

Supplementary Material for the Manuscript entitled “Unsupervised Learning on U.S. Weather Forecast Performance”

Chuyuan Lin¹, Ying Yu¹, Yifan Wu¹, and Jiguo Cao^{1,*}

¹Department of Statistics and Actuarial Science, Simon Fraser University, 8888
University Drive, Burnaby, BC V5A 1S6, Canada

*Corresponding Author: jiguo_cao@sfu.ca

1 More Result in Real Application

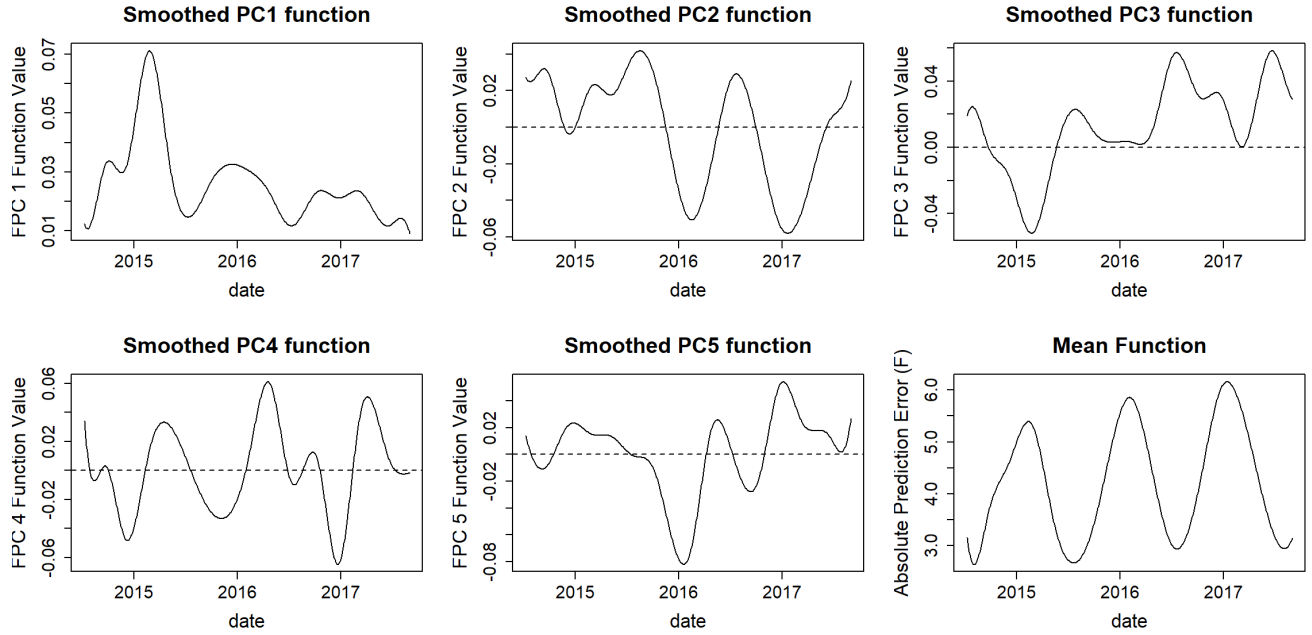


Figure 1: The first 5 smoothed FPC functions and mean function with CV-achieved $\lambda = 18500$.

