Some more worked examples from Ch 2

p 83 Exercise 45. (Note that a good understanding of conditioning would allow you to read the answers almost directly from the table)

a) P(A) = how many ways can someone be A? = .106+.141+.200 = .447
   P(C) = .215+.200+.065+.020 = .500
   P(A and C) =.200
b) P(A|C)=P(A and C)/P(C) = .200/.500 = .400 proportion of group 3 that are type A
   P(C|A)= /P(A) = .200/.447 =.447 proportion of type A that are group 3
c) Given B', what is prob of group 1? i.e P(group 1|B') =?

P(group 1|B') = P(group 1 and B')/P(B')
P(B') = sum of columns O, A and AB = .432 + .447 + .030 = .909
P(group 1 and B') = .082+.106 + .004 = .192
So P(group 1|B') = .192/.909 = .211

p 84 Exercise 57. Explain in words why this must be true!

p 85 Exercise 65.	500 in Mean,	300 in Median,	200 in Mode
Satisfied	200	150	160

P(Mean|Satis) = P(Satis|Mean)P(Mean)/P(Satis)

P(Satis) = (200+150+160)/1000 = .510

So P(Mean|Satis) = (200/500) (500/1000)/.510=.4*.5/.51=.392
Similarly
   P(Median|Satis) = (150/300) (300/1000)/.510 = .5*.3/.51 = .294
and P(Mode|Satis) = (160/200) (200/1000)/.510=.8*.2/.51 = .314

So most likely is Mean and Least Likely is Median (for the randomly selected student that was satisfied).

Note we did not need Bayes Theorem, only the twice applied definition of P(A|B).

p 91 Exercise 77

O: the event old one fails
Y: the event young one fails

$P(O \cap Y') = .10 = P(O)(1-P(Y))$ by independence, so $P(O)=.10/(1-P(Y))$
Similarly, $P(O' \cap Y) = .05 = P(Y)((1-P(O)) = P(Y)(1-.10/(1-P(Y)))$
Solve for x=P(Y): .05=x-.10x/(1-x)
$.05(1-x)=x(1-x)-.1x$ so $.05=.95x-x^2$ or $x^2-.95x+.05=0$
$x=.95/2 \pm (1/2)(.95^2-.2)^{1/2}$

So x= .0559 or .894 and the only feasible solution is P(Y)=.0559
So P(O)=.10/(.9441)=.1059 and finally P(Y)*P(O) = .1059*.0559 = .0059
The probability that the system fails is .0059. (harder than it looked!)