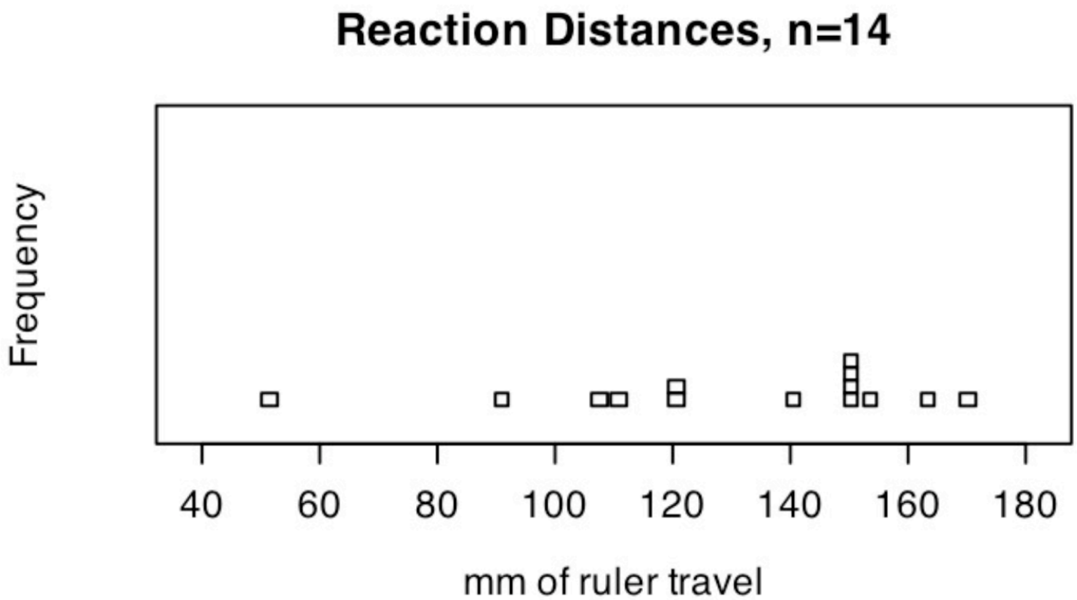
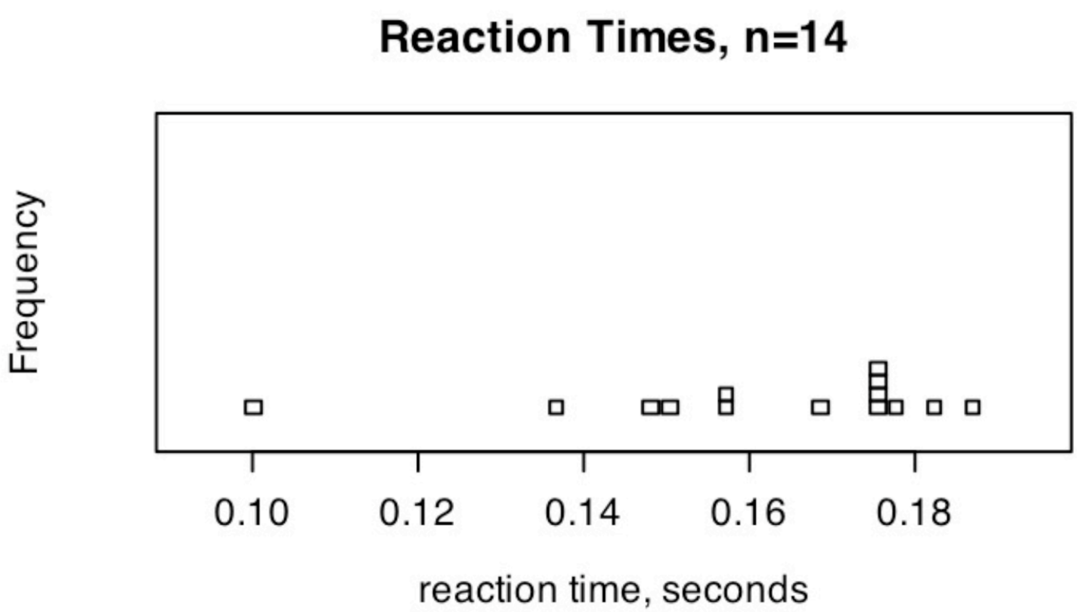
Analysis of the Reaction Time Data

Test of Claimed Average (of "population")

The first thing to do in data analysis is to plot the data. Here is the reaction time "distances" (i.e. before conversion to times).



This is not lognormal (which is right skewed) but maybe after conversion to times?



Still not logNormal! So before we proceed with our analysis, we need to give up the assumption that the population is logNormal. Looks more like Normal, no matter whether we use distance or time.

The hypothesis, using the raw time data, was that the mean would be 0.166 seconds and the SD would be 0.16 seconds. Now the actual mean and SD of the sample reaction times are, respectively, 0.162 and 0.016 seconds, and so it looks like the hypothesis was pretty close to the truth. But for demonstration purposes, lets consider testing this hypothesis with our data.

