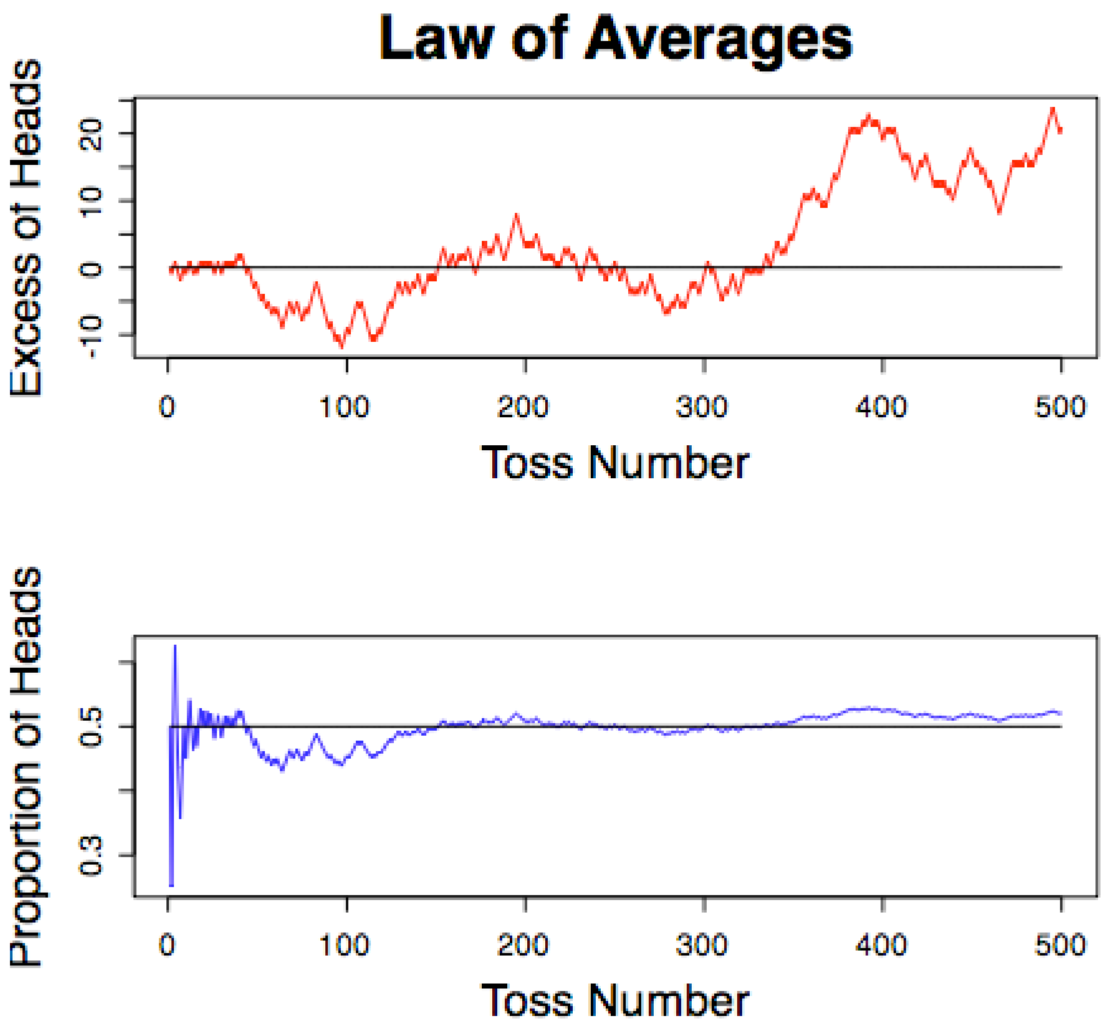
Making use of Probability Models to Study the Markets