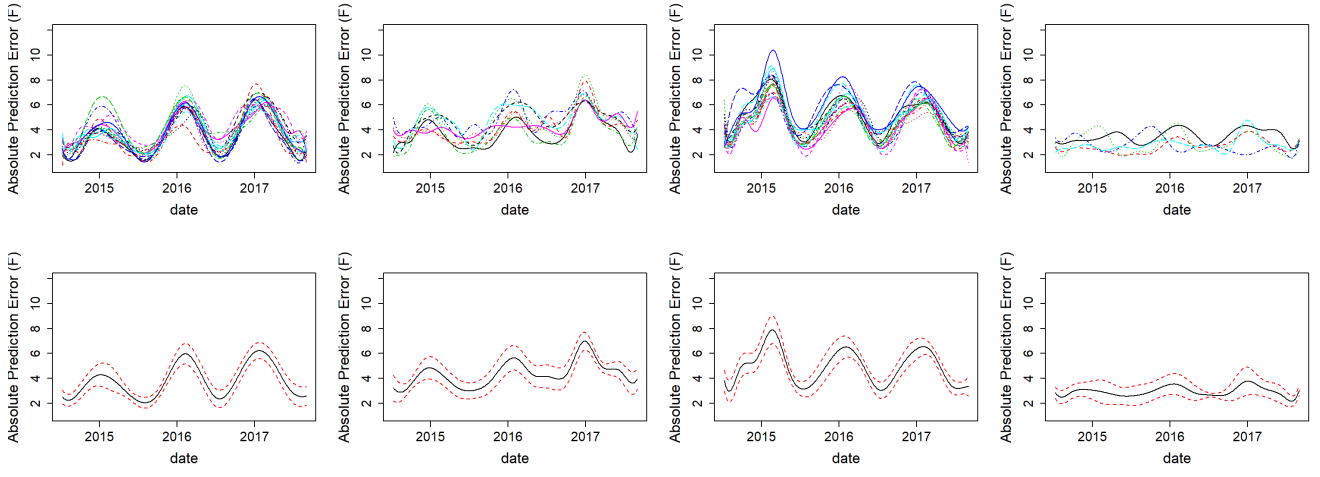


Figure 2: Cluster results from the K -means clustering on B-spline coefficients

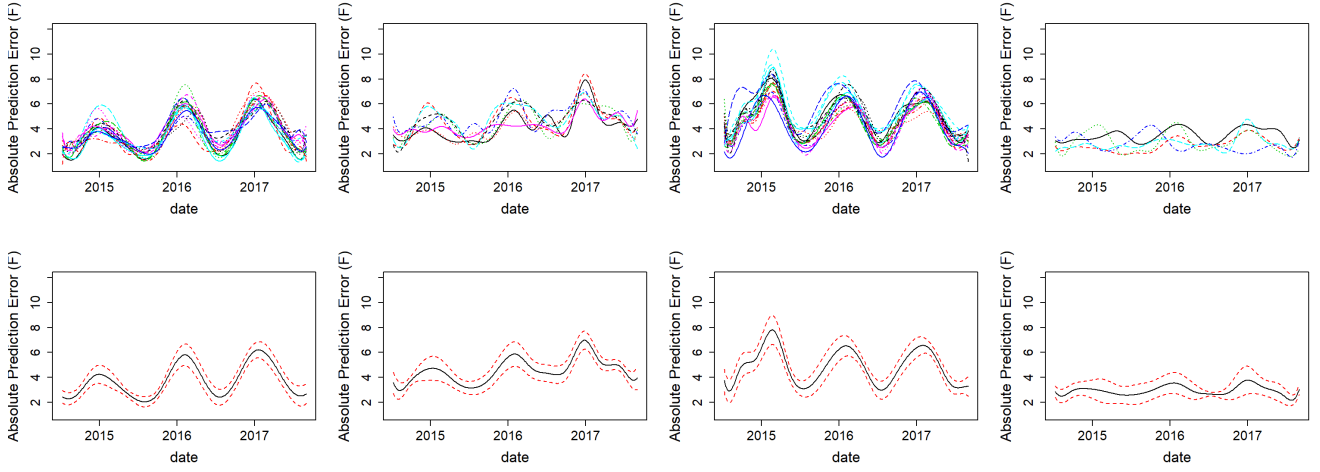


Figure 3: Cluster Result from K -means Clustering on Smoothed FPC Scores

2 More Result in Simulation Study

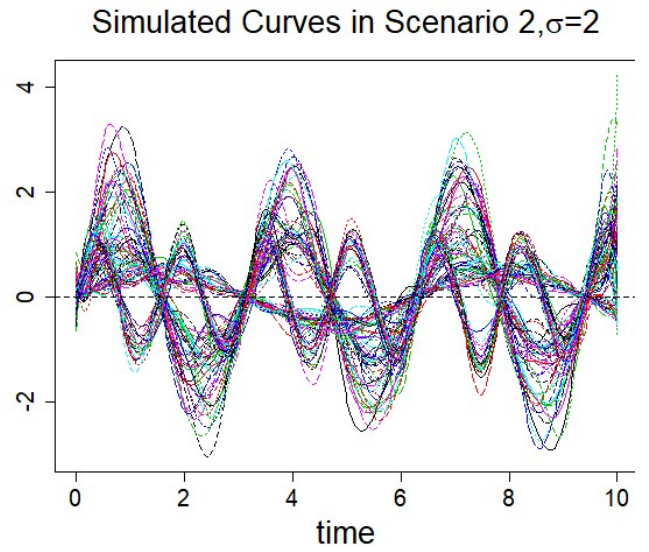
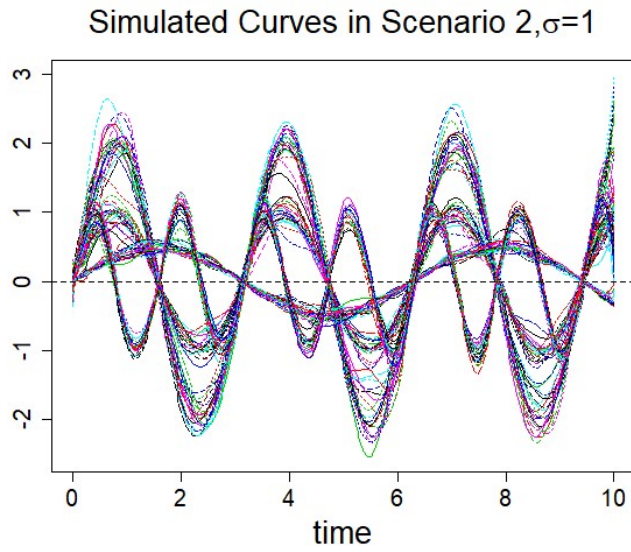
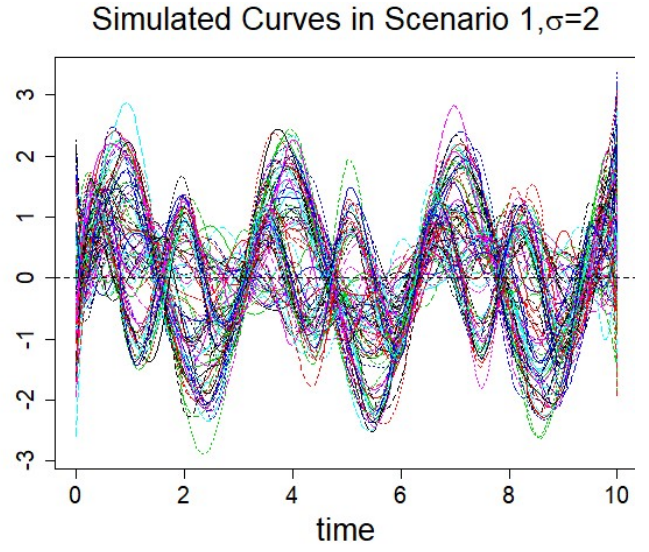
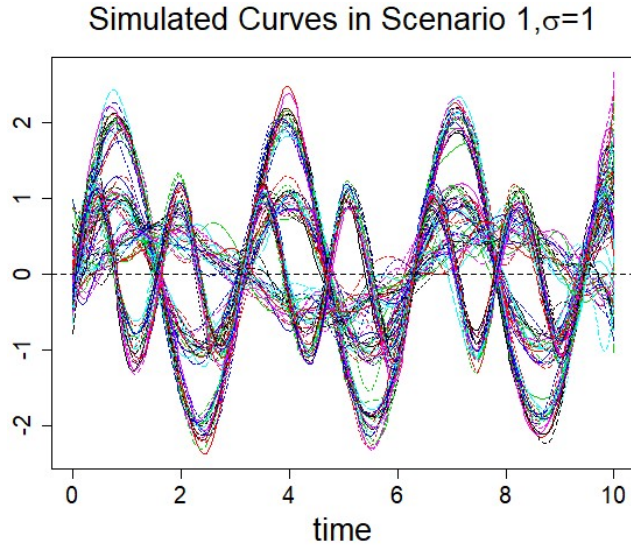


Figure 4: Simulated curves in two different scenarios with $n = 20$ and $\sigma = 1$ or 2

		Selected Number of Clusters									
		2		3		4		5		6	
		20	50	20	50	20	50	20	50	20	50
Scenario	Clustering Method										
1	K-means on B-spline coefficients	0	0	200	200	0	0	0	0	0	0
	K-means on FPC scores	0	0	156	133	25	40	18	27	1	0
	FunFEM (BIC)	0	0	0	0	149	3	44	71	7	126
	FunFEM (ICL)	0	0	0	0	139	4	54	54	7	142
2	K-means on B-spline coefficients	0	0	200	200	0	0	0	0	0	0
	K-means on FPC scores	0	0	160	156	30	35	10	9	0	0
	FunFEM (BIC)	0	0	0	0	149	7	43	72	8	121
	FunFEM (ICL)	0	0	0	0	137	6	53	74	10	120

Table 1: Frequency of the number of clusters selected over 200 simulations ($n = 20$ or 50) using different clustering methods in 2 different scenarios under $\sigma = 1$. The true number of clusters is 4.

		Selected Number of Clusters									
		2		3		4		5		6	
		20	50	20	50	20	50	20	50	20	50
Scenario	Clustering Method										
1	K-means on B-spline coefficients	0	0	200	200	0	0	0	0	0	0
	K-means on FPC scores	1	0	85	70	95	97	18	32	1	1
	FunFEM (BIC)	0	0	0	0	152	7	44	60	3	133
	FunFEM (ICL)	0	0	1	0	154	2	31	58	14	140
2	K-means on B-spline coefficients	0	0	200	200	0	0	0	0	0	0
	K-means on FPC scores	2	0	105	108	82	87	10	5	1	0
	FunFEM (BIC)	0	0	0	0	132	7	55	64	13	129
	FunFEM (ICL)	0	0	0	0	139	4	56	64	5	132

Table 2: Frequency of the number of clusters selected over 200 simulations ($n = 20$ and 50) using different clustering methods in 2 different scenarios under $\sigma = 2$. The true number of clusters is 4.