Now for a start we are really only interested in the population mean (actually, there is more that is interesting but the theory of Ch 7-9 is mostly concerned with estimating and testing population means, so we will stick with this focus here.) This means that we are not going to test the claim that pop SD = 0.016 – but the question is whether we are going to use it when we test pop mean = 0.166. Remember that we have a "**Case 1**" that says that if we know the population SD we should use it, especially if we can assume Normality of the population distribution (which seems reasonable here). So lets do a first analysis using SD=0.016 as a given population mean.

So our hypothesis about the population mean time is that it is 0.166 seconds. If this were not credible, we need to state what we would then believe, and in this case the belief would be pop mean time ≠ 0.166. (Sounds obvious, but in some situations it might have been pop mean >0.166, for example, and this would have slight implications for the following calculations.) The shorthand for this is:

H_0: $\mu = 0.166$
H_a: $\mu \neq 0.166$

We sometimes call H_0 the "null" hypothesis since it states that there is no difference from the theory, and H_a the "alternative" hypothesis.

Now the Case 1 calculation is to compute $z = \frac{(0.162 - 0.166)}{\left(0.016 / \sqrt{14}\right)}$ = -0.94 and what we need to do next is to decide if a z value of -0.94 (based on the observed sample mean of 0.162 and the assumed SD of 0.016) is a credible value of the N(0,1) distribution. We do this by computing a P-value. In this case the probability that a value this far (-0.94) from the mean of 0, or further away, would have resulted as a random sample mean from the N(0,1) population. We look up -0.94 in the normal table and find that P(Z<-0.94) = 0.1736. So it looks like our observed mean reaction time of 0.162 is not too surprising to come from a Normal population with mean 0.166. The probability of something this unusual in the lower direction is .1736 which is something that would happen in about 1/6 of the samples from the hypothesized distribution. This 0.1736 is essentially the P-value on which we decide to accept and reject the null hypothesis. In this case we accept it.

One small complication – we need to double the .1736 to get the real P-value, since deviations in either direction must be taken into account to judge credibility. So the P-value is really .3472. But this does not change anything in this case – it is not small either. (if the P-value hade been .01 for example, we would have doubted the null hypothesis, even "rejected" it.)

So we conclude that the hypothesis that the mean reaction time for the population of which our 14 students are a random sample, is 0.166, is credible – we "accept" it.

Now lets try a **Case 2** analysis, but eliminate all the stuff that is the same as Case 1.

The sample mean and SD were 0.162 and 0.023. Now suppose we do not trust the theory that the pop SD is 0.016 – now we would have to use 0.023. We do the same calculation but now we have to look up that statistic in the t-table – in fact the t-table with 14-1 or 13 degrees of freedom.

$$t = \frac{(0.162 - 0.166)}{\left(0.023 \big/ \sqrt{14}\right)} = -.65$$

and now looking at Table A.5, p 743, we see that $P(t<-.65)$ is tough to read!
Note the symmetry though, so $P(t<-.65)=P(t>+.65)$. Now we can get a bit of information from the row labeled $\nu = 13$, namely that $P(t>+.65)$ must be a lot bigger than .10. So whatever the P-value is in this case, we know it is not small, and so we accept the null hypothesis as credible. (Yes, if we did find $P(t<-.65)$, the P-value in this case would be twice this.)

Now lets see what would have happened if we decided that n=14 were large enough the use the CLT approximation. (I called this **Case 3**) What we would get is

$$z = \frac{(0.162 - 0.166)}{\left(0.023 \big/ \sqrt{14}\right)} = -.65$$ and the P-value would be $2*P(z<-.65) = 2*.2578 = .5032$ so we would again accept the null hypothesis. But of course this calculation would not be justified in this case since n=14 is NOT a large sample. However, the conclusion turned out to be correct – we were lucky!

Ch 9 Test?

There is one more thing to do with this data – compare the two tables reaction times.

```
reacs grp
 [1,]  140   1
 [2,]  170   1
 [3,]  110   1
 [4,]  150   1
 [5,]  155   1
 [6,]   50   1
 [7,]  120   1
 [9,]  163   2
[10,]  150   2
[11,]   90   2
[12,]  150   2
[13,]  151   2
[14,]  109   2
```

[8,]   120   1

The mean and SD for table 1 was 0.159, 0.026 and
The mean and SD for table 2 was 0.165, 0.019

Is there a difference in the population means (the sample means differ by 0.165-0.159 = 0.006)? To judge whether this is attributable to sampling error, or only explainable by the fact the populations underlying the two tables have different means, we need to know how variable this difference of sample means is. More precisely, we need the SD of the sampling distribution of $\left(\overline{X}_1 - \overline{X}_2\right)$. This leads us to the procedures of section 9.2, which we will delay for a while.