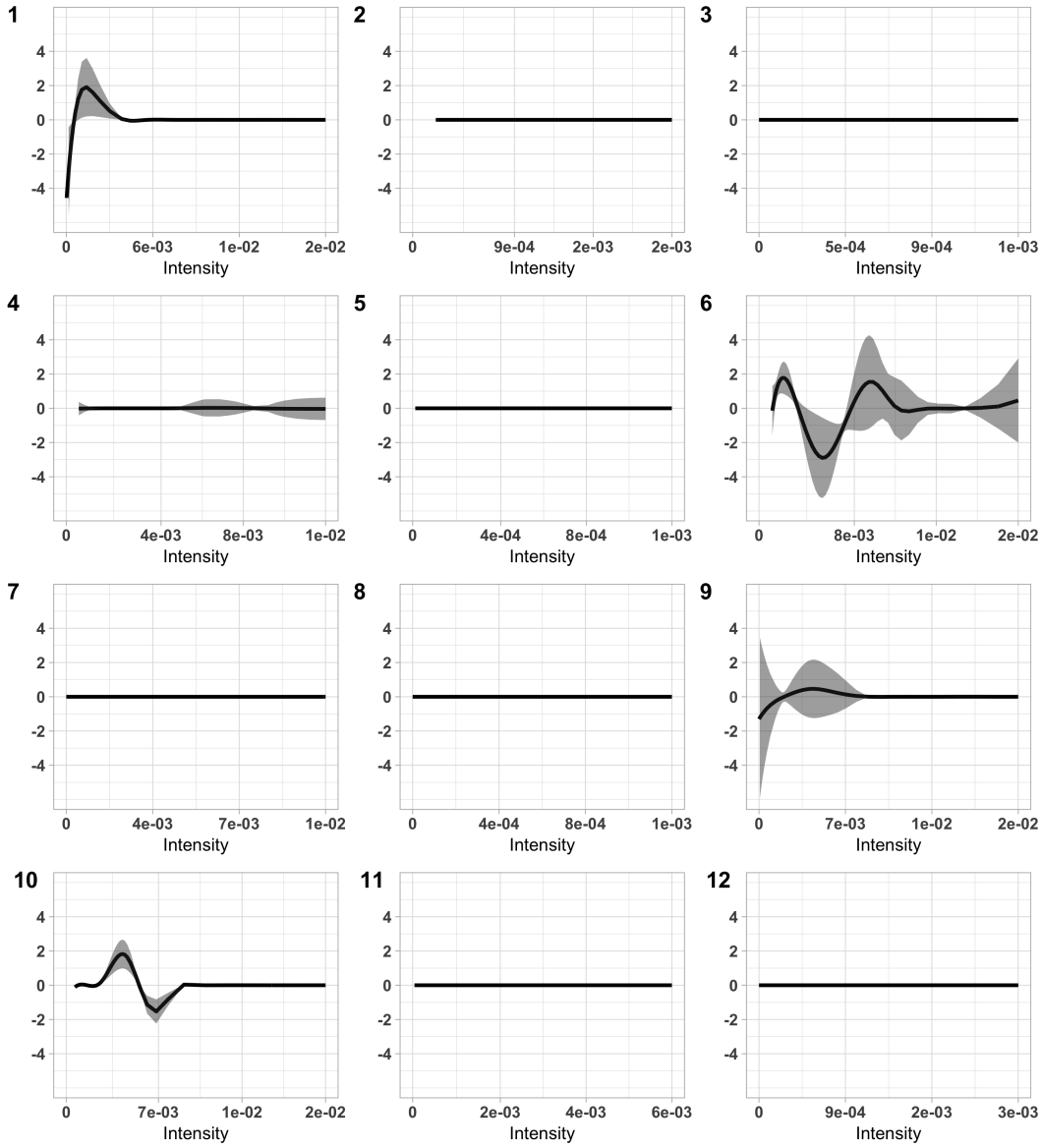

Figure 3: Regulation functions for the target neuron 3. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

