Chapter 2 Probability - A summary of the theory January 15, 2007

Basic Definitions:

Experiment – a process of producing outcomes subject to uncertainty.
Sample Space – All possible outcomes of an experiment
Event – a set of one or more outcomes from the sample space

Equally likely Outcomes – in which each item in the sample space has the same chance (Example 2.2, and many more examples in section 2.3).
Note: Many experiments do not have equally likely outcomes. (Example 2.3)

Set Theory: Union $A \cup B$ means A or B (inclusive "or": includes both A and B)
Intersection $A \cap B$ means A and B
Complement A' means not A

when $A \cap B$ is the empty set, we say A and B are "disjoint" sets. When A and B refer to events, it is customary to say A and B are "mutually exclusive".

Venn Diagrams (p 56) are very helpful for understanding the "calculus of probabilties" which is to come (section 2.2)

Calculus of Probabilties

The axioms of probability doe not constitute a definition of probability, but rather a set of constraints which a probability must have. The probability of an event A is denoted P(A) and is best understood as the long run relative frequency (LRRF) for the outcome A. It is true that LLRF does satisfy the axioms on p 58. Some theorists are troubled by the fact that a notion of probability is useful even when there is not an implied infinite sequence of experiments, and then LRRF would seem not to apply. For example, the probability that it will rain January 13 may be .6 say, even though there is only one instance of this experiment. A way out of this dilemma is to say such probabilities are estimates, and the underlying true probability has the property that in an infinite sequence of such instances, with possibly different true probabilities, the expected long run proportions obtain.

One situation where understanding probability is fairly easy is in a repeatable experiment in which the possible outcomes are all equally likely. If there are n such possible outcomes to one experiment, the probability of each outcome is 1/n. Clearly this would be the LRRF.

Note the following

i) $P(A')=1-P(A)$
ii) If $A \cap B = \Phi$, the empty set, then $P(A \cup B) = P(A) + P(B)$
iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
i) & ii) are obviously true when you think in terms of LRRF. iii) needs Venn diagram p 62

iii) can be extended to the union of many (not just two) events. You can see how this works by looking at the Venn diagram on p 63.

Counting possibilities in equally likely outcomes:

mn rule. If there are m ways something can happen, and m ways something else can happen, then there are mn ways for both things to happen. e.g.1 deck of cards – four suits, 13 denominations, so 52 combinations e.g.2 four hair colors (blond, brown, black, red) and three eye colours (brown, blue, green) so there are 4 x 3 = 12 hair-eye combinations.

The mn rule obviously extends to a $n_1 n_2 \ldots n_k$ rule (p 69)

$P_{k,n}$ represents the number of permutations of k distinct things from n objects

Note: why "distinct"? Consider three objects A,B,B note that there are only 3 permutations. With objects A,B,C there are 6 permutations.

$P_{n,n}$ is special and is denoted n! ("n factorial" or "factorial n")

With the factorial notation it can be shown that $P_{k,n} = n!/(n-k)!$

Note that n-k of the terms in numerator and denominator divide out.

$P_{7,3} = 7!/(7-3)! = 7!/4! = 7.6.5$ This is useful for large n and k.

$C_{k,n}$ Combinations refers to selections of k things from n. From A,B,C,D the selection ABC is considered the same as the selection BAC or CBA. Order does not distinguish a selection in a "combination". Note the notation using () that is customary but hard to do in a word processor.

It can be shown that $C_{k,n} = P_{k,n}/k! = \frac{n!}{k!(n-k)!}$. We will discuss these $C_{k,n}$ and $P_{k,n}$ formulae in class.