The goodness of estimated mean curve of each cluster is another interest in the simulation study. The main idea of the assessment of the mean curve estimation is measuring the difference between the estimated mean curves and the original mean curves. An average distance of cluster-specific mean function between estimated and real clusters (ADCMF) is defined to assess the mean curve estimations. Given K clusters and N discrete time observations t_1, t_2, \dots, t_N , as well as the real and estimated mean curves of each cluster, say $C_1(t), C_2(t), \dots, C_K(t)$ and $\hat{C}_1(t), \hat{C}_2(t), \dots, \hat{C}_K(t)$ respectively, the average distance of cluster-specific mean function (ADCMF) is then defined as

$$\text{ADCMF} = \frac{1}{K} \frac{1}{N} \sum_{j=1}^K \sum_{i=1}^N (\hat{C}_j(t_i) - C_j(t_i))^2 \quad (1)$$

When ADCMF is smaller, the average difference between the estimated and real within-cluster mean curves is smaller among all observed time points, which indicates that the clustering method performs better in the view of better estimation of mean curve within each cluster.

		Accuracy (SD)		AWS D		ADCMF	
		(Curve Number)		(Curve Number)		(Curve Number)	
Scenario	Methods	20	50	20	50	20	50
1	K-means on B-spline Coefficients	0.810 (0.188)	0.802 (0.189)	2.46	2.51	0.052	0.053
	K-means on FPC Scores	0.802 (0.173)	0.813 (0.179)	2.15	2.09	0.038	0.033
	FunFEM with BIC	0.997 (0.033)	0.993 (0.051)	1.55	1.58	0.002	0.003
	FunFEM with ICL	0.992 (0.053)	0.997 (0.032)	1.58	1.56	0.004	0.002
2	K-means on B-spline Coefficients	0.788 (0.190)	0.801 (0.190)	2.17	2.03	0.053	0.048
	K-means on FPC Scores	0.802 (0.176)	0.782 (0.176)	1.84	1.75	0.042	0.036
	FunFEM with BIC	0.997 (0.032)	0.993 (0.051)	1.04	1.07	0.002	0.003
	FunFEM with ICL	0.987 (0.063)	0.978 (0.085)	1.11	1.18	0.006	0.008

Table 3: Summary table of average accuracy, AWS D and ADCMF from the clustering results over 200 simulations in 4 different scenarios under $\sigma = 1$ and number of clusters $K = 4$.

		Accuracy (SD)		AWS D		ADCMF	
		(Curve Number)		(Curve Number)		(Curve Number)	
Scenario	Methods	20	50	20	50	20	50
1	K-means on B-spline Coefficients	0.852 (0.179)	0.864 (0.181)	3.68	3.55	0.102	0.112
	K-means on FPC Scores	0.839 (0.175)	0.808 (0.176)	3.39	3.46	0.033	0.039
	FunFEM with BIC	0.987 (0.065)	0.998 (0.024)	3.12	3.08	0.009	0.002
	FunFEM with ICL	0.983 (0.072)	0.998 (0.024)	3.14	3.08	0.011	0.002
2	K-means on B-spline Coefficients	0.793 (0.180)	0.782 (0.192)	3.15	2.97	0.090	0.095
	K-means on FPC Scores	0.816 (0.174)	0.820 (0.177)	2.60	2.63	0.033	0.036
	FunFEM with BIC	0.979 (0.080)	0.976 (0.086)	2.16	2.19	0.010	0.009
	FunFEM with ICL	0.987 (0.062)	0.971 (0.094)	2.11	2.21	0.008	0.016

Table 4: Summary table of average accuracy, AWS D and ADCMF from the clustering results over 200 simulations in 2 different scenarios under $\sigma = 2$ and number of clusters $K = 4$.

Scenario	Clustering Method	Selected Number of Clusters				
		1 (real)	2	3	4	5 or 6
$K = 1$ with constant variance noise	K -means on B-spline coefficients	95	5	0	0	0
	K -means on FPC scores	100	0	0	0	0
	FunFEM (BIC)	0	41	53	5	1
	FunFEM (ICL)	0	21	61	16	2
$K = 1$ with time-varied variance noise	K -means on B-spline coefficients	77	22	0	0	1
	K -means on FPC scores	100	0	0	0	0
	FunFEM (BIC)	0	20	60	16	4
	FunFEM (ICL)	0	11	64	20	5

Table 5: Frequency of the number of clusters selected over 100 simulations with $n = 40$ curves using different clustering methods in 2 different scenarios under $\sigma = 0.5$. The true number of clusters is 1 (Null Case).