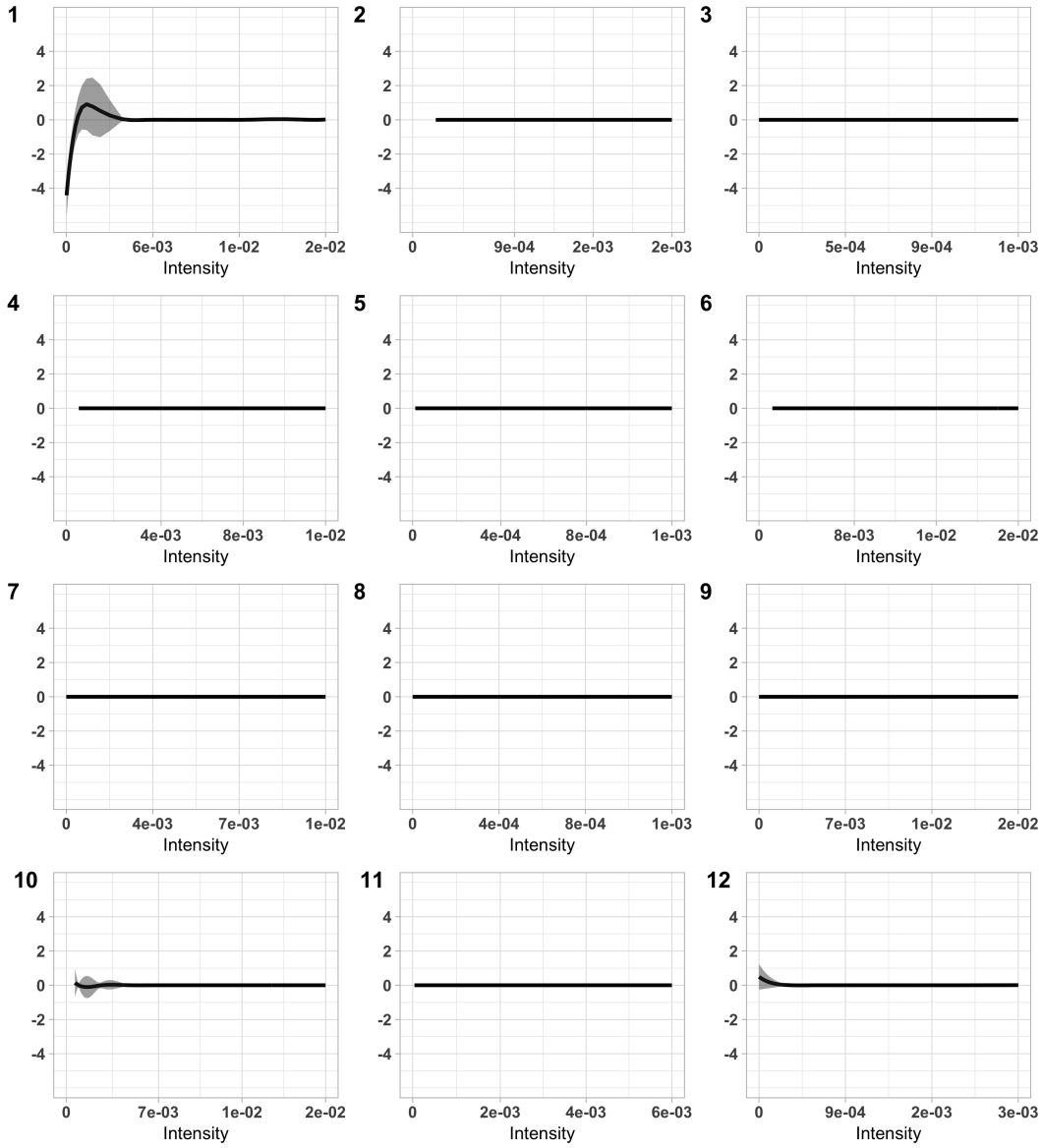

Figure 4: Regulation functions for the target neuron 4. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