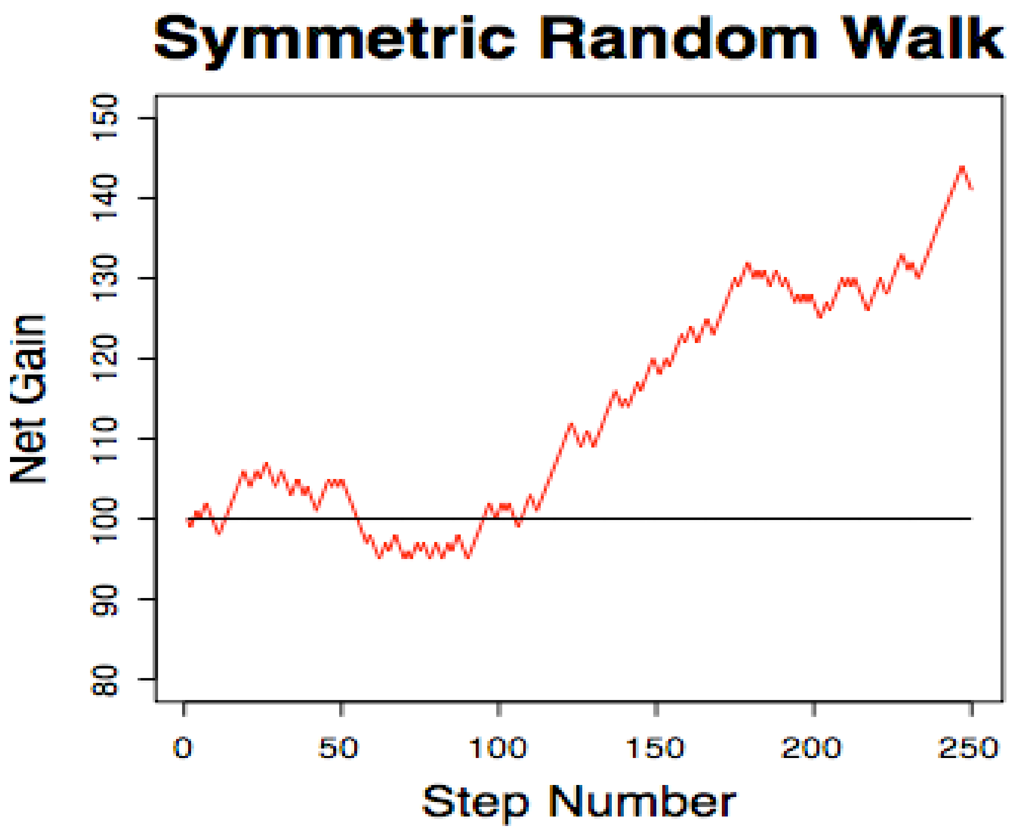
Recall our use of "simulation" to demonstrate the "law of averages" – what it is and what it is not. This simulation showed what happened in a symmetric random walk.



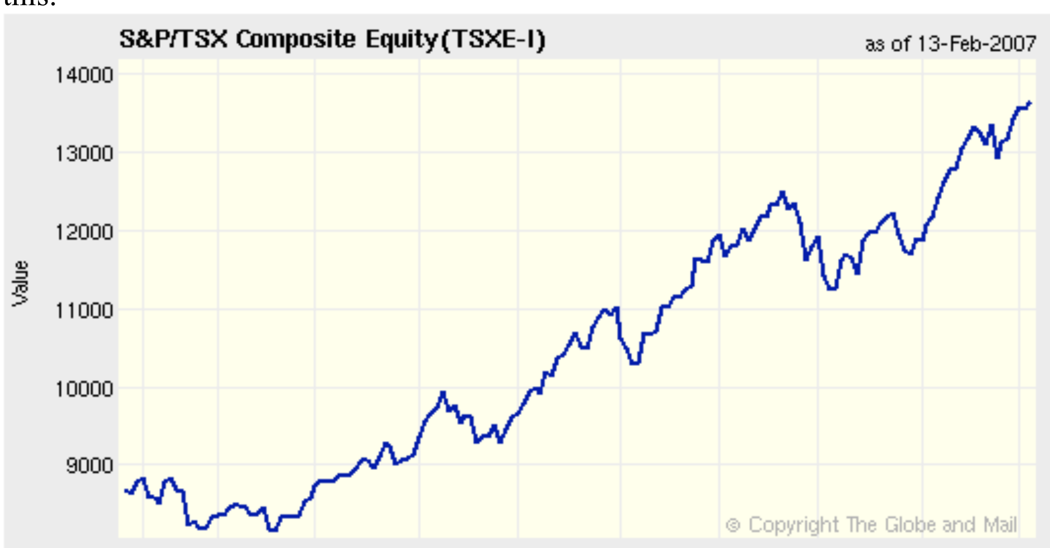
We now focus on the top graph – the cumulative impact of unit changes that can be ± 1 with equal probability (as might be indicated by a fair coin toss).

