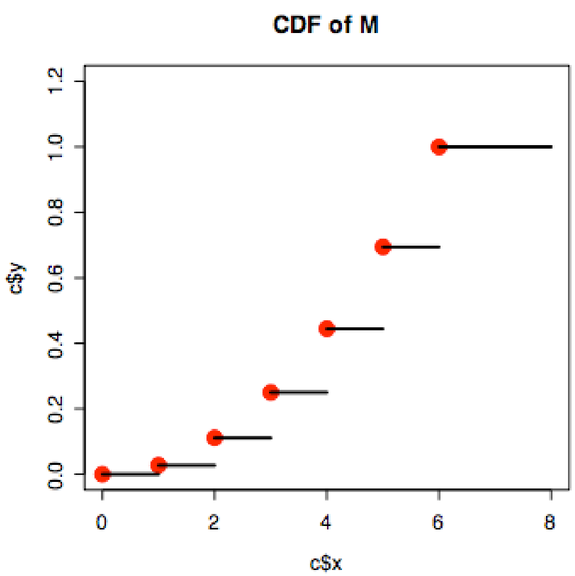
Some Ch 3 Exercises

p 110. 3.2.18. M(dice outcome) is a random variable since it produces a number as a outcome of an experiment. To know the probability distribution of M, we need to think about the probability distribution of the dice outcomes.

To specify the dice outcomes unambiguously, lets suppose one is green and one is red. Then the possible outcomes of (green,red) are (i,j) where i and j can each range over 1,2,3,4,5,6. These 36 outcomes are all equally likely (prob = 1/36). How many of them produce a 1? Only (1,1) so p(1)=P(M=1)=1/36. Similarly, M=2 occurs for (1,2),(2,1) or (2,2) so p(2)= P(M=2)=3/36. Similarly p(3)=5/36,p(4)=7/36,p(5)=9/36,p(6)=11/36

So $P(M \le x)=$ 1/36 for x=1
4/36 for x=2
9/36 for x=3
16/36 for x=4
25/36 for x=5
36/36 for x=6

and the cdf graph (of $F(x)=P(M \le x)$) is



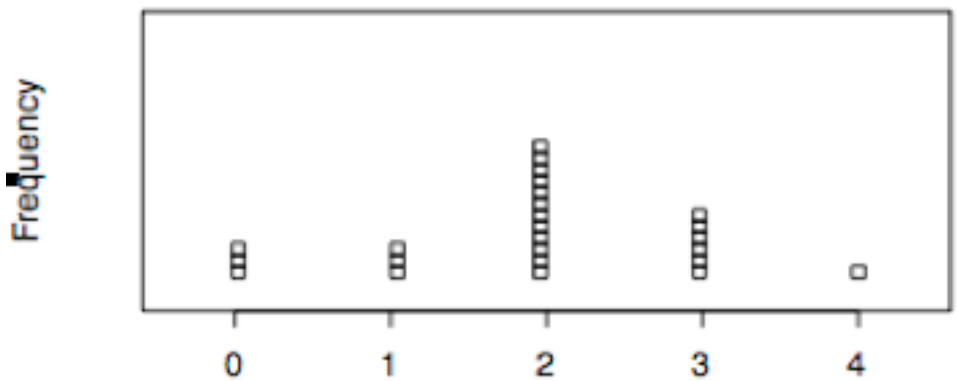
p 118 3.3.28

a) E(X)=0*.08+1*.15+2*.45+3*.27+4*.05 = 2.06
b) V(X)=.08*(0-2.06)^2+.15*(1-2.06)^2+.45*(2-2.06)^2+.27*(3-2.06)^2+.05*(4-2.06)^2=.9364
c) sd=$.9364^{1/2}$ = .9677
d)V(X)= (.08*0^2+.15*1^2+.45*2^2+.27*3^2+.05*4^2=5.18)– 2.06^2 =.9364

Extra: simulation of the prob dist in ex 28

x=0,1,2,3,4
P(X=x) = .08,.15,.45,.27,.05 respectively

In R use
x=c(0:4)
a=sample(x,25,c(.08,.15,.45,.27,.05),replace=T) to sample from this distribution
my.dotplot(a) gives you, in one simulation



If you try this simulation a few times, you will appreciate how difficult it would be to estimate the probabilities from a single sample of size 25. Note though, that the mean of the sample does not vary too much. It is a lot easier to estimate a mean from a sample than to estimate the actual probabilities of the distribution.

Ex 55. p 127

X = number of homes with detectors among the 25 sampled

a) When p=.8 , P(X≤15) =? Note X is Binomial (n=25,p=.8) so we need
to cumulate the probabilities that X=0,1,2,....,15
In R use
pbinom(15,25,.8)
[1] 0.01733187
or use Table A.1 on p 738 of text. No need to add up 15 Binomial probs!
b) .81, for p=,7 and .42 for p=.6
c) .006 for a), .90, and .59 for b)

There is a lesson in this exercise. There are two kinds of wrong decisions you can make based on the sample of 25:
One is deciding that p<.8 when it is not. The other is deciding p≥.8 when it is not.

Part a) says that the first error will only happen with prob .017. In part c) we try to reduce this error by making the condition for deciding $p<.8$ harder to reach (by changing 15 to 14). But in doing so we risk missing a case of $p<.8$ with higher probability than in part a (given in part b).

So there is a trade off between the ability to detect $p<.8$ when it is true and the ability to accept $p>.8$ when it is true. Of course, larger samples reduce both errors but may be expensive.

Ex 71 p 134

a) This is a negative binomial probability with k=2, p=.5, where X is the number of male children until two females are born.
b) exactly 4 children means exactly 2 males so P(X=2) is
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> dnbinom(2,2,.5)
[1] 0.1875
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c) dnbinom(0,2,.5)+dnbinom(1,2,.5)+dnbinom(2,2,.5)=.6875
or pbinom(2,2,.5)=.6875
d) P(X=x)=dnbinom(x,2,.5) neg binomial with k=2 and p=.5
so E(X) = (formula p 133)=2(.5)/.5=2 male children so expected number of children is 4.

Ex 84 p 139

a) This is a binomial X so mean =200*.01=2 and sd is $(200*.01*.99)^{1/2} = 1.41$
(Note that you do not really "expect 2 to fail" but it is correct to say the "expected number" is 2. This is a jargon failure! If 150 had been tested, the so-called "expected number to fail" would be 1.5 – see the problem?)
b) One could compute binomial probs for 0,1,2,3 and subtract the sum from 1. This would give 0.1420. Usually for these large n cumulative probabilities the Poisson approximation is adequate. Use X Poisson with mean 2, P(X=0,1,2,3) = .8571 and 1-.8571 = .1429. So the Poisson approximation yields .1429 while the actual Binomial is .1420. This approx is not always so good. n large is needed for this.
c) A board works when all five diodes work. P(diode works)=.99 so P(200 diodes work)=$.99^{200}$=.134 and use binomial again to compute 4 or 5 of 5 boards when p(one board works)=.134, is $5*.866*.134^4 + .134^5 = 0.00144$. The Poisson approximation would be P(0 diodes fail in 200 diodes) = .135 and $5*.865*.135^4 + .134^5 = 0.00148$

Again, the Poisson approximation is very good.

Note that if the manufacturer guarantees that at least 4 of the 5 boards will work, a bankruptcy is in the near future!