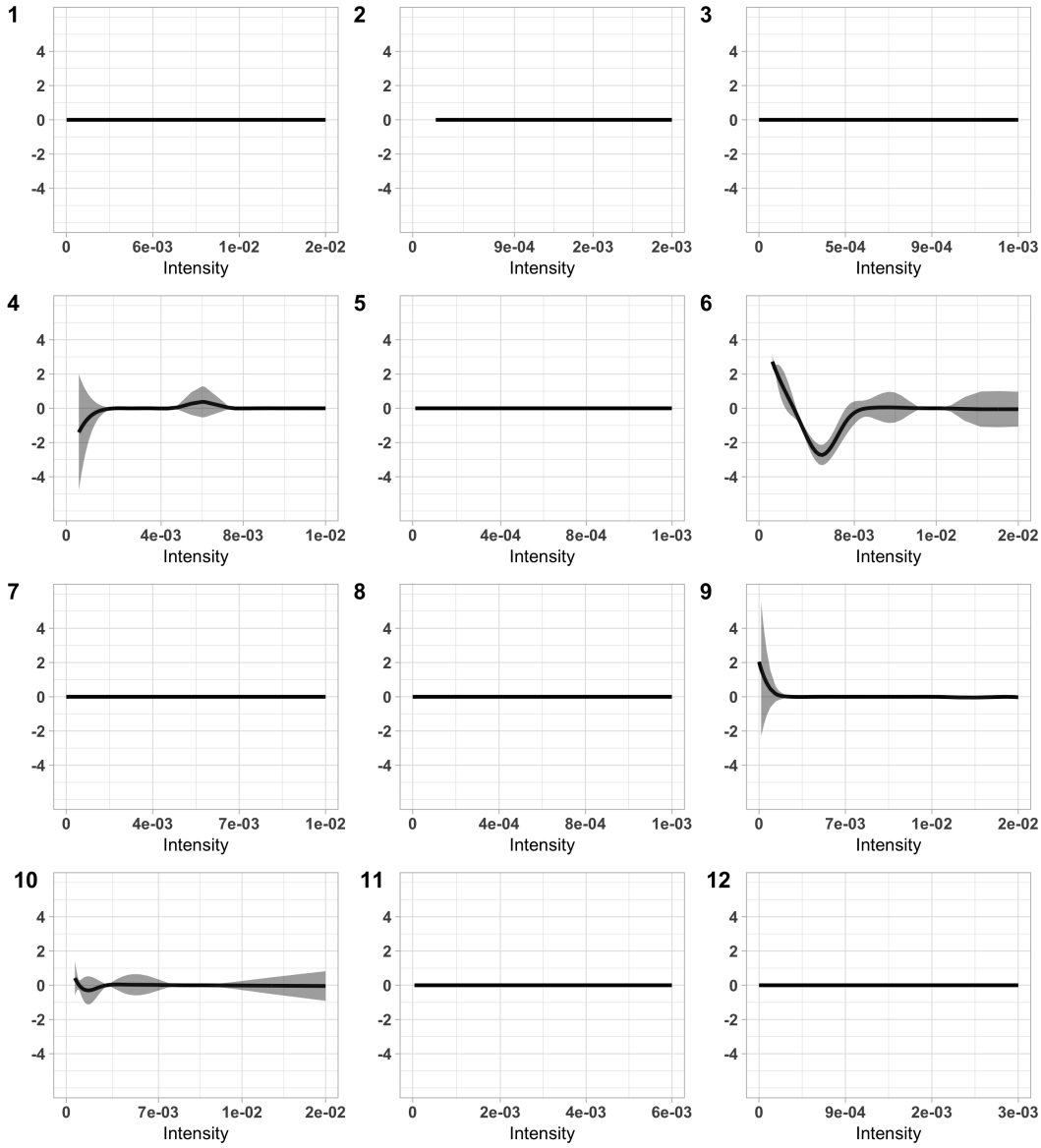

Figure 5: Regulation functions for the target neuron 5. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