Suppose that we start with a stock portfolio worth $100, and we experience daily impacts of ± $1 to this market value over a one year period (about 250 business days).

A typical simulation outcome from this simulation would look like this :



Now lets compare this with what really happens in the Toronto Stock Exchange. The S&P/TSX Index is a composite of many stocks and over the last 12 months looks like this:



Apparently, the stock market behaves very like a symmetric random walk. So what? Well suppose you could use the trend in the market index to predict the next few days – if you could do that, you could make a lot of money: if it is going up, buy now sell later, and if it is going down, sell now and buy later!

BUT, if the market really is like a symmetric random walk, the future has no trend up or down – or to put it another way, no matter what has happened in the past, the future is as likely to be up as down.

The lesson in this? **Do not assume that forecasting the stock market is as easy as projecting recent trends.**

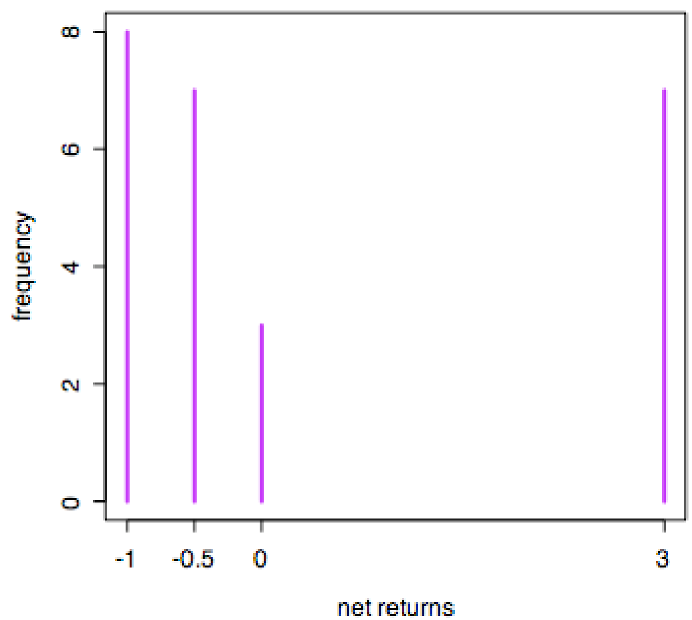
This is kind of a negative result. Isn't there something positive in these simulations? Consider the following:

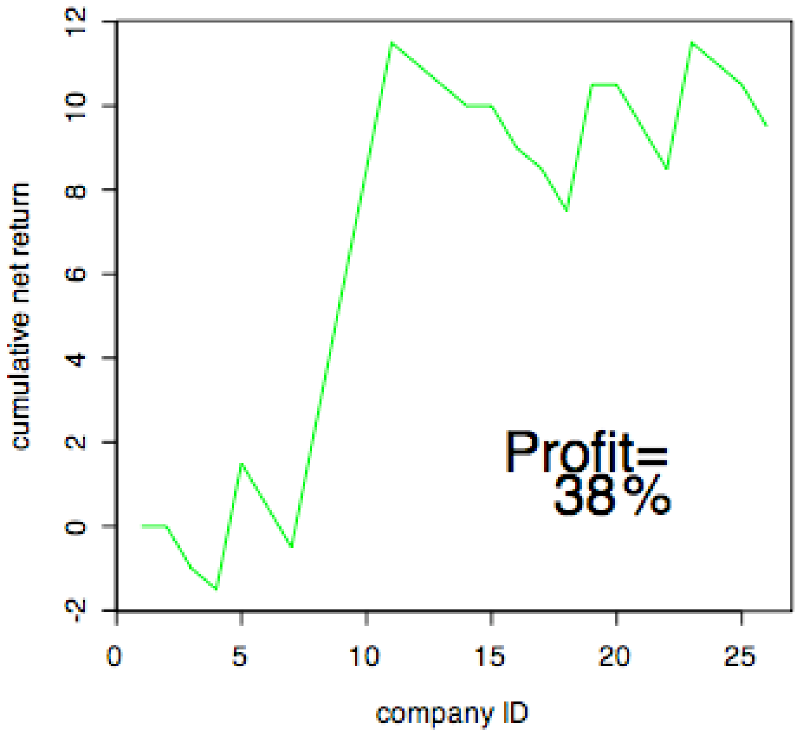
Many investors like to reduce what they call "risk" by investing in large, stable corporations – they are willing to forgo the small chance of a large gain by accepting a large chance of a small gain. For example, few investors would be interested in putting a lot of money into a company whose prospects over the next year were as follows:

For each $1 invested, the company is expected to pay back, at the end of the year, the following amounts, with the associated probabilities:

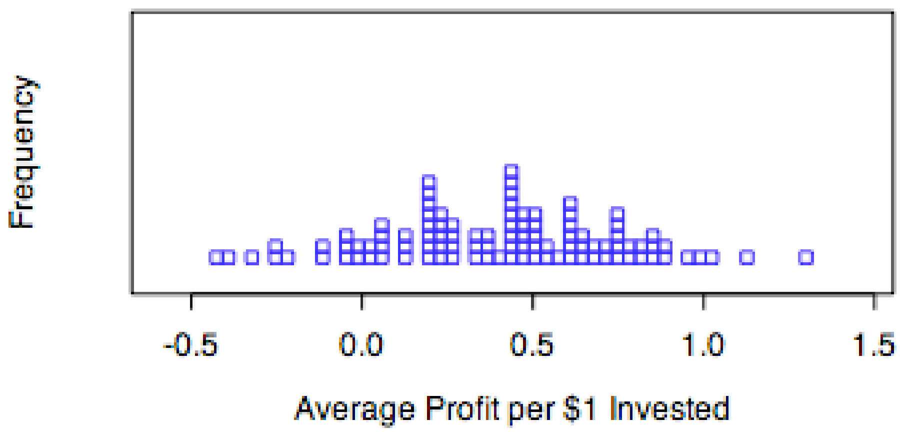
Payback($)	Probability	Net Profit($)
0.00	0.25	-1.00
0.50	0.25	-0.50
1.00	0.25	0.00
4.00	0.25	3.00

This company would be considered a risky investment, since 75% of the time you make no money at all and 50% of the time you lose money. However, suppose you invested in 25 such companies, and further suppose that the outcomes of one company are independent of the outcomes of any other company (for example, suppose they are in different industries in a variety of countries). Here is the simulation result of what could (typically) happen to the portfolio of 25 companies:


frequency
0
2
4
6
8
-1
-0.5
0
3
net returns


cumulative net return
-2
0
2
4
6
8
10
12
0
5
10
15
20
25
company ID
Profit=
38%

And if we redo this simulation 100 times, we will see that years when money is lost are fairly rare, and losses are small even in those cases.



In other words, for the $25 invested ($1 in each of 25 companies), the profit varies from a loss of about $40 to a gain of about $130 – in fact an average gain of about $38. The simulation shows a profit 87% of the time, even though each individual company had only a 25% chance of making money.

This is the phenomenon of diversification, and it is a phenomenon that is easily demonstrated mathematically. Lets compute the expected value and statndard deviation of the investors prospects when he/she invests in 25 companies as suggested above.

E(Profit) = (-1.00)*.25 + (-0.50)*.25 + (0.00)*.25 + (3.00)*.25 = .375 per company or $37.50 for the 25 companies.
$V(Profit)=(.375-(-1))^{\wedge 2}*.25 + (.375-(-0.50))^{\wedge 2} *0.25 + (.375-0)^{\wedge 2} * 0.25 + (.375-3.00)^{\wedge 2} * 0.25=2.42$ so
SD = 1.56

So the mean and SD for one company is $0.38 and $1.56 respectively. The mean for 25 companies is also $0.38. **BUT, the SD for the average profit from the 25 companies**

combined is obviously less than \$1.56. It turns out it is $1.56/25^{1/2}$ = \$0.31. Check that the distribution in the dotplot does have an SD of about 0.31. So with a mean of 0.375 and an SD of 0.31, the profit will usually be positive. Does the distribution look normal? (YES). Is it true that 68% of the profit is within 1 SD of the mean, and 95% within 2 SD?

We have demonstrated two useful theoretical outcomes:

1. Averages tend to have normal distributions (more carefully – see "Central Limit Theorem".
2. The SD of an average of n things is the SD of the individual things divided by the square root of n (Square Root Law)

Of course we have also suggested how to make money on the stock market – however, be forewarned, the independence of companies is hard to achieve. The stack market tends to affect all stocks the same way. To paraphrase the drug advertisers, "Speak to your investment professional about diversification".