Solutions for STAT 270 assignment 2

38.

a. P(selecting 2 - 75 watt bulbs) $= \dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{15 \cdot 9}{455} = .2967$

b. P(all three are the same) $= \dfrac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \dfrac{4+10+20}{455} = .0747$

c. $\binom{4}{1}\binom{5}{1}\binom{6}{1} = \dfrac{120}{455} = .2637$

d. To examine exactly one, a 75 watt bulb must be chosen first. (6 ways to accomplish this). To examine exactly two, we must choose another wattage first, then a 75 watt. (9×6 ways). Following the pattern, for exactly three, $9 \times 8 \times 6$ ways; for four, $9 \times 8 \times 7 \times 6$; for five, $9 \times 8 \times 7 \times 6 \times 6$.

P(examine at least 6 bulbs) = 1 – P(examine 5 or less)
= 1 – P(examine exactly 1 or 2 or 3 or 4 or 5)
= 1 – [P(one) + P(two) + … + P(five)]

$$= 1 - \left[\frac{6}{15} + \frac{9\times 6}{15\times 14} + \frac{9\times 8\times 6}{15\times 14\times 13} + \frac{9\times 8\times 7\times 6}{15\times 14\times 13\times 12} + \frac{9\times 8\times 7\times 6\times 6}{15\times 14\times 13\times 12\times 11}\right]$$

= 1 – [.4 + .2571 + .1582 + .0923 + .0503]
= 1 - .9579 = .0421

44. $\binom{n}{k} = \dfrac{n!}{k!(n-k)!} = \dfrac{n!}{(n-k)!k!} = \binom{n}{n-k}$

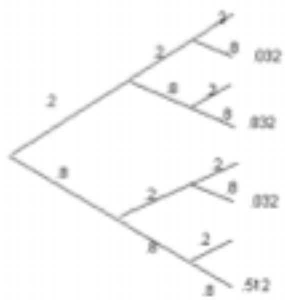
The number of subsets of size k = the number of subsets of size n-k, because to each subset of size k there corresponds exactly one subset of size n-k (the n-k objects not in the subset of size k).

77. Let A_1 = older pump fails, A_2 = newer pump fails, and $x = P(A_1 \cap A_2)$. Then $P(A_1) = .10 + x$, $P(A_2) = .05 + x$, and $x = P(A_1 \cap A_2) = P(A_1) \bullet P(A_2) = (.10 + x)(.05 + x)$. The resulting quadratic equation, $x^2 - .85x + .005 = 0$, has roots $x = .0059$ and $x = .8441$. Hopefully the smaller root is the actual probability of system failure.

92.

a. (.8)(.8)(.8) = .512

b.



.512+.032+.023+.023 = .608

c. P(1 sent | 1 received) = $\dfrac{P(1sent \cap 1received)}{P(1received}$ = $\dfrac{.4256}{.5432}$ = .7835

98.

a. P(both + ve) = P(carrier ∩ both + ve) + P(not a carrier ∩ both + ve)
=P(both + ve | carrier) x P(carrier)
+ P(both + ve | not a carrier) x P(not a carrier)
= $(.90)^2(.01) + (.05)^2(.99)$ = .01058

P(both – ve) = $(.10)^2(.01) + (.95)^2(.99)$ = .89358

P(tests agree) = .01058 + .89358 = .90416

b. P(carrier | both + ve) = $\dfrac{P(carrier \cap both.positive)}{P(both.positive)} = \dfrac{(.90)^2(.01)}{.01058}$ = .7656