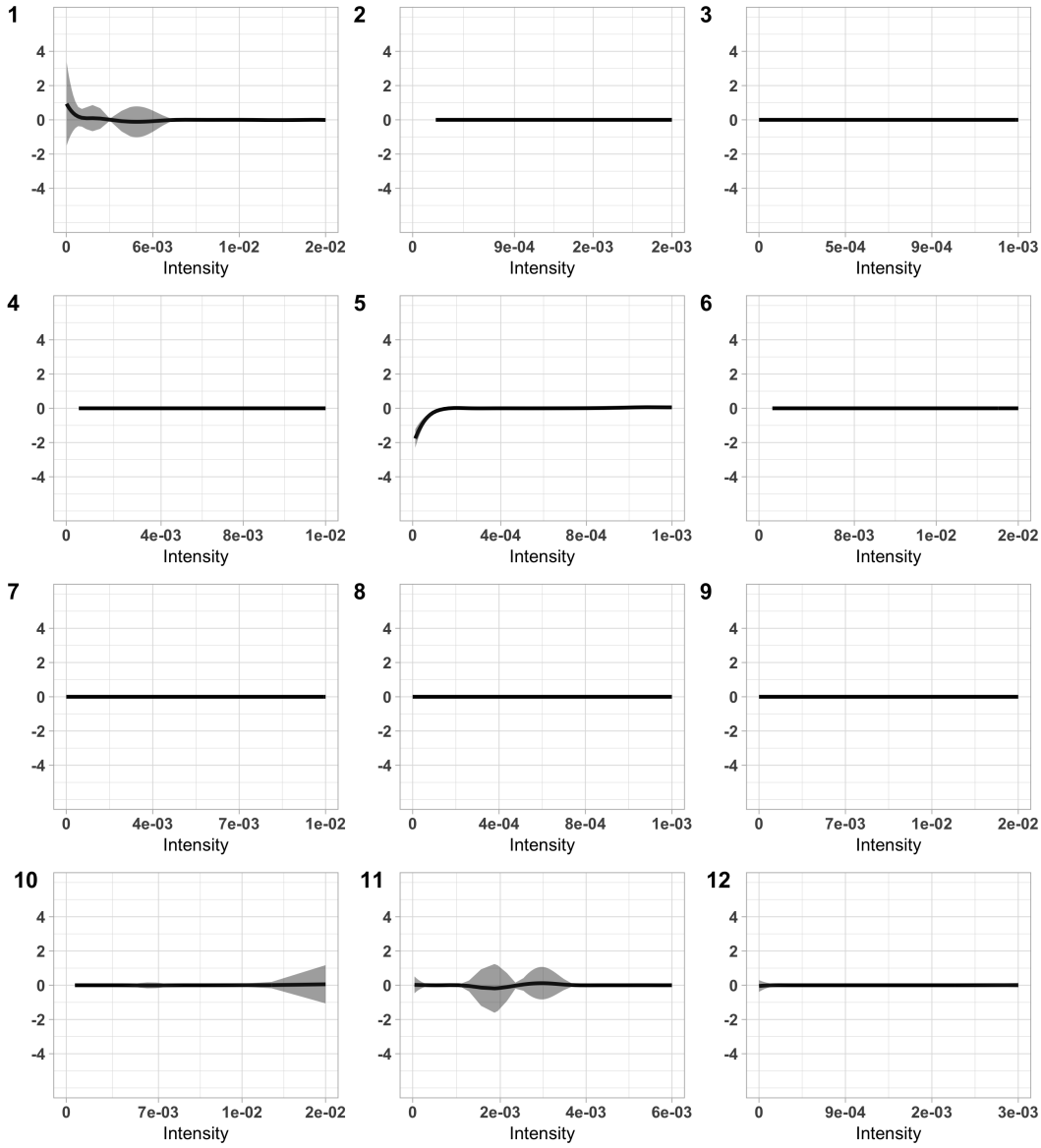

Figure 6: Regulation functions for the target neuron 6. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

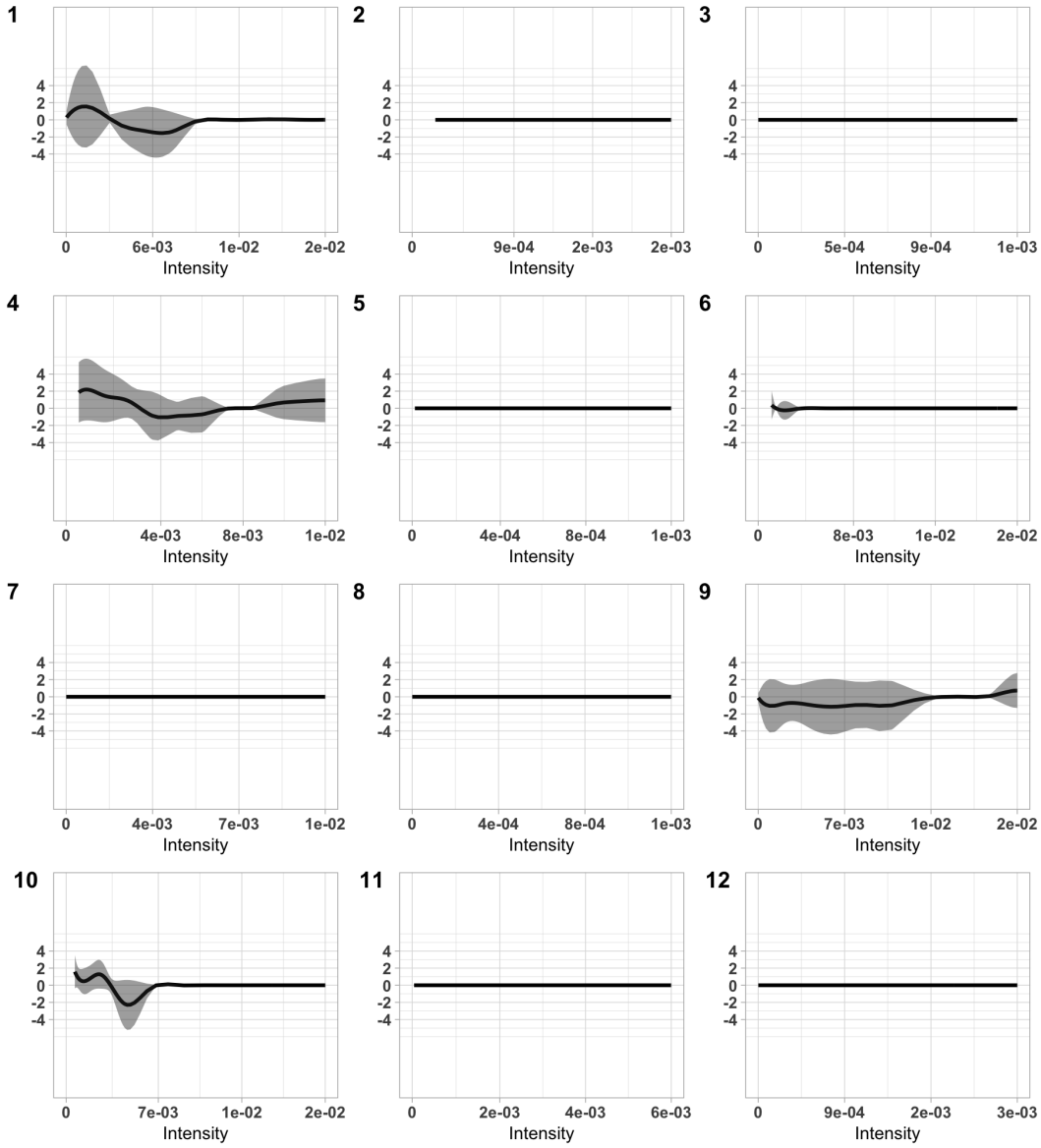

Figure 7: Regulation functions for the target neuron 7. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

