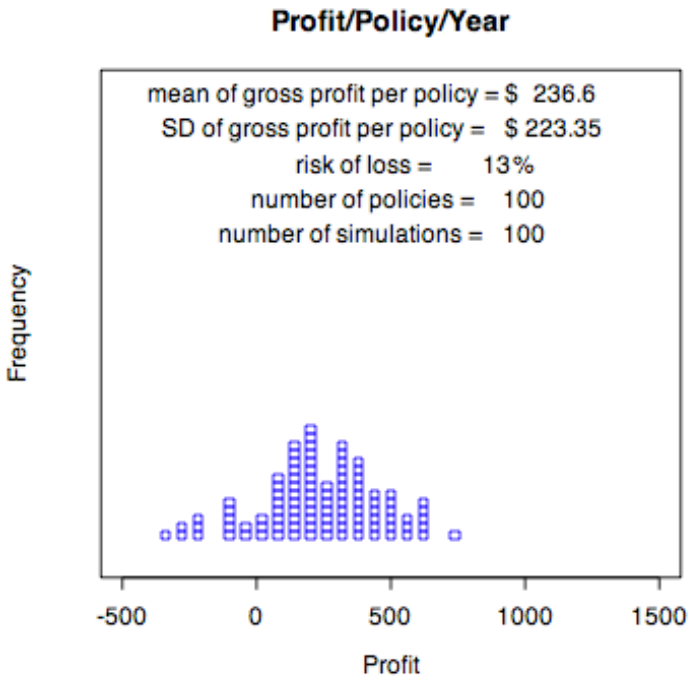
Auto Insurance

If you drive a car in BC, you, or someone who loves you, pays a lot of money each year for casualty insurance. If you do have an accident, the property damage paid by your insurance company (if it is your fault) is often several thousand dollars, and if you injure someone else, the insurance liability will usually be much more. Does the insurance company make any money? Isn't the number and size of accident claims soemthing out of control of the insurance company?

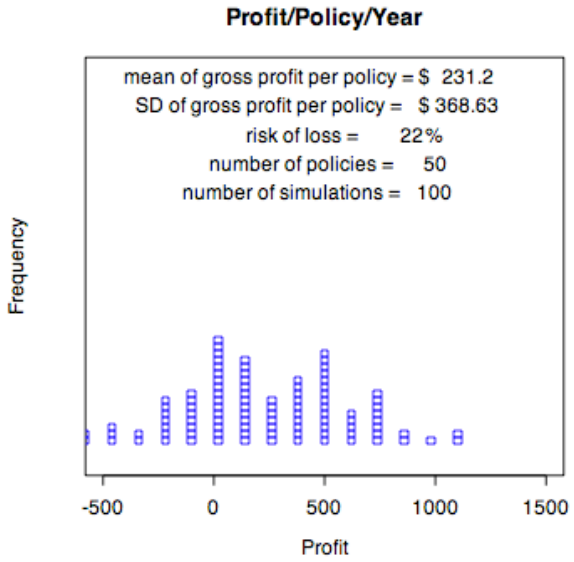
We can investigate this situation with a simulation program. In our case we need to make some economic assumptions about the average accident payout and frequency, and the variety of driving care among the insured population, but in a real life application there is data available on these things. So we will simplify the situation considerably to illustrate how the cash flow in such a company can work to create a profitable business.

Suppose the insurance policy costs $1460 per year ($4 per day). And suppose the average accident claim is $6000. Also, suppose that the chance of a policy holder having one accident in a year is 0.2, and lets ignore the chance of more than one accident. Now lets suppose this upstart insurance company only has 100 such policies. Would it have a profit at the end of the year?

Using simulation we can run this companies experience several times, 100 say, and here is a typical output of the net profit per policy:

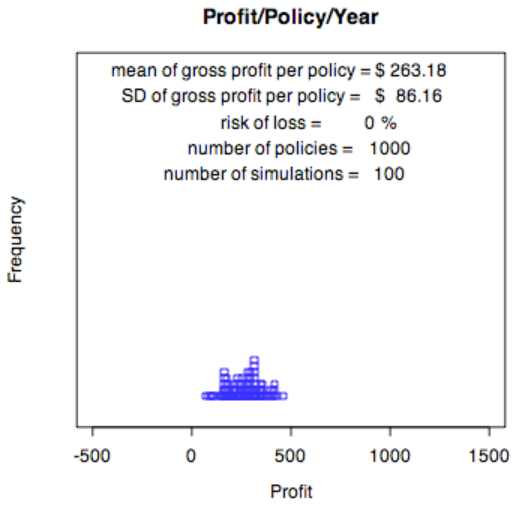


The company is usually profitable, but sometimes loses money.
Now suppose the main salesman quits and the next year the company only has 50 policies: Here is what would likely happen (100 simulations of the one-year):