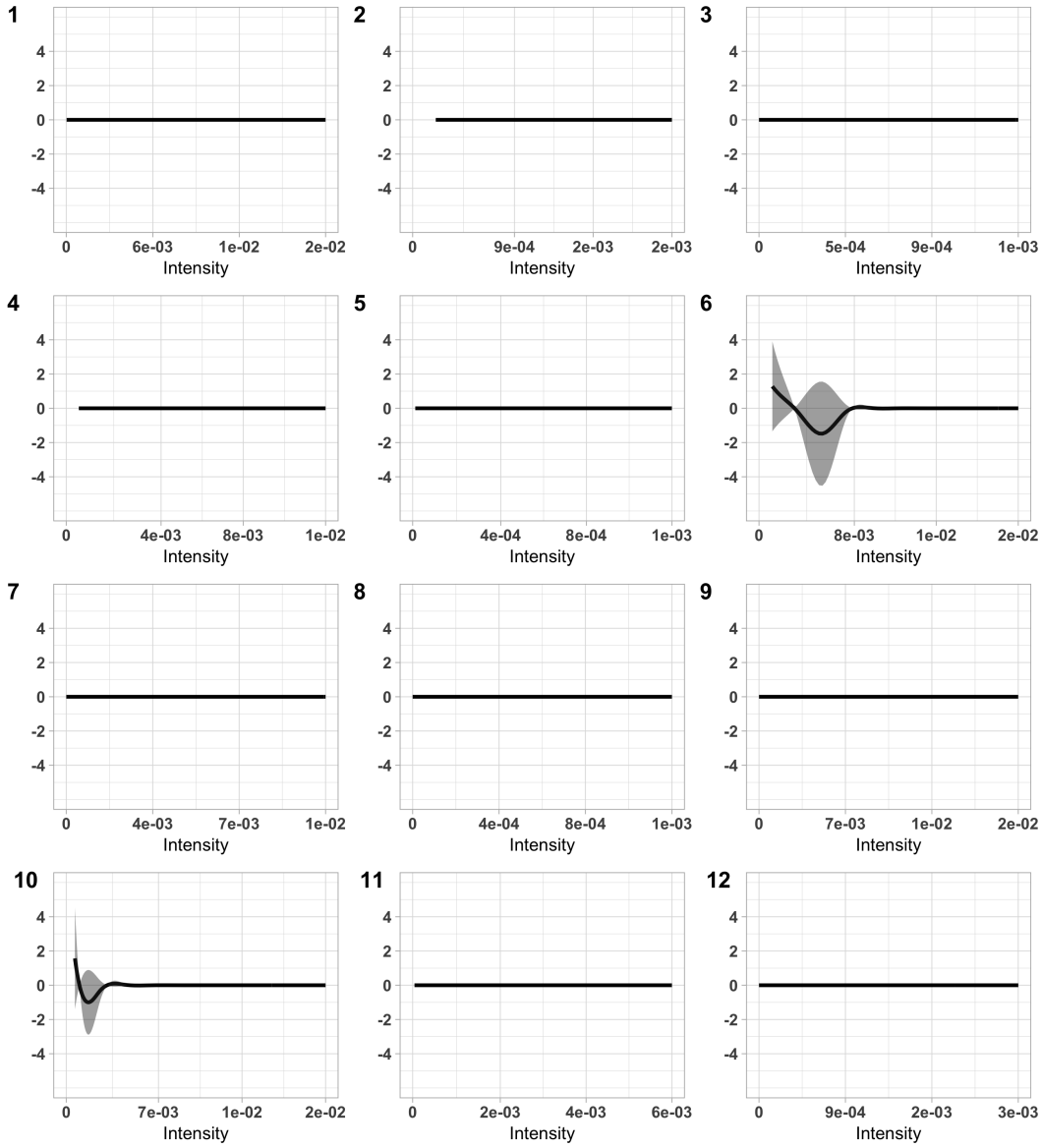

Figure 8: Regulation functions for the target neuron 8. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

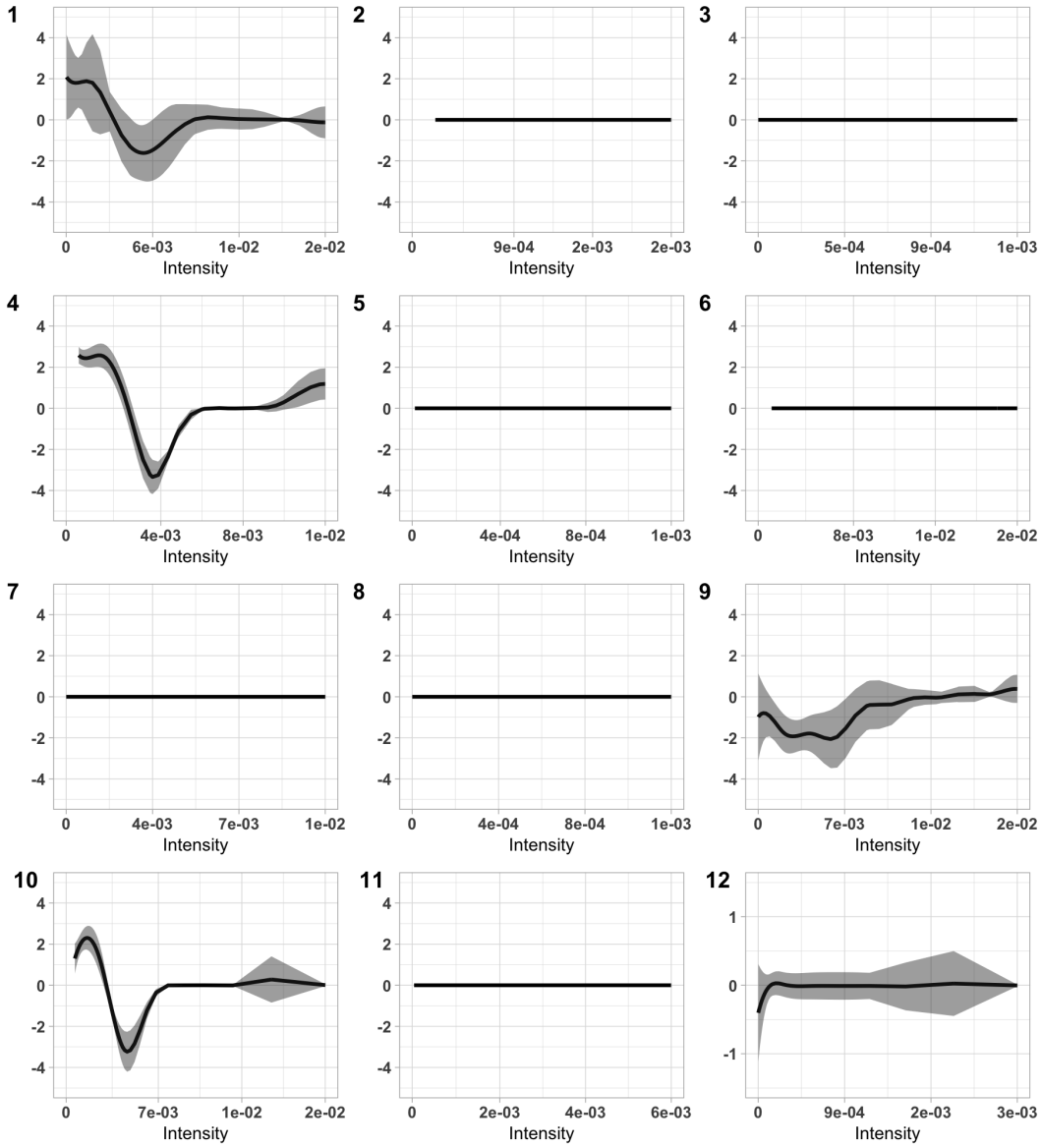

Figure 9: Regulation functions for the target neuron 9. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

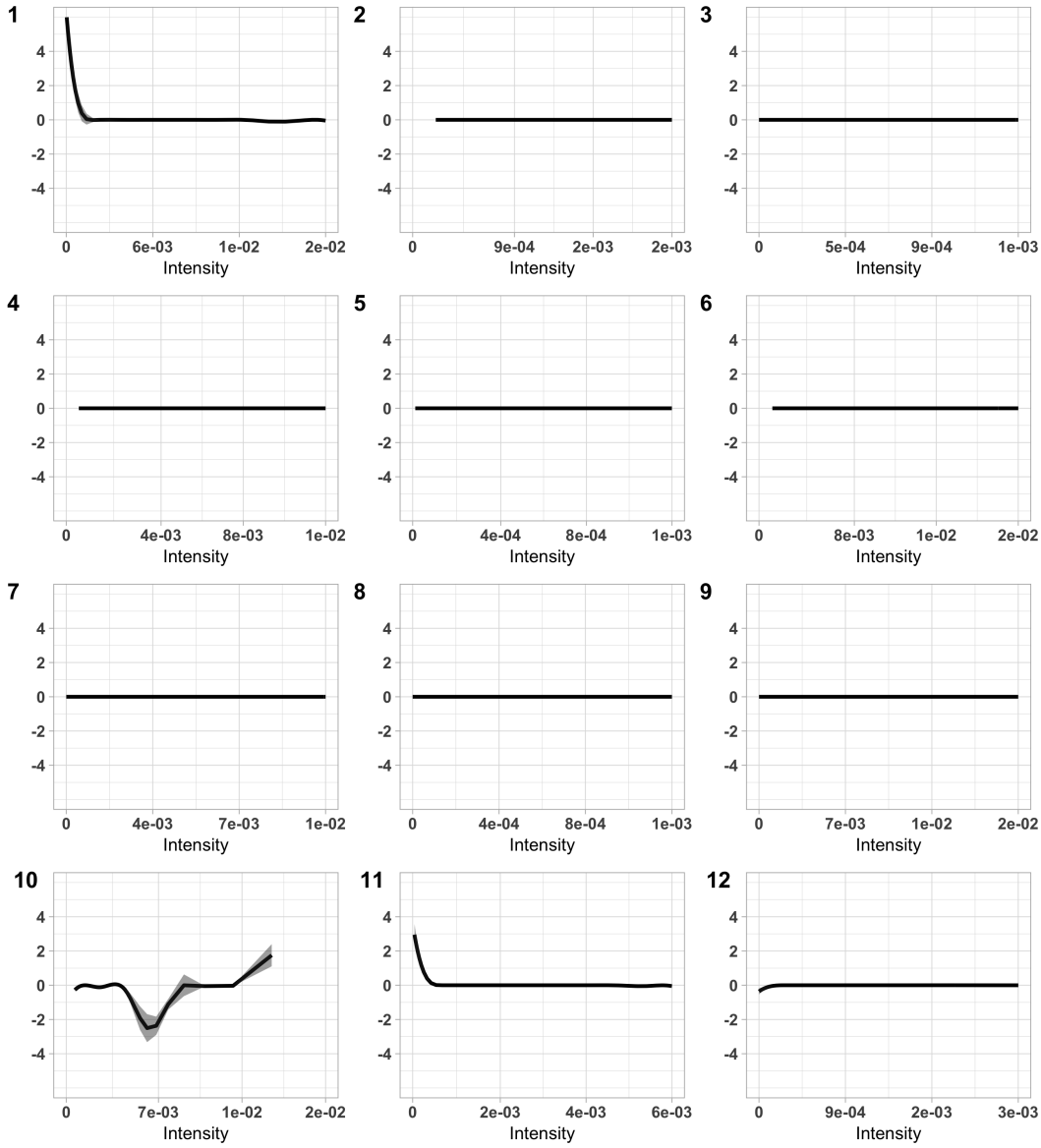

Figure 10: Regulation functions for the target neuron 10. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

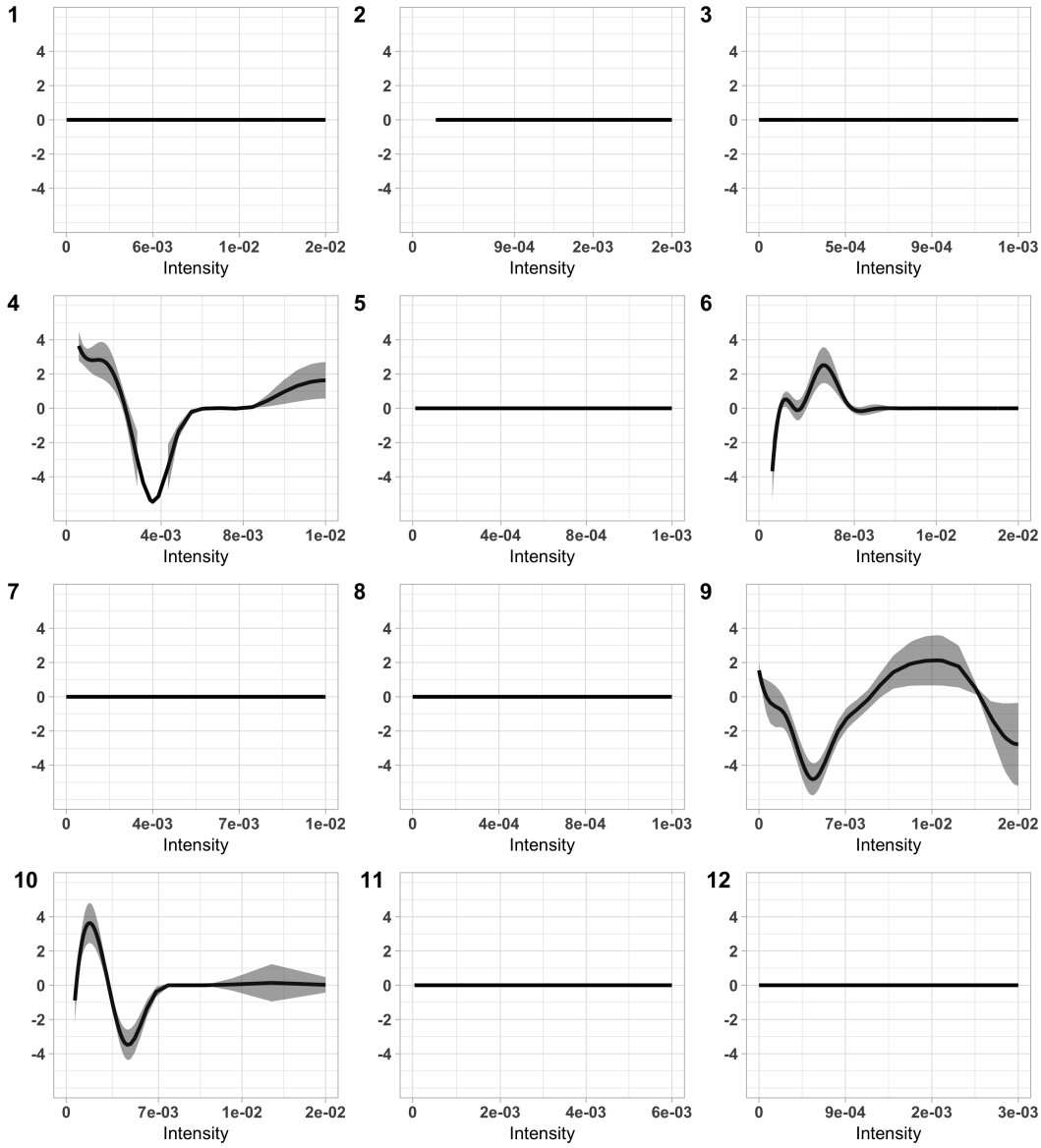

Figure 11: Regulation functions for the target neuron 11. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.

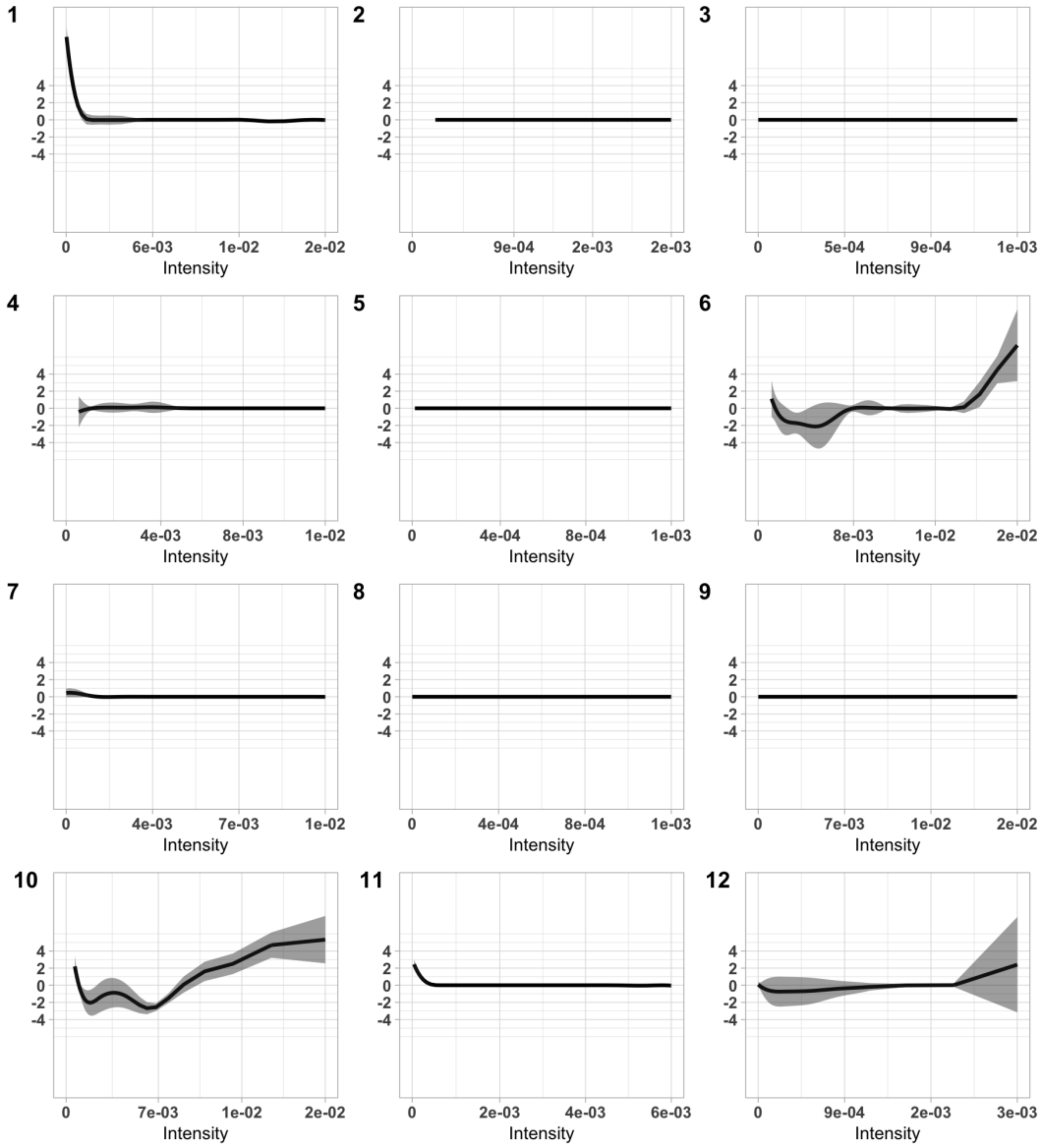

Figure 12: Regulation functions for the target neuron 12. Bold numbers on the top-left corner indicate the number of regulation neuron. Shaded bands correspond to the pointwise 95% confidence intervals.