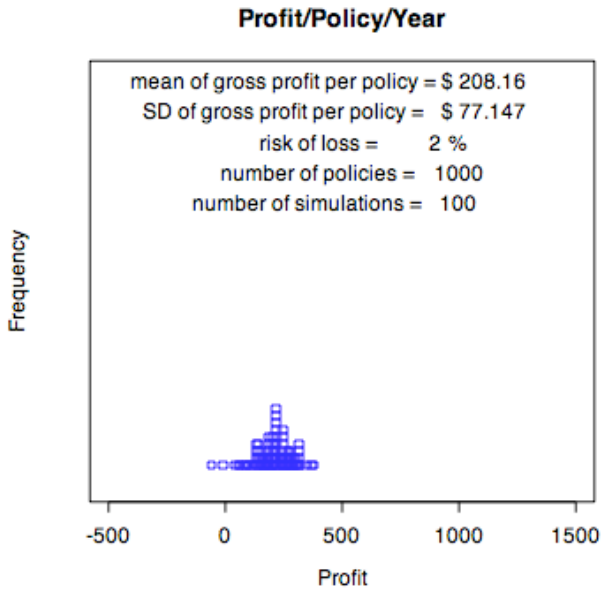
This company could go bankrupt fairly quickly with only this many policies, even though on average it is still making quite a good profit per policy.

Can you see that this is a phenomenon of "the sampling distribution of the sample mean" and that the more policies the company has, the narrower profit margin it can risk – that is a big company can lower premiums more safely than a small company. For example, look at the company when it has 1000 policies.

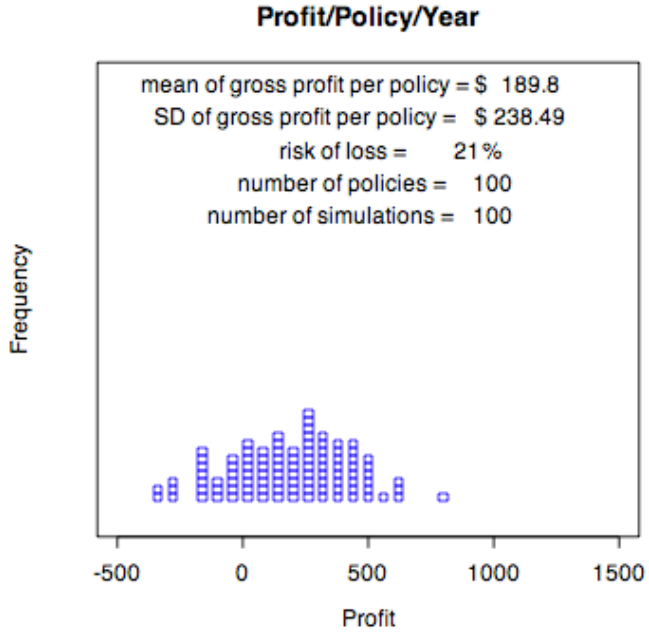


The company with 1000 policies is almost always profitable. Policy holders are gamblers, but the insce company is not!

Note that with 1000 policies, the insurance company can lower premiums and still make a profit, driving out the smaller companies. For example if the premium is $1400 per year instead of $1460, the big company would experience something like



whereas the company with only 100 policies would experience



The average profit is similar (equal if enough simulations were done), but the chance of a loss is much greater for the smaller company.

This is caused by the fact that a sample mean has an SD that is inversely proportional to the square root of the sample size.

R-program for this demo:

```
> insce
function (reps=100,policies=100,prem=1460,cost=6000,prob=.2,sd=T,...)
{
        a=1:policies
        b=1:reps
        for (j in 1:reps) {
                for (i in 1:policies) {if (runif(1)>prob) {a[i]=prem} else {a[i]=prem-cost}}
                b[j]=sum(a)/policies
                }
                risk=100*(length(b[b<0]))/reps
                print(risk)
                avgprofit=format(mean(b),digits=5)
                sdprofit=format(sd(b),digits=5)

        stripchart(b,method="stack",main="Profit/Policy/Year",xlab="Profit",at=.1,ylab=
"Frequency",xlim=c(-500,1500),col="blue",...)
                text(400,1.55,"number of policies =")
                text(1000,1.55,policies)
                text(350,1.4,"number of simulations =")
                text(1000,1.4,reps)
                text(250,2.0,"mean of gross profit per policy =")
                text(950,2.0,"$")
                text(1150,2.0,avgprofit)
                text(400,1.7,"risk of loss =")
                text(900,1.7,round(risk,))
                text(1000,1.7,"%")
                if (sd==T) {text(250,1.85,"SD of gross profit per policy =")
                        text(950,1.85,"$")
                        text(1150,1.85,sdprofit)
                        }
                invisible(b